ŚIṢYADHĪVṚDDHIDA TANTRA OF LALLA

PART II

TRANSLATION AND MATHEMATICAL NOTES

लल्लाचार्यविरचितं

शिष्यधीवृद्धिदतन्त्रम्

मल्लिकार्जुनसूरिविरचित-व्याख्योपेतम्

ŚIṢYADHĪVṚDDHIDA TANTRA OF LALLA

With the Commentary of Mallikārjuna Sūri

Critical edition with Introduction,
English Translation, Mathematical Notes and Indices

By

BINA CHATTERJEE

PART II

TRANSLATION AND MATHEMATICAL NOTES

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FOREWORD

The publication of original texts in Sanskrit, Arabic, Persian etc. and their translation into English is one of the main objectives of the National Commission for the Compilation of History of Sciences in India. Texts like *Āryabhaṭīya* with commentaries (3 vols.) and *Rasārṇava-kalpa* are some of the prestigious publications of the Commission in this direction. The present text, the *Śiṣyadhīvṛddhida Tantra* ('A Treatise for increasing the Intellect of Students') by Lalla, a very important work in astronomy in the pre-Bhāskara (II) period, belongs to this category of texts.

The work of editing and translating into English of *Śiṣyadhīvṛddhida* with critical annotations was taken up by the late Dr. (Mrs) Bina Chatterjee whose scholarship in this field is already well known. While the work of printing was in progress, and 32 pages had been printed, Dr. Chatterjee passed away in January, 1978. After her demise we took the help of two other scholars in the field, namely, Prof. K. S. Shukla and Prof. K. V. Sarma towards the publication of the work. They have taken utmost care in their work and have given dedicated service towards the press-processing, printing, proof reading, preparing indices etc. of the work. The Academy is indeed very grateful to these two scholars for their valuable co-operation in the work. Without their help, the work could not have been published so early. We hope that this publication also will receive the approbation of scholars like the other publications of the Commission.

F. C. AULUCK
Vice-chairman

National Commission
For the Compilation of
History of Sciences in India

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PREFACE

The first time I had a glimpse of Lalla's *Śiṣyadhī-vṛddhida* was in 1937 when I began studying Indian astronomy and mathematics with Pandit Muralidhar Thakur. Since then, while at Oxford preparing a critical edition of the *Khaṇḍakhādyaka* by Brahmagupta, I cherished the intention of editing the *Śiṣyadhīvṛddhida* after completing the work in hand. Under the auspices of the International Federation of University Women, whose Fellowship I held from 1941-43, my main subject for study was Indian and Greek trigonometry and planetary motions. However, I managed to glance through the *Śiṣyadhīvṛddhida* as well, though perfunctorily.

The *Khaṇḍakhādyaka* was published by me in October 1970 and, soon after, my study of the *Śiṣyadhīvṛddhida* commenced. This plan was facilitated by the timely financial assistance extended to me by the Indian National Science Academy with effect from November 1971, for which I am extremely grateful to the Academy.

The *Śiṣyadhīvṛddhida* had already been published by Sudhakara Dvivedi in A.D. 1886, but that edition was based only on one manuscript and could not, therefore, be called a critical edition. The present edition has been prepared after collating twenty manuscripts, of which one is from the Bodleian Library, Oxford, and another from the Statsbibliothek, Berlin (W. Germany). The remaining manuscripts are from different libraries in India. The present edition gives also the commentary by Mallikārjuna Sūri, available only for the First Part. The translation provided is intended to give an intelligible rendering of the text and the Notes to interpret, in terms of modern

science, the mathematical calculations involved in the textual verses.

I take this opportunity to express my gratitude to Professor David Pingree, Brown University, U.S.A., who after learning that I desired to bring out an edition of the *Śiṣyadhīvṛddhida*, very kindly sent me a list of the manuscripts of the work. I thank all the libraries which extended their cooperation not only by sending me copies of the manuscripts required but also by making it possible for me to visit those institutions to collect materials for my study. I am also grateful to Dr. K. S. Shukla, Professor of Mathematics, Lucknow University, who was always ready to help me to solve my difficulties with the text. I must also mention with gratitude the assistance offered by Dr. M. L. Sharma, Head of the Department of Astronomy, Sampurnanand Sanskrit Vishvavidyalaya, Varanasi, in the matter of checking some of the examples in the commentary. My thanks are due also to Dr. K.V. Sarma, Director-Professor of the Vishveshvaranand Institute, Hoshiarpur, for the wholesome suggestions and advice he gave me towards the presentation of the volume.

In all humility I place this work before the learned world and fervently hope that in spite of its short-comings it will still be of some use to the study of the history of Indian astronomy.

Bina Chatterjee

New Delhi,

31 May, 1975

INTRODUCTION

I. LALLA: HIS LIFE AND DATE

Lalla does not mention his date either in his astronomical treatise, *Śiṣyadhīvṛddhida* (henceforth *ŚD*), or in his astrological work, *Ratnakośa*, the latter available only in manuscript form. His date has, therefore, to be determined on the basis of internal and external evidences, that may be forthcoming.

Āmarāja's commentary on the *Khaṇḍakhādyaka* of Brahmagupta mentions Lalla as a commentator of that work. In the very beginning Āmarāja says: *Lalla-Utpala-Someśvara-viracita-bhāṣyāṇi tattvato buddhvā*, 'having studied the essence of the commentaries by Lalla, Utpala and Someśvara' (p. 1). Later on, he quotes from the *Khaṇḍakhādya-paddhati* (most probably the name of the commentary) of Lalla (p. 27). Once again, Āmarāja refers to the commentaries of Lalla and Utpala, when he says, *Lallotpalabhāṣyayoḥ*, 'thus in the commentaries of Lalla and Utpala' (p. 57). Āmarāja also quotes profusely from the *ŚD*. In fact, he uses the astronomical constants given by Lalla while proving the formulas in the *Khaṇḍakhādyaka*. Every quotation is introduced with phrases like, *tathā ca Śiṣyādhīvṛddhidatantre*, 'thus in the *ŚD*'. It is true that Āmarāja nowhere says that the *ŚD* is by Lalla. It will not be wrong to surmise that, during his time, *ŚD* was so well known that the mention of the name of its author would have been redundant. So, it is safe to conclude that the author Lalla and the commentator Lalla are the same. This would mean that Lalla succeeded Brahmagupta; that is, he came after the seventh century A.D.

There is another and very strong evidence in favour of placing Lalla after Brahmagupta. There is much similarity between Lalla's *ŚD* and Brahmagupta's *Brāhmasphuṭasiddhānta*. Many of Brahmagupta's special methods have been borrowed by Lalla. It is unbelievable that so learned though arrogant an astronomer like Brahmagupta,

who scorns even Āryabhaṭa, would borrow from Lalla, a scholar of lesser calibre. The cases of similarity have been specified in the mathematical notes.

P. C. Sen Gupta in the Introduction to his translation of the *Khaṇḍakhādyaka* has compared the polar longitudes of various stars given by Brahmagupta in the *Brāhmasphuṭasiddhānta* and by Lalla in the *ŚD* and has come to the conclusion that Lalla should be placed after Brahmagupta (pp. xxiv-xxvi).

So far, the main evidence in support of Lalla's date has been the two verses '*śake*...*vṛddhim*' (text, p. 15), 'subtract 420 from the *Śaka* year elapsed. Multiply the remainder severally by 25, 114, 96, 47 and 153. Divide each product by 250. The quotients in minutes should be subtracted respectively from the mean longitudes of Moon, its apogee and node, Jupiter and the *śīghrocca* of Venus. Again multiply the above remainder severally by 48, 20 and 420. Divide each product by 250. Add the quotients in minutes respectively to the mean longitudes of Mars, Saturn and the *śīghrocca* of Mercury.' What could 250 signify ? It shows that Lalla corrected the positions of the planets calculated with the constants of Āryabhaṭa, by his own observations. According to the formula, 250 years after 420 Śaka, which is Āryabhaṭa's time, or in 670 *Śaka* (A.D. 748), corrections were necessary. Thus, A.D. 748 may well denote Lalla's time. The same argument has been put forward by Sen Gupta in his Introduction (p. xxvii), to the translation of the *Khaṇḍakhādyaka*, referred to above.

Of the twenty MSS collated for the present edition,[1] M_1 and M_2 giving the text and Mallikārjuna Sūri's commentary and A_1 and S_1 giving the text (upto the ninth chapter) and Bhāskara II's commentary, do not contain these two verses. V_1, also giving Bhāskara's commentary, has these two verses along with five others but with the introduction, *ślokasaptakaṃ spaṣṭārthe vilikhyate*, 'these seven verses are written as explanatary ones' (text, p. 25). And there is no

1. For a description of there manuscripts, see Introduction to the Text volume.

commentary by Bhāskara on these even in this MS. V_1 when compared with the above four MSS, has a few more extra verses interspersed in the first chapter but without any commentary of Bhāskara. The remaining complete MSS excepting M_8 also give these two verses, some, as the critical apparatus (*vide* Introduction to Pt. II) would show, in the first and thirteenth chapters and some only in the first chapter. Are then the two verses under reference to be considered as interpolations ? In any case, these cannot now constitute an *unchallenged evidence* for Lalla's date.

It will, however, not be out of place to say a few words about the authorship of these verses. Sūryadeva Yajvā (12th cent.), Parameśvara (14th cent.) and Yallaya (15th cent.), while commenting on the tenth verse in the *Kālakriyāpāda* of *Āryabhaṭīya*, where Āryabhaṭa gives the date of this work, quote the above two verses and state that they were composed by Lalla. Nīlakaṇṭha (15th-16th cent.) in his commentary on the forty-eighth verse in the *Golapāda* expresses the same view. Sūryadeva and Yallaya even mention that Lalla wrote these verses in his *Śiṣyadhīvṛddhidamahātantra*.[1] Could then these be interpolations ? Yet the manuscripts giving Mallikārjuna's and Bhāskara's commentaries do not have them. Anyhow, even without these verses it has been shown that Lalla came after Brahmagupta.

Sūryadeva Yajvā, Parameśvara and Yallaya in their respective commentaries on *Āryabhaṭīya, Kālakriyāpāda*, 10, call Lalla a '*śiṣya*' of Āryabhaṭa. *Śiṣya* need not necessarily mean a direct pupil. It could well denote a 'follower'. And Lalla is a follower of Āryabhaṭa. to a certain extent as he himself acknowledges in his first two verses (text, p. 1). He says that he would explain the theory of Āryabhaṭa. If he were Āryabhaṭa's direct pupil he would not use the expression *tadīyaśiṣyaiḥ*, 'his disciples'. And, he would also not contradict his Guru's theory of rotation of earth (text, xx. 43, p. 238). Moreover, could he, a direct pupil of Āryabhaṭa have escaped the criticism of Brahmagupta in *Tantraparīkṣādhyāya* of the *Brāhmasphuṭasiddhānta* ?

1. See pp. 93-94, *ĀBh.* with Sūryadeva Yajvan's Com, ed. K. V. Sarma, New Delhi, 1976.

Again, in *Ratnakośa*, there is a section called *Varāhamihirakṛtaṃ strīlakṣaṇaikadeśaḥ*, 'Some of the auspicious marks on a woman according to Varāhamihira'. So, obviously, Lalla had read Varāha's works on astrology.

Thus Dvivedi's contention that Lalla's date is 421 Śaka or A.D. 499 (*GT*, p. 10), and Dikshit's observation that his date is 560 Śaka or A.D. 638 (*BJ*, p. 313) cannot be maintained in view of the evidence given above. It is baffling as to why both these eminent scholars did not consider the similarities in the *Brāhmasphuṭasiddhānta* and *ŚD*. Not only this : Dikshit goes further and says that since the *Turīya* instrument of Brahmagupta is not mentioned in the *ŚD*, Lalla never saw Brahmagupta's work (*BJ*, p. 315).

Lalla's upper limit should, therefore, be the beginning of the eighth century.

As regards the lower limit Lalla, so far as we know at present, is first mentioned by name in Śrīpati's *Siddhāntaśekhara* (xviii. 18) and *Ratnamālā*. In the beginning of the *Ratnamāla* Śrīpati acknowledges his indebtedness to Lalla and so also in the end of the *Ratnakośa*. Moreover, in the *Siddhāntaśekhera* there are many formulas which not only occur in the *ŚD* but appear to be more or less reproductions of Lalla's words. Thus, Lalla's works were well known by the early eleventh century A.D., which is Śrīpati's time. The limits of Lalla's dates are, therefore, to be fixed between the eighth century and early eleventh century A.D. If Sen Gupta's calculations are taken into consideration and also the two verses referred to above, Lalla may be placed in the beginning of the eighth century.

At the end of *Gaṇitādhyāya* Lalla gives the name of his grandfather as Tāladhvaja and that of his father, Sāmba, better known as Trivikrama. In the beginning and end of the *Ratnakośa* also Lalla gives his father's name as Trivikrama. There he calls himself *Trivikramasuta*, 'son of Trivikrama'. Lalla belonged to a Brahmin family. He offers homage to Brahmā, Viṣṇu and Śiva before he begins the *ŚD* and ends the work with obeisance to Sarasvatī, Śiva and Pārvatī.

What part of the country did Lalla belong to ? The author is silent about it. In ix. 19 of the *ŚD*, Lalla compares the half Moon with the forehead of a woman of Lāṭadeśa, which is the present south Gujarat. His words are *Lāṭīlalāṭataṭarūpadharaḥ śaśāṅkaḥ*. Again, in the *Ratnakośa*, in the section on seasons called *Ṛtucarcā* he says, *lalāṭe lāṭīnāṃ luṭhitam alakaṃ tāṇḍavayati*, 'wisps of hair play havoc with the foreheads of the women of Lāṭa'. While commenting on the first simile Mallikārjuna observes that Lalla shows partiality to his country women or *deśapakṣapāta* (text, p. 144). And, Mallikārjuna must have made this remark with some knowledge. Lāṭa was well known in the Deccan region and is often mentioned in the history of these parts.[1] On the basis of this fact Lalla appears to have belonged to the Lāṭadeśa.

II. WORKS OF LALLA

1. Śiṣyadhīvṛddhida-Tantra

Śiṣyadhīvṛddhidatantra is the only work on astronomy by Lalla, which is at present available. The title means a 'treatise, that increases the knowledge of students of astronomy'. In the very beginning of this work Lalla writes :

Ācāryāryabhaṭoditaṃ suviṣamaṃ vyomaukasāṃ karma yac-
chiṣyāṇāmabhidhīyate tadadhunā Lallena dhīvṛddhidam / (text, p. 1)

'Lalla now describes the motion, etc. of the dwellers of the sky, a subject, which is very difficult indeed. This subject has already been dealt with by Ācārya Āryabhaṭa. Lalla hopes that this work of his will increase the astronomical knowledge of students'.

Lalla further states that many pupils of Āryabhaṭa studied his works and wrote several *Tantras* or astronomical treatises, but their works are not comprehensive. So he, Lalla, undertakes the task of writing a treatise fully and systematically. Again, at the end of the *ŚD*, *Gaṇitādhyāya*, he says that he has now completed the First Part of his *Tantra* which would give the same results as those obtained from the work of Āryabhaṭa.

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1. *A history of South India* by K. A. Nilakanta Sastri, Oxford, 1966.

The *ŚD* itself shows that Lalla uses the astronomical parameters of the *Āryabhaṭīya* and, perhaps, borrows a few of the formulas but his work is definitely more comprehensive and explanatory. And, if the two verses *śāke nakhābdhi* etc. (Text, p. 15), referred to above are his, he also gave corrections to Āryabhaṭa's results.

The work is known as *Śiṣyadhīvṛddhidatantra, Dhīvṛddhidatantra* or even *Śiṣyadhīvṛddhidamahātantra.* It is divided into two sections, *Gaṇitādhyāya*, 'Mathematics' and *Golādhyāya*, 'Spherics'.

The first section has thirteen chapters, (i-xiii), on Mean planets, True Sun and Moon, True planets, Three problems, Lunar eclipse, Solar eclipse, Possibility of an eclipse, Rising and setting of planets, Cusps of the Moon, Conjunction of planets, Conjunction of stars and planets, Vyatīpāta and Vaidhṛta and Rationale of corrections. The thirteenth chapter is also called *Uttarādhikāra* or the last chapter. Mallikārjuna calls it *Bījavāsanā* or 'Rationale of corrections'. The second section has nine chapters (xiv-xxii), on Graphical representation of the Motion of planets, Construction of the armillary sphere, Rationale of rules re. mean motion, Sphere of earth, Motion of the Celestial sphere, Description of the earth, False notions, Astronomical instruments and Astronomical problems.

Lalla uses Āryabhaṭa's constants with one significant difference. What Āryabhaṭa calls the number of rotations of the earth, Lalla terms as the number of rotations of the celestial sphere or the sphere of constellations round the earth.

Lalla borrows the table of R sines from Āryabhaṭa, using 3438 as R and dividing the quadrant into arcs of 225′ each. But in the last chapter he gives another table, called *Laghujyā.* There he uses 150 as R and divides the quadrant into arcs of 10°. Some of the methods of astronomical calculations are the same as those in the *Āryabhaṭīya*; *e.g.*, the determination of the true places of the Sun, Moon and planets, use of spherical trigonometry to obtain declination, earth-sine, etc., calculation of obscured portions in a solar or lunar eclipse, etc. etc. These have been pointed out in the mathematical notes. Lalla uses R versed sine for determining *āyanavalana, āyanadṛkkarmāsu* (visibility correction) and the illuminated portion of

the Moon. Āryabhaṭa uses R versed sine for calculating the *āyanadṛkkarmāsu* but perhaps not for *āyanavalana*.[1] He does not give any formula for the illuminated portion of the Moon. Bhāskara I uses R versed sine in all the three cases, in both the *Mahābhāskarīya* and the *Laghubhāskarīya*. As is shown in the notes, R versed sine for the cusp of the Moon gives a more correct result than R sine ; but in the other two cases the results are not correct. As regards the calculation of parallax in a solar eclipse, Lalla like Āryabhaṭa uses the meridian ecliptic point.

A historically important point of observation regarding the *ŚD* is that so many of the special rules here are found in the *Brāhmasphuṭa-siddhānta*. Calculations of *śuddhi, laghvahargaṇa* and hence the mean longitudes of the Sun, Moon, etc., application of parallax in a solar eclipse, correct time of conjunction of planets and that of *pāta*, are only a few such cases. These have been pointed out in the mathematical notes. Lalla's method to find the correct circumference of an epicycle and then the *mandaphala* and *śīghraphala* also occur in the *Brāhmasphuṭasiddhānta*.

Lalla gives special methods to calculate the correct true places of Mercury and Venus. He, however, does not say whose methods these are. Lalla, in all probability, borrowed Brahmagupta's special rules and incorporated them in his work. It is a good synthesis of Āryabhaṭa's and Brahmagupta's works.

Was Lalla acquainted with the works of Bhāskara I ? As the mathematical notes would show, some of the formulas in the *ŚD* are found also in the *Mahābhāskarīya* and the *Laghubhāskarīya* ; but, then, these give the common astronomical methods. There is nothing special about them. In the beginning of the work Lalla does mention *tantrāṇi*, 'astronomical treatises' composed by Āryabhaṭa's pupils. These may or may not include Bhāskara's works.

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1. There are different readings ; some give R sine and some R versed sine.

Lalla, in his turn, was followed by Śrīpati and Bhāskara II. Śrīpati names Lalla as *Trivikramasuta*, 'son of Trivikrama' in *Siddhāntaśekhara* (ii. 58), and, as much as acknowledges that he followed Āryabhaṭa, Brahmagupta, Lalla and others in determining the orbits of the planets. And, again, at the end of the section on the sphere of universe (xviii. 18) he says that when Āryabhaṭa, Brahmagupta, Lalla, Vaṭeśvara and others did not always give rational explanations for all the phenomena of the universe, how can a much less able astronomer like himself be successful. Moreover, when one reads *Siddhāntaśekhara* of Śrīpati one cannot but be struck with its close affinity with the *ŚD.*

Bhāskara II wrote a commentary on the *ŚD*, now available in manuscript form. He also has borrowed profusely from Lalla. He, however, does not hesitate to criticise Lalla in his *Siddhāntaśiromaṇi* (henceforth *SŚi*), wherever the latter went wrong. Some of the criticisms are very correct ; *e.g.*, Lalla's formula for the true motion of a planet ; his use of R versed sine for calculating *valana* and visibility correction ; equating a civil day to a sidereal day plus Sun's motion in *asus* ; and so on. These also have been pointed out in the mathematical notes.

Golādhyāya, being the second section of the *ŚD*, is of great historical significance. The chapters on the 'Sphere of earth' and 'Description of the earth' give an idea of the geography of that time. The chapter on 'False notions' acquaints the readers with the current astronomical beliefs. The chapter on 'Astronomical instruments' gives a glimpse of the practical methods used for observations. Lalla is the first astronomer whose work on astronomy contains a section on the 'Celestial sphere' also. This is not given either in the *Āryabhaṭīya* or in the *Mahābhāskarīya* or *Laghubhāskarīya*. Brahmagupta also writes a chapter on the 'Celestial sphere', but this is not so well defined as that of Lalla. So far this should be taken as Lalla's originality and he should be given credit for it.

2. Siddhāntatilaka

Another work of Lalla on astronomy is mentioned by Yallaya in his commentary on the *Sūryasiddhānta* (in MS). While explaining

i. 45-47, relating to *yuga* he quotes Lalla as saying in his *Siddhāntatilaka* that 2,65,68,00,00,00,C00 years elapsed since Brahmā's birth. He also says (commentary on *SS*, i. 53 mean planets) that Lalla gave only a few methods to calculate the mean positions of planets in his *ŚD*, but in the *Siddhāntatilaka* he gave a greater variety of methods. Again, Rāmakṛṣṇa Ārādhya in his commentary on the *Sūryasiddhānta* (in MS), says, towards the end of the first chapter, that Lalla gave a large number of methods to calculate the mean longitudes of planets in his *Siddhānta-tilaka* and he quotes the relevant passage.

It thus appears that Lalla must have written a *Siddhānta* also, which was, perhaps, again a synthesis of Āryabhaṭa's *Āryasiddhānta* (now lost, but references available) and Brahmagupta's *Brāhmasphuṭa-siddhānta*.

3. Ratnakośa

Lalla wrote on astrology as well. His work *Ratnakośa* is available in manuscript. It is a fair-sized work with astrological topics systematically arranged. To start with, the effects of *tithis, karaṇas*, week-days, *muhūrtas*, lunar mansions and stars are given ; then the different stages of child-birth are described ; details of house-building, temple-building, setting up of idols, sowing plants, etc. are stated ; the good and bad features of a man and woman are described—these are according to Nārada, Māṇḍavya and Varāhamihira ; certain other topics such as times for journey, types of elephants and horses to be bought, the signs of rain, etc. are also set out.

The historically important points to note are that the words *Kuhū* and *Sinīvālī*, being names of the New Moon day and the day before that, respectively, used in the Vedic literature, also occur here. *Madhu*, *Mādhava* and *Nabhasya* are used as names of months, the same in the Vedic literature. Atri, Garga, Parāśara and Vasiṣṭha have been named as 'Munis' or 'sages'. Yavana, meaning a foreign astrologer, has also been mentioned. Here, again, as in the *ŚD*, Lalla speaks of the beauty of the foreheads of *Lāṭī* women. Occasionally, one has a glimpse of some poetically beautiful verses in this work.

The authors of the *ŚD* and *Ratnakośa* are the same, because in both the works Lalla calls himself the son of Trivikrama. Moreover, this work is profusely quoted under the name Lalla.

4. Astrological works

It would appear that Lalla wrote more than one treatise on astrology. There are many quotations from Lalla introduced with *athāha Lallaḥ, tathā ca Lallaḥ* ; etc., or 'so says Lalla', in the *Ratnasaṃgraha* by Govinda, and in his commentary *Pīyūṣadhārā* on the *Muhūrtacintāmaṇi*. But all of these quotations are not available in the *Ratnakośa*. *Vidyāmādhavīyam* of Vidyāmādhava (14th century) also quotes from Lalla's astrological works but always refers to him as Ralla. As this work quotes from the *ŚD* also under the name of Ralla, there is no doubt that Ralla and Lalla are the same. Lalla is also quoted in *Mīnarājajātaka*. Nṛhari, the author of *Jātakasāra*, states that Lalla wrote on the *Jātaka* branch of astrology (*BJ*, p. 637).

Śrīpati acknowledges that he based his *Muhūrtagrantha*, *Ratnamālā* on *Ratnakośa* and also on works of Garga, Varāha, and Lalla. His words are (i. 2) :

vilokya Gargādimunipraṇitaṃ Varāhalallādikṛtaṃ ca śāstram
Viracyate jyotiṣaratnamālā

In the last chapter he writes :

udaharat Ratnakośāt, or 'illustrated from *Ratnakośa*'.

As quoted by Dvivedi, Lalla was an expert in all the three branches of *Jyotiṣa, viz., Gaṇita, Saṃhitā* and *Jātaka*. He is popularly known as *Triskandha-vidyākuśalaikamallaḥ*, or '*versed in all the three branches*' (*GT*, p. 10).

5. Works on Mathematics

Lalla must have written on mathematics also. No such work is at present available. Bhāskara II in the *SŚi*, *Goladhyāya*, *Bhuvanakośa*, while commenting on verse 57, quotes Lalla as saying in his *Gaṇita* (*svagaṇite*) that the area of the surface of a sphere is $2\pi r. \pi r^2$ and then contradicts him. He is also quoted by Nārāyaṇa in his *Gaṇitakaumudī*, *Kṣetravyavahāra*. In his commentary on 138-ab he

says that Lalla's formula for the length of the hypotenuse of a quadrilateral is wrong.

Lalla must have been a very popular author. In addition to the references made above there are quotations from the *ŚD* in Sūryadeva Yajvā's and Parameśvara's commentaries on the *Āryabhaṭīya* and in Yallaya's and Rāmakṛṣṇa Ārādhya's commentaries on the *Śuryasiddhānta*. Munīśvara's commentary (*Marīci*) and Lakṣmīdāsa's commentary on the *SŚi* quote Lalla. Lalla is quoted in many other works, today available only in manuscripts, such as *Parāśaratulya* by Gaṅgādhara, *Daivajñavilāsa* by Kañcayallayārya, *Āryasiddhāntatulyakaraṇa* and *Anūpamahodadhi* by Vīrasiṃha, *Trivikramaśataka*, commentary by Gopīnātha, commentaries on the *SS* by Nṛsiṃha, Caṇḍeśvara, Bhūdhara, etc.

Now a word about *Karaṇaprakāśa* by Brahmadeva (early 11th century A. D.). In the beginning, the author promises that his work would give the same results as those according to the treatise of Āryabhaṭa or '*Āryabhaṭaśāstrasamam*'. The corrections to be given to the Sun, Moon, etc. have taken from Lalla's correction given in *śāke nakhābdhi-rahite* etc. (i. 13-14). *Bhaṭatulyakaraṇa* by Dāmodara (14th cent. A.D.) is another work which has borrowed Āryabhaṭa's parameters with Lalla's corrections (*BJ*, p. 275).

The above account illustrates that Lalla's contribution to Indian astronomy and astrology cannot be ignored. It is correct that there is not much of originality. But there is definitely a systematic treatment of both the subjects. He has borrowed profusely from Āryabhaṭa, Bhāskara I (perhaps ?) and Brahmagupta and has been closely followed by Śrīpati and Bhāskara II. Lalla has a historical link which is important in Indian astronomical tradition.

III. COMMENTATORS OF ŚIṢYADHIVṚDDHIDA

Two commentaries on the *ŚD* are now available, one by Bhāskara II and the other by Mallikārjuna Sūri.[1] Bhāskara's commen-

1. There is evidence to show that Viṣṇu Yajvā was a third commentator of the *Śiṣyadhīvṛddhida*, but his commentary has not been discovered so far. —*KSS*

tary extends only up to the ninth chapter *viz.*, *Sṛṅgonnatyadhikāra* of *Gaṇitādhyāya*, and Mallikārjuna's covers all the thirteen chapters of this section. It is most unfortunate that no commentary on *Golādhyāya* has yet come to light.

1. Bhāskara II

Bhāskara II, popularly known as Bhāskarācārya, is the best known among the ancient Indian astronomers and is the most widely read. Towards the end of *Golādhyāya* of his *SŚi* (verses 58-63), he has given his year of birth as 1036 Śaka or A.D. 1114. According to him, he was born in the town of Vijjaḍaviḍa (perhaps modern Bijapur) amidst the Sahyādri range. His father was Maheśvara, a famous astronomer. Bhāskara belonged to a Brahmin family of Śāṇḍilya.

His works are *Līlāvatī*, a treatise on arithmetic, *Bījagaṇita*, a treatise on algebra, *SŚi*, an astronomical work in two parts, called *Gaṇitādhyāya* and *Golādhyāya*, and *Karaṇakutūhala*, another astronomical work. Possibly, he also wrote a treatise on astronomical instruments called *Bhadrayantra*. While describing *Nāḍīvalayayantra* (*SŚi*, II. xi. 5-7, commentary) he refers to it. Nṛsiṃha, a commentator of the *SŚi*, call him *bhadrādiyantranirmātā*, 'a maker of the instruments *Bhadra*, etc.' in his commentary on the first verse of *SŚi*. Bhāskara's commentary on the *ŚD* was also known to other astronomers ; *e.g.*, Nṛsiṃha in the above passage calls him *Dhīvṛdddhidopāyakartā*, 'who has shown the methods in the *ŚD*'. From his remarks it appears that Nṛsiṃha had seen the whole commentary.

Bhāskara starts his commentary on the *ŚD* with obeisance to the Sun. His words are, *bhāskaraṃ bhāskareṇātra natvotpattiḥ sphuṭocyate*, 'after making obeisance to the Sun, Bhāskara starts explaining the formulas', He calls Lalla's work *Dhīvṛddhida* and *Śiṣyadhīvṛddhida*. He refers to Lalla as Ācārya, and on several occasions while explaining the rules he remarks *śobhanamuktam* and *sādhūktam* 'correctly said' by the Ācārya. He has proved most of the formulas by means of simple ratios ; and occasionally by practical methods and observations with the *yaṣṭi-yantra*, especially some of the rules in the chapter iv on 'direction, time and shadow'. The special features in his commentary has been brought out in the mathematical notes.

It is rather surprising that while explaining the formulas on *valana* and *dṛkkarma* (visibility correction), Bhāskara does not criticise Lalla's use of 'R versed sine' instead of 'R sine' though he has repeatedly condemned this use in the *SŚi*. In fact, in his commentary, he supports the use of R versed sine (vide Mathematical Notes).

The style of the commentary shows that, perhaps, this is one of the earliest works of Bhāskara. It is immature when compared with his own commentary on the *SŚi*. Anyhow, it was important enough to attract the attention of some astronomers as stated above. Bhāskara has occasionally mentioned Brahmagupta. These cases have also been explained in the Mathematical Notes.

2. Mallikārjuna

The second commentator is Mallikārjuna Sūri. The credit of discovering this commentary should perhaps be attributed to the late Bibhuti Bhushan Datta, who wrote an article in No. 2 of the Bengali *Sahitya Parisat Patrika* of Bangabda, 1340 (A.D. 1934). Datta got a copy of the MS from the Durbar Library, Mysore. This is probably Ms. M_1 used for the present edition of the work.

Mallikārjuna does not mention his date anywhere in the commentary on the *ŚD*. But he uses 1100 and 1107 Śakas for illustrating rules. The same years he uses for calculating the *ahargaṇa* in his commentary on the *Sūryasiddhānta* and illustrating the rule on *vyatīpāta*. Again, for determining the *ahargaṇa* in his work called *Tithicakra*, 1100 Śaka is subtracted from the total number of Śaka years. Warren speaks of a Telugu astronomer, Mallikārjuna, who is supposed to have flourished in 4279th year of the Kaliyuga (1100 Śaka) and referred his calculations to the meridian of Rāmeśvara (*Kālasaṅkalita*, p. 369). According to Datta, Caṇḍeśvara, an astronomer of Mithila also uses 1100 and 1107 Śakas in the examples in his commentary on the *Sūryasiddhānta*. He refers to the commentary on the same work by Mallikārjuna and praises his explanation of the chapter on 'astronomical instruments'.[1] If *these* could be used as indications of his

1. Datta, *ibid*.

time, it may be surmised that Mallikārjuna lived somewhere in the twelfth century, probably at the same time as Bhāskara II, though one does not appear to know the other's existence ; at least, one does not mention the other in the commentary.

In the colophon at the end of his commentary Mallikārjuna writes *Veṅgideśasamudbhavena,* 'by one born in the Veṅgi Deśa' (text, p. 196). This shows that he belonged to Veṅgi, a state south of the Godāvarī river, now in the Andhra State. This state played an important part in the history of South India from the sixth till the twelfth century A.D.[1]

It may be observed here that the MS M_1 of which a copy was in the possession of Dr. Datta reads 'Vaṅga' instead of 'Veṅgi' and this, led him to state, in his article mentioned above, that Mallikārjuna was a Bengali. The very name contradicts this statement. Mallikārjuna explains that he is named after God (*Jyotirliṅga*), Śrīśailamallikārjuna. There is a temple dedicated to this God in the northern Mysore.

In the said colophon, Mallikārjuna says that his grandfather was Anantanārāyaṇa and his elder brother, Naṅganārya. In the beginning of the commentary (text, p. 1) also he describes himself as the younger brother of Naṅganārya and once again in the body of the commentary (text, p. 24). He does not give the name of his father. Mallikārjuna was born in the Kauṇḍinya family. He makes obeisance to the goddess Caṇḍikā before he begins the commentary. And, in all probability, he refers to the same goddess whom he calls only Devī, at the end of his commentary (text, p. 196).

In his commentary, Mallikārjuna has given, on different occasions, fortyfour verses of his own, either explanatory or giving his own rules or corrections. These have been noted in the Mathematical Notes. He calls Lalla's work *Śiṣyadhītantra* or *Śiṣyadhīmahātantra.* Nowhere he calls it *Śiṣyadhivṛddhida*. Mallikārjuna in his commentary mentions *Sūryasiddhinta*, *Somasiddhānta* and *Khaṇḍakhādyaka*. He has spoken about Āryabhaṭa also.

1. K.A.N. Sastri, *A history of South India*, pp. 146-94.

Mallikārjuna has also written a commentary on the *Sūrya-siddhānta*, which is available in MS form. This extends up to the end of the eleventh chapter. While explaining i. 8-10, wherein the representative of the Sun God speaks to Maya, he writes, *eṣām uktaślokānām artho asmābhir Āndhraṭīkāyāṃ vyākhyātaḥ*, 'I have explained these verses in my commentary in Telugu'. This shows that he wrote the commentary first in Telugu, and then in Sanskrit. That also would support the theory that Mallikārjuna was a Telugu and not a Bengali as claimed by Datta. Here, in the beginning of the commentary, he repeats the first three lines of the benedictory stanza as given in the commentary on the *ŚD*.

He quotes several times from Lalla's *ŚD* and twice from *Ratnakośa*. There are also quotations from *Āryabhaṭīya* and *Ārya-siddhānta*, *Siddhāntasāra*, *Siddhāntaprakāśa*, *Vaṭeśvara-siddhānta* and *Prabhākaratantra*.

In both the commentaries, Mallikārjuna has discussed *Āyurdāya* or longevity, a topic in astrology. The verse quoted in this connection is ascribed to Parāśara in this commentary, but in the commentary on the *ŚD* he simply says that it is given in the *Bṛhajjātaka* (of Varāha). It does not, however, seem to occur in the *Bṛhajjātaka*. Anyhow, Rāmakṛṣṇa Ārādhya in his commentary on the *Sūryasiddhānta* quotes this same verse and ascribes it to Parāśara.

Another work of Mallikārjuna is *Tithicakra*, of which there are two MSS, SP 4771 and SP 2588, in the Oriental Research Institute, Mysore. The first gives the text only and the second a commentary and tables in Telugu, probably by Mallikārjuna himself, in addition to the text. There are rules to calculate the *ahargaṇa* and then the *tithi*, *karaṇa* and *nakṣatra*. The tables are for preparing a *Pañcāṅga*.

In the beginning of the *Tithi-cakra* also Mallikārjuna repeats the first three lines of the benedictory stanza in the *ŚD*. It appears from all these works that Mallikārjuna was experienced both in astronomy and astrology. He was an observer also as his commentaries on the *ŚD* and the *Sūryasiddhānta* testify. In both the

commentaries Mallikārjuna gives several lengths of equinoctial midday shadow from which the latitudes and longitudes of these places can be calculated. These references should, therefore, be of geographical and historical importance. Mallikārjuna's style of writing is simple and lucid, though occasionally there are long compounds which are detrimental to the understanding of the explanatory notes. But, on the whole, he should be considered a good commentator.

IV. TRANSLATION AND MATHEMATICAL NOTES

The main object of the translation of the text into English has been to express clearly the formulas which are couched in the text in terse language. And, so, it has not always been possible to give a literal version of the verses. Moreover, words and sentences had to be added to make the English rendering intelligible. Construction of sentences in both these languages is so different that sometimes literal translation would have verged on an arrangement of words without sense. These points have been kept in view while translating the work. It was not considered necessary to translate the commentary as well. Any special point therein has been mentioned in the Mathematical Notes.

As far as possible equivalents of the Sanskrit technical terms in English have been used in the translation. An attempt has been made to explain the formulas in the Mathematical Notes. The proofs are strictly traditional. The notes indicate a comparative study of the works of Āryabhaṭa, Bhāskara I, Brahmagupta, Lalla, Śrīpati and Bhaskara II. Their astronomical parameters have been shown in tables. The diagrams are meant merely to convey an idea of the proof of the formulas, and are by no means intended to be precise. The theory of epicycles and eccentrics representing the motion of planets graphically plays a very important part in the Indian astronomy. It has, therefore, been explained separately in Appendices XIII and XIV (pp. 311 ff.).

ABBREVIATIONS

ĀBh	*Āryabhaṭīya*
BJ	*Bhāratīya Jyotiḥśāstra* (translation in Hindi)
BS	*Bṛhatsaṃhitā*
BSS	*Brāhmasphuṭasiddhānta*
GT	*Gaṇakataraṅgiṇī*
KK	*Khaṇḍakhādyaka*
KP	*Pūrvakhaṇḍakhādyaka*
KU	*Uttarakhaṇḍakhādyaka*
LB	*Laghubhāskarīya*
MB	*Mahābhāskarīya*
PS	*Pañcasiddhāntikā*
ŚD	*Śiṣyadhīvṛddhida*
SŚe	*Siddhāntaśekhara*
SŚi	*Siddhāntaśiromaṇi*
SS	*Sūryasiddhānta*

ROMAN TRANSLITERATION OF DEVANAGARI

VOWELS

Short :	अ	इ	उ	ऋ	ऌ		
	a	i	u	ṛ	ḷ		
Long :	आ	ई	ऊ	ए	ओ	ऐ	औ
	ā	ī	ū	e	o	ai	au

Anusvāra : ं = ṁ

Visarga : ः = ḥ

Non-aspirant : ' = ऽ

CONSONANTS

Classified :	क्	ख्	ग्	घ्	ङ्			
	k	kh	g	gh	ṅ			
	च्	छ्	ज्	झ्	ञ्			
	c	ch	j	jh	ñ			
	ट्	ठ्	ड्	ढ्	ण्			
	ṭ	ṭh	ḍ	ḍh	ṇ			
	त्	थ्	द्	ध्	न्			
	t	th	d	dh	n			
	प्	फ्	ब्	भ्	म्			
	p	ph	b	bh	m			
Un-classed :	य्	र्	ल्	व्	श्	ष्	स्	ह्
	y	r	l	v	ś	ṣ	s	h
Compound :	क्ष्	त्र्	ज्ञ्					
	kṣ	tr	jñ					

ŚIṢYADHĪVṚDDHIDA-TANTRA

OF LALLĀCĀRYA

TRANSLATION AND NOTES

ŚIṢYADHĪVṚDDHIDA-TANTRA

OF LALLĀCĀRYA

MATHEMATICS SECTION

CHAPTER I

MEAN PLANETS

[Invocation]

1. Having made obeisance to Brahmā, Hari, Śiva, Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn and Gaṇeśa, Lalla now depicts the motion etc. of the heavenly bodies, a subject which has been dealt with by Ācārya Āryabhaṭa. But the subject is rather difficult. Therefore, Lalla is composing the work *Dhīvṛddhida*, 'Enlightener of the Intellect', towards enhancing the (astronomical) knowledge of students.

2. It is true that the Āryabhaṭa's pupils have made a complete study of the works of their teacher and have written treatises

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MATHEMATICAL NOTES

1-2. Lalla indicates here that the astronomical treatise he is going to write will be based on Āryabhaṭa's work, or, in other words, he will be using the astronomical constants of Āryabhaṭa's *Audayika* ('sunrise') system, as given in the *Āryabhaṭīya* (*ĀBh*). He also hints that his work will be known as *Śiṣyadhīvṛddhida*, or 'that which increases the knowledge of students', and will certainly be more descriptive of the procedures of computation than any so far written by Āryabhaṭa's pupils.

Tantra is a special type of astronomical treatise. There are, in general, three kinds of Indian astronomical works, *Siddhānta*, *Tantra* and *Karaṇa*. In a *Siddhānta* the astronomical calculations or the epoch start from *Kalpa* or the beginning of creation. In a *Tantra* the epoch is the beginning of *Kaliyuga* or 3102 B. C. In a *Karaṇa* any convenient epoch is selected by the astronomer. The present work is a *Tantra*.

on astronomy ; but they have not dealt fully with the procedures of computation. Therefore, I shall deal, in proper order, with the procedures of what has been enunciated by Āryabhaṭa.

[Revolutions of the planets]

3. 432 multiplied by 10,000 (=43,20,000) gives the number of solar years in a *yuga*. This is also the number of revolutions of the Sun, Mercury and Venus and that of the *śīghroccas* of Mars, Jupiter and Saturn during the same period. This has been specified by astronomers.

4-6. (The number of revolutions) of the Moon (in a *yuga*) is 5,77,53,336 ; that of Mars is 22,96,824 ; that of the *śīghrocca* of Mercury is 1,79,37,020 ; that of Jupiter is 3,64,224 ; that of the *śīghrocca* of Venus is 70,22,388 ; that of Saturn is 1,46,564 ; that of the Moon's node is 2,32,226 in the opposite direction ; and that of the Moon's apogee is 4,88,219. (The number of) intercalary months (in a *yuga*) is 15,93,336 and (the number of) omitted *tithis* is 2,50,82,580.

3-8. Before setting down the astronomical constants, some of the technical terms used for measurement of time are first explained. A solar day (*sauradina*) is the interval of time during which the Sun moves through 1° of the ecliptic. A solar month (*sauramāsa*) is the interval of time during which the Sun moves through 30° or 1 sign of the zodiac. Thus, a solar month is equal to 30 solar days. The day when a solar month ends is called *Saṅkrānti*. A solar year (*sauravarṣa*) is equal to 12 solar months.

A lunar day (*Tithi* or *Cāndradina*) is the interval of time during which the Moon gains 12° of longtitude over the Sun. A lunar month (*Cāndramāsa*) is equal to 30 lunar days or *tithis*. A lunar month ends on the day of conjunction of the Sun and Moon, *i. e.*, New Moon or *amāvasyā*. According to another school it also ends on a Full Moon day or *pūrṇima*. *Cf.* : '*darśāntāvadhikaḥ pūrṇimāntāvadhikaś ca*', Raṅganātha's commentary on *Sūryāsiddhānta* (*SS*), i.13.

Intercalary months (*Adhimāsas*) are the total number of months in the difference between solar and lunar months during any period and the residue or the fraction is called *Adhiśeṣa*. An *Adhivarṣa* is equal to 12 intercalary months. 1/30 of an intercalary month is *Adhidina*.

[Lunar and Solar days]

7. It is said that the number of solar days in a *yuga* is 1,55,520 multiplied by 10,000 (=1,55,52,00,000). This divided by 30 gives the number of solar months. The number of lunar days or *tithis* in a *yuga* is 1,60,30,00,080.

[Civil and Siderial days]

8 (The number of) civil days (in a *yuga*) is 1,57,79,17,500 and the number of revolutions of the asterisms is 1,58,22,37,500.

A civil day (*Sāvanadina, Kudina*) is the interval of time between one sunrise and the next. *Cf.* : *udayād udayaṃ bhānor bhūmisāvanavāsarāḥ, i. e.*, 'Civil days are measured from one sunrise to to the next'. (*SS*, i. 36 cd). The number of civil days between any two dates is called *Ahargaṇa.* If one of those dates is the first day of the current solar year and the other date is within the year, the number of civil days is called *Laghvahargaṇa.*

Omitted *tithis* (*Avamādina, Avamarātra*) are the total number of days in the difference between the lunar days and the civil days during any period, and the residue or fraction is called *Avamaśeṣa.*

A sidereal day (*Nākṣatradina*) is the time during which the celestial sphere makes one revolution round the earth.

A *Mahāyuga*, or briefly *Yuga*, is a period 43,20,000 solar years. The following are the revolutions of the Sun etc. during, this period as used by Lalla.

Sun, Mercury, Venus	43,20,000
Moon	5,77,53,336
Mars	22,96,824
Mercury (*śīghrocca*)	1,79,37,020
Jupiter	3,64,224
Venus (*śīghrocca*)	70,22,388
Saturn	1,46,564
Moon's Node (opposite direction)	2,32,226

Moon's Apogee	4,88,219
Asterisms	1,58,22,37,500

The following are the solar years, etc. in a *yuga* of 43,20,000 solar years :

Solar years	43,20,000
Solar months	5,18,40,000
Solar days	1,55,52,00,000
Lunar months (difference between solar and lunar revolutions)	5,34,33,336
Lunar days	1,60,30,00,080
Intercalary months	15,93,336
Omitted *tithis*	2,50,82,580
Civil days	1,57,79,17,500

The same constants are given by Āryabhaṭa in *ĀBh, Daśagītikā*, 3-4. But instead of saying that the celestial sphere moves, he says that in a *yuga* the rotations of the earth eastward are 1,58,22,37,500.

For similar constants in other astronomical works, see below Tables I and II.

It will be useful to indicate here the smaller units of a civil day and those of a circle.

Civil day :

60 *vipalas* = 1 *pala* or *vināḍī*
60 *palas* = 1 *ghaṭikā, ghaṭī* or *nāḍī*
60 *ghaṭikās* = 1 *dina* or *divasa*
6 *asus* or *prāṇas* = 1 *vināḍī*

Cf. : *ṣaṭpramāṇair asubhiḥ pānīyapalaṃ bhavati.* (Commentary by Raṅganātha on *SS*, i. 11).

Circle :

60 *viliptās, vikalās*
or seconds = 1 *liptā, kalā* or minute
60 *liptās* = 1 *aṃśa, bhāga* or degree

[Civil days of the planets and Lunar months]

9. When the second number (given above) is diminished by the number of revolutions of a planet (in a *yuga*), the remainder, it is said, is the number of civil days of that planet (during the same period).

The wise say that the difference between the revolutions of the Sun and Moon (in a *yuga*) is the number of lunar months (in the same period).

[Intercalary months and Omitted tithis]

10. Or, the revolutions of the Moon diminished by 13 times the revolutions of the Sun (during a given period or *yuga*), give the number of intercalary months (during the same period).

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60 *aṃśas* = 1 *rāśi* or sign
12 *rāśis* = 1 *bhagaṇa* or revolution

In his commentary, Mallikārjuna says (text, p. 2), that the *yuga* of 43,20,000 solar years is divided into 4 periods as follows :

Kṛtayuga	17,28,000 solar years
Tretāyuga	12,96,000 solar years
Dvāparayuga	8,64,000 solar years
Kaliyuga	4,32,000 solar years
Total	43,20,000 solar years

This is the traditional division of 1 *yuga*, But Āryabhaṭa, in his *ĀBh, Daśagītikā*, 5, divides it into 4 equal parts. Brahmagupta in *Brāhmasphuṭasiddhānta* (*BSS*), i. 9, criticises Āryabhaṭa for having ignored tradition.

10. Intercalary months
= lunar moths — solar months
= Moon's revolutions — Sun's revolutions — 12 Sun's revolutions
= Moon's revolutions — 13 Sun's revolutions

And, remember, the sum of the *tithis* and the revolutions of the Sun (during a given period or *yuga*), when diminished by the revolutions of the asterisms, gives the number of omitted *tithis* (during the same period).

[Planetary conjunctions and Revolutions of the apogees]

11. The difference between the revolutions of two planets in a *yuga* is the number of their conjunctions during the same period.

The difference between the revolutions of a planet and those of its apogee (*mandocca*) or *śīghrocca* gives the number of revolutions of the mean anomaly (in the first case) and of the *śīghra* anomaly in the second case (during the same period).

[Calculation of days elapsed]

12-14. The Śaka year (when the civil days are required) added to 3179 gives the solar years elapsed since the beginning of the *Kaliyuga*. Multiply the number by 12 and add the months (beginning from *Caitra*) passed in the current year. Multiply the sum by 30 and add the number of *tithis* elapsed (since the last *amāvasyā*). Put down the result at two places. (At one place), multiply it by the number of intercalary months (in a *yuga*) and divide by the number of solar days. The result is the corresponding number of intercalary months.

See Śrīpati's *Siddhāntaśekhara*, (*SŚe*), i. 38 ab, and Bhāskara II's *Siddhānta Śiromaṇi*, (*SSi*), I. i. b. 14 ab.

Omitted *tithis*

= lunar days — civil days

= lunar days — (revolutions of asterisms — Sun's revolutions)

= lunar days (*tithis*) + Sun's revolutions — revolutions of asterisms

(See *ŚSe*, i. 38 cd).

12-14. These verses give the general method followed by Indian astronomers to calculate the civil days between the beginning of the *Kaliyuga* and any day during the current solar year.

Change it into days and add to the result at the second place. Put down the sum at two places. (At one place), multiply it by the number of omitted *tithis* (in a *yuga*) and divide by the number of *tithis* (in the same period). The result is the corresponding number of omitted *tithis*. Subtract it from the sum at the second place. The remainder is always the number of civil days or *ahargaṇa* elapsed since the beginning of the *Kaliyuga* (till the given day) and is counted from Friday at (mean) sunrise at Laṅkā.

The Śaka era began 3179 years after the beginning of the *Kaliyuga*. So, the given Śaka year *plus* 3179 is the the number of complete solar years between the two dates. This number multiplied by 12 is converted into solar months. To this is added the number of elapsed lunar months beginning with the light half of the month *Caitra* and the sum assumed as solar months. This sum multiplied by 30 becomes the number of solar days. To this is added the number of lunar days or *tithis* elapsed since the last *amāvasyā* and assumed as solar days. Thus the given number of years is converted into solar days between the epoch and the time when the last solar day ends or *saurānta*. Let it be A.

Now, to find the corresponding number of intercalary months together with the remainder from the proportion :

Solar days in *yuga* : intercalary months in yuga :: A : B (say). Then, A+ 30 B gives the lunar days between the epoch and the end of the solar day or *saurānta*. The residue of the intercalary months is the time between the end of the last *tithi* and the *saurānta*. So, when this element is subtracted from A+30 B, the remainder is the number of lunar days between the epoch and the end of the last *tithi*. Let it be C.

Note that the fraction relating to the intercalary months is first added and then subtracted. Or, in other words, it is not used to calculate the civil days.

Then, to convert the lunar days into civil days, the number of omitted *tithis* corresponding to C is obtained from the proportion :

Lunar days in *yuga* : omitted *tithis* in *yuga* : : C : D (say).

Then C—D is the number of civil days between the epoch and the end of the last *tithi*. The residue relating to the omitted *tithis* is the time between the end of the *tithi* and the sunrise following. So, when this element is added to C—D, the result is the number of civil days between the epoch and the sunrise of the given day and is called *Ahargaṇa*.

Note that the fraction relating to omitted *tithis* is first subtracted and then added. Or, in other words, it is not used to calculate the civil days.

In the above method, the lunar months and days are added to the solar months and days. But this apparently wrong method does not affect the final result, because neither the residue relating to the intercalary months nor that relating to the omitted *tithis* is used. Bhāskara II has explained it clearly in his commentary on *ŚSi*, II, iv, 15-18. His commentary on Lalla's verses, however, is not lucid.

In his *Tantra*, Lalla calculates the civil days for sunrise at Laṅkā. This is called *audayika* system and is used by Āryabhaṭa in *ĀBh*, Bhāskara I in *Mahābhāskarīya* (*MB*), i. 4-6, and *Laghubhāskarīya* (*LB*), i. 4-8, Brahmagupta in *BSS*, i. 30-31, Śrīpati in *SŚe*, ii. 1-2 ab and Bhāskara II in *SŚi*, I, i.c. 1-3 use the same method for calculating the *ahargaṇa*. See also *SS*, i. 48-51, which, however, uses the midnight system.

Āryabhaṭa uses the midnight (*ārdharātrika*) system also, when the calculations start from midnight. The treatise composed by Āryabhaṭa on this system is lost, but the astronomical constants used therein are preserved by Bhāskara I in *MB*, vii. 21-35 and these constants have been used by Brahmagupta in his work, *Khaṇḍakhādyaka*. In this system a civil day would be from midnight to midnight.

In this connection, it is worth noting what Varāhamihira says in *Pañcasiddhāntikā* (*PS*), xv. 17-20 :

दिनवारप्रतिपत्तिर्न समा सर्वत्र कारणं कथितम् ।
नेहापि भवति यस्माद्विप्रवदन्तेऽत्र दैवज्ञाः ॥ १७ ॥

द्युगणाद् दिनवाराप्तिर्द्युगणोऽपि देशकालसंबन्धात् ।
लाटाचार्येणोक्तो यवनपुरेऽर्द्धास्तगे सूर्ये ॥ १८ ॥

रव्युदये लङ्कायां सिंहाचार्येण दिनगणोऽभिहितः ।
यवनानां निशि दशभिर्गतैर्मुहूर्त्तैश्च तद् गुरुणा ॥ १९ ॥

लङ्कार्द्धरात्रसमये दिनप्रवृत्तिं जगाद चार्यभटः ।
भूयः स एव सूर्योदयात् प्रभृत्याह लङ्कायाम् ॥ २० ॥

'The reckoning of the weekdays is not the same everywhere. As there is no particular reason for it, the astronomers disagree about it. The day of the week is obtained from the *ahargaṇa*, which again depends on time and place. Lāṭācārya says that the *ahargaṇa* should be counted from Yavanapura (Alexandria) at the time of sunset, whereas Siṃhācārya says that it should be counted from sunrise at Laṅkā. Again, according to the Guru (teacher) of the Yavanas, it is to be reckoned at night when 10 *muhūrtas* have passed. Āryabhaṭa has said that the day begins at midnight at Laṅkā ; but even he again says that day begins from sunrise at Laṅkā.''

These verses have been quoted by Makkibhaṭṭa in his commentary on *SŚe*, ii. 10, which also gives the various times of reckoning, sunrise, midday, sunset, 10 *muhūrtas* passed in the night, and according to Lāṭācārya in his work, from midnight. Śrīpati, however, suggests (*SŚe*, ii. 11) the *audayika* system.

Laṅkā is an imaginary place on the equator (latitude 0°) whose longitude is considered 0° by the Indian astronomers for applying corrections due to the difference in terrestrial longitudes.

The *ahargaṇa* is counted from Friday, since the *Kaliyuga* began on a Friday. So says Bhāskara II in his commentary on the above verses, *yato bṛhaspatidine dvāparānto ataḥ sitavārādikaḥ*, '(the *yuga* preceding Kali), that is, Dvāpara ends on a Thursday. So the present *ahargaṇa* starts from Friday'. Āryabhaṭa also has said that *Kaliyuga* started on a Friday (*ĀBh, Daśagītikā*, 5).

[Alternate method for the calculation of days]

15-16. Or, the solar months elapsed since the beginning of the *Kaliyuga* multiplied by 22,26,389 and divided by 21,60,000 give the corresponding lunar months. Convert them into days and add the *tithis* elapsed (since the last *amāvasyā*). Multiply the sum by 7,88,95,875 and divide by 8,01,50,004. The result increased by 1 is the *ahargaṇa* required.

15-16. Another method is given to calculate the *ahargaṇa*. Find the solar years elapsed since the beginning of the *Kaliyuga* till the end of the year previous to the current year and convert them into months. Let the number be A, after adding the months of the current year. Then from the proportion,

Solar months in yuga : lunar months in *yuga* : A : B (say)

or 5,18,40,000 : 5,34,33,336 :: A : B

or 21,60,000 : 22,26,389 :: A : B

$$B = \frac{22,26,389 \text{ A}}{21,60,000} \text{ lunar months}$$

$$= \frac{22,26,389}{21,60,000} \text{A. 30 } \textit{tithis.}$$

Add to B the *tithis* elapsed since the last *amāvasyā* and let the sum be C. Then, from the proportion :

Lunar days in *yuga* : civil days in *yuga* :: C : D (say)

or 1,60,30,00,080 : 1,57,79,17,500 :: C : D

or 8,01,50,004 : 7,88,95,875 :: C : D,

the number of civil days corresponding to C *tithis* is obtained. The result, when increased by 1, as there is a remainder, gives the required *ahargaṇa*.

Bhāskara I says :

त्रैराशिकागतदिनेषु च रूपमेकं
व्यावर्णयन्ति गणका भटशास्त्रचित्ताः ॥ (*MB*, i. 40 cd)

'Add 1 to the *ahargaṇa* calculated by proportion, (if there is a

[Mean longitudes of planets]

17. The *ahargaṇa* multiplied by the number of revolutions of a planet and divided by the number of civil days (in a *yuga*) gives the mean longitude of the planet in revolutions, etc.

Or, the mean longitude of the Sun expressed in minutes, multiplied by the number of revolutions of a planet and divided by the number of revolutions of the Sun (in a *yuga*), gives the mean longitude of the planet in minutes (at that time).

remainder). Thus say the astronomers devoted to Āryabhaṭa's system.' It is quoted by Makkibhaṭṭa in his commentary on *ŚSe*, ii. 3.

Bhaskara II in his commentary on the above two verses has also said, *yadi savikalās tadā rūpasaṃyutāḥ kartavyāḥ*, 'If there is a remainder, 1 should be added'. While commenting on these verses, Mallikārjuna talks of *Āyurdāya* or longevity of a man (text p. 9). He says : 'Man's age is measured in sidereal units. This multiplied by the solar years in a *yuga* and divided by the sidereal years (in the same period) gives his age (in solar units)'. According to Mallikārjuna this verse is given in the *Bṛhajjātaka*. But it is not found there. Mallikārjuna, however, attributes it to Parāśara in his commentary on *SS*, i. 34. And so does Rāmakṛṣṇa Ārādhya in his commentary on the same verse in *SS*.

The above method is given in *MB*, i. 7; BSS, xiii. 18; and in *SSe*., ii. 3.

17. The first rule follows from the proportion :

Civil days in *yuga* : planet's revolutions in *yuga* :: *ahargaṇa* : ?

This rule is given in *MB*, i. 8 ; *LB*, i. 15 ; *SS*, i. 53 ; *BSS*, i. 32 ; *ŚSe*, ii. 14; and in *SŚi*, I, i. c. 4.

The second formula follows from :

Sun's revolutions in *yuga* : planet's revolutions in *puga* :: Sun's longitude (in minutes) : ?

This rule is given in *MB*, i. 9.

[Alternate method for Mean longitude]

18. Or, the *ahargaṇa* multiplied by 576 and divided by 2,10,389, gives the longitude of the Sun, Mercury or Venus and that of the *śīghrocca* of Mars, Jupiter or Saturn.

[Mean Moon]

19. Or, the *ahargaṇa* multiplied by 78,898 and divided by 21,55,625, gives the mean longitude of the Moon in revolutions, etc.

[Alternate method for Mean Moon and Mean Sun]

20. When the *ahargaṇa* is severally multiplied by the lunar years and the number of intercalary years (in a *yuga*), and each product divided by the number of civil days (in a *yuga*), and the difference of the two results taken, the final result is the mean longitude of the Sun.

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18. This gives the formula for the Sun's mean longitude on any day.

It follows from the proportion :

Civil days in *yuga* : Sun's revolutions in *yuga* :: *ahargaṇa* : ?

or 1,57,79,17,500 : 43,20,000 :: *ahargaṇa* : ?

or 2,10,389 : 576 :: *ahargaṇa* : ?.

The number of revolutions of the Sun in a *yuga* is the same as that of Mercury or Venus and that of the *śīghrocca* of Mars, Jupiter or Saturn.

19. This gives the formula for the mean longitude of the Moon. It follows from the proportion :

Civil days in *yuga* : Moon's revolutions in *yuga* :: *ahargaṇa* : ?

or 1,57,79,17,500 : 5,77,53,336 :: *ahargaṇa* : ?

or 21,55,625 : 78,898 :: *ahargaṇa* : ?.

The formula is given in *SŚe*, ii. 18, though the figures are different, the epoch being different.

20. Here is given other formulas for the Sun and Moon.

13 times the mean longitude of the Sun added to 12 times the number of intercalary years during the *ahargaṇa* gives the mean longitude of the Moon.

The Sun's longitude on any day

$$=\frac{\text{Sun's revolutions in } yuga}{\text{civil days in } yuga}\cdot ahargaṇa$$

$$=\frac{\text{solar years}}{\text{civil days}}\cdot ahargaṇa$$

$$=\frac{ahargaṇa}{\text{civil days}}\cdot\frac{\text{solar months}}{12}$$

$$=\frac{ahargaṇa}{\text{civil days}}\cdot\left(\frac{\text{lunar months}}{12}-\frac{\text{intercalary months}}{12}\right)$$

$$=\frac{ahargaṇa}{\text{civil days}}\cdot(\text{lunar years}-\text{intercalary years}).$$

The moon's longitude on any day

$$=\frac{\text{Moon's revolutions in } yuga}{\text{civil days in } yuga}\cdot ahargaṇa$$

$$=\frac{ahargaṇa}{\text{civil days}}\ (\text{Sun's revolutions}+\text{lunar months})$$

$$=\frac{12\ ahargaṇa}{12\text{ civil days}}\ (\text{Sun's revolutions}+\text{solar months}+\text{intercalary months})$$

$$=\text{Sun's longitude}+12\ \frac{ahargaṇa}{\text{civil days}}\cdot\frac{\text{solar months}}{12}+12\ \frac{ahargaṇa}{\text{civil days}}\cdot\frac{\text{intercalary months}}{12}$$

$=13$ Sun's longitude $+12$ intercalary years during the *ahargaṇa*.

This rule is given in *MB*, i. 11 ; *BSS*, xiii. 33 ; and *SŚe*, ii. 19.

21. Divide the *adhimāsaśeṣa* (the remainder corresponding to the intercalary months obtained while calculating the *ahargaṇa*) and the *avamaśeṣa* (the remainder corresponding to the omitted *tithis* obtained while calculating the *ahargaṇa*) by the respective intercalary months and the omitted *tithis* in a *yuga*. (The first quotient) gives solar days, etc. and (the second) *tithis*, etc. The first divided by 1006 is in terms of signs, etc. ; the second diminished by its twelfth part is in minutes, etc.

22. The number of months from *Caitra* (elapsed in the current year) is equivalent to the same number of signs, and the number of *tithis* (elapsed since the last *amāvasyā*) is equivalent to the same number of degrees. These signs, etc. when decreased by the signs, etc. obtained previously and added to the minutes thus obtained (verse 21), give, in terms of signs, degrees and minutes, the mean longitude of the Sun. To the longitude add degrees equal to 12 times the *tithis* and also $\frac{25}{2}$ times the minutes obtained above. The result is the mean longitude of the Moon.

23. The *adhimāsaśeṣa* divided by the number of lunar months (in a *yuga*) yields subtractive result. The *avamaśeṣa* when divided by the number of civil days gives additive result. The sum of the number of *tithis* elapsed (since the *amāvasyā* in *Caitra*) and the second result, when diminished by the first, gives (also) the mean longitude of the Sun in degrees, etc. The same sum multiplied by 13 and diminished by the first, gives the mean longitude of the Moon in degrees, etc.

24. The *ahargaṇa* multiplied by the number of sidereal days and divided by the number of civil days (in a *yuga*) gives the corresponding

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21-22. These give a method to determine the longitudes of the Sun and Moon from the residue of intercalary months or *adhimāsaśeṣa* and that of omitted *tithis* or *avamaśeṣa*, corresponding to a certain *ahargaṇa*.

23. This gives another method to ascertain the longitudes of the Sun and Moon from the residues of intercalary months and omitted *tithis*, corresponding to a certain *ahargaṇa*.

24. This gives the formulas for the Sun and Moon.

number of sidereal days. This, diminished by the *ahargaṇa,* results in the mean longitude of the Sun (in revolutions etc.).

When to this longitude is added the excess of the Moon's longitude calculated from the number of lunar months (in a *yuga*), the sum is the mean longitude of the Moon.

[Alternate method for Mean planets]

25. Multiply the *ahargaṇa* by the difference of the number of revolutions of any two planets and divide by the number of civil days (in a *yuga*). Subtract the result from the greater longitude (of the two planets). The remainder is the lesser longitude. Again add the result to the lesser longitude and the sum is the greater longitude.

Civil days in *yuga* = sidereal days in *yuga* — Sun's revolutions in *yuga*

or, Sun's revolutions = sidereal days — civil days.

So, the Sun's longitude on any day in terms of revolutions, etc. is equal to the number of sidereal days in the *ahargaṇa minus* the *ahargaṇa.* The first term follows from the proportion :

Civil days in *yuga* : sidereal days in *yuga* :: *ahargaṇa* : ?

Now, for the Moon. The Moon's revolutions in a *yuga* are equal to the Sun's revolutions *plus* the lunar months. So, the Moon's longitude on any day is equal to the Sun's longitude *plus* the excess, which is obtained from the proportion :

Civil days in *yuga* : lunar months in *yuga* :: *ahargaṇa* : ?

Śrīpati gives these two formulas in *ŚSe,* ii. 17, 24.

25. The difference between the longitudes of any two planets is obtained from the proportion :

Civil days in *yuga* : difference in revolutions in *yuga* :: *ahargaṇa* : ?

The rest is clear.

This is given in *SŚe,* ii. 27 and *SŚi,* I, i. c. 13.

26. The number of revolutions of a planet, the longitude of which is required, when multiplied by the given longitude of another planet, expressed in minutes, and divided by the number of revolutions of the latter, gives its mean longitude in minutes.

The longitude of a planet is the sum of its apogee and mean anomaly. It is also equal to the difference of its *śīghrocca* and *śīghrakendra.*

[**Lord of the year**]

27. The number of solar years elapsed (since the beginning of the *Kaliyuga*) multiplied by 149 and divided by 576, gives the result in

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26. This verse explains how to calculate the longitude of a planet from the given longitude of another planet. From the proportion :

Known planet's revolutions in *yuga* : unknown planet's revolutions

in *yuga* :: given longitude : ?

The result obtained is the longitude.

This rule occurs in *MB,* i 10 ; *BSS,* xiii. 27 ; *ŚSe,* ii. 25 ; and *SŚi,* i. c. 14 cd.

In the above verses Lalla has given various methods to calculate the mean longitudes of the Sun, Moon and planets.

In his commentary on *SS,* i. 53, Yallaya says, 'Lallācārya has indicated only a few methods in the *Śīṣyadhīvṛddhidatantra,* for calculating the mean longitudes of the Sun, Moon etc. but he has given numerous methods in *Siddhāntatilaka.*' The same observation has been made by Rāmakṛṣṇa Ārādhya towards the end of his commentary on the first chapter of the *SS.* He also quotes from the *Siddhāntatilaka* indicating the methods.

27. The following is a formula for determining the lord of the year. In a *yuga,* the civil days are 1,57,79,17,500 and the solar days are 1,55,52,00,000. So the difference is 2,27,17,500, during 43,20,000 solar years. Thus the difference in 1 solar year is $\frac{2,27,17,500}{43,22,000}$ or $5\frac{149}{576}$

days, etc. Add it to the number of solar years and divide the sum by 7. The remainder counted from Friday gives the Lord of the year.

[Calculation of Śuddhi]

28. The number of solar years (elapsed since the beginning of the *Kaliyuga*) multiplied by 58,043 and divided by 72,C00, gives as quotient the number of omitted *tithis* at the end of the year. Again divide the remainder by 1C4. The result is the *avamaśeṣa* at the end of the last year (or, at the beginning of the current year).

29. Find the sum of the days (verse 27), the omitted *tithis* (verse 28), and 10 times the solar years (elapsed since the beginning of the

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So, in a solar year of 360 solar days, there are $365\frac{149}{576}$ civil days or 52 complete weeks and $1\frac{149}{576}$ days.

Now, let A be the number of solar years since the epoch. Then, during that period there are $A+\frac{149}{576}A$ extra civil days besides the complete number of weeks. When this is divided by 7, the quotient is the number of weeks and the remainder is the number of extra civil days. This remainder counted from Friday gives the *Sāvana Lord* of the year. It is counted from Friday as the *Kaliyuga* began on a Friday.

28. This gives the method to find the omitted *tithis* and the residue or *avamaśeṣa* at the end of the year.

In 1 solar year the omitted *tithis* are $\frac{25082580}{4320000}=5\frac{58043}{72000}$. So, if A is the number of solar years since the epoch, the omitted *tithis* at the end of the year, considering the fraction only, are $\frac{58043}{72000}$. A. Let the residue or *avamaśeṣa* be $\frac{C}{72000}$, which, when expressed in *sāvana* or civil measurement, will be $\frac{C.692}{72000}=\frac{C}{104}$, nearly.

Similar rules to those in verses 27-28 are given by Brahmagupta in *BSS*, i. 40 and by Śrīpati in *ŚSe*, ii. 34-35. Bhāskara II deals with the topic in *SŚi*, I. i. e. 1-4.

29. This explains the method to calculate the *śuddhi*. The

Kaliyuga). Divide it by 30. The quotient gives the number of intercalary months till the end of the year. The remainder converted into days plus the hours (*ghaṭikās*) etc. (written) underneath the days (verse 27), is called *Śuddhi.*

[Calculation of Grahadhruva]

30. When the number of solar years (elapsed since the beginning of the *Kaliyuga*) is multiplied by the number of revolutions of a planet and divided by the number of solar years in a *yuga*, (the quotient gives the revolutions). The successive divisors should be 43,20,000 first divided by 12, (the quotient) then by 30 and, (the resulting quotient) the by 60. (The result, omitting the

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difference between the number of solar days and the civil days in 1 solar year is $5\frac{149}{576}$ (verse 27) and the *avamadina* or difference between the number of lunar days and civil days is $5\frac{58043}{72000}$ (verse 28). So, in 1 solar year, the difference between the number of solar days and lunar days is $10+\frac{149}{576}+\frac{58043}{72000}$.

If X is the solar years elapsed since the beginning of the epoch, then in X years the difference will amount to,

$$10X+\frac{149}{576}X+\frac{58043}{72000}X=Y.$$

When Y is divided by 30, the integral part of the result is the complete number of intercalary months and the fractional part is the interval between the end of the *amāvasyā* in the month of *Caitra* and the end of the solar year. When this fractional part is multiplied by 30, the integral part of the result is called *adhidina* and the fractional part *adhiśeṣa*.

Brahmagupta gives the same method in *BSS*, i. 41 ; Śrīpati in *SŚe*, ii. 36 ; and Bhāskara II in *SŚi*, I. i. e. 5.

30. This gives the method to find the longitude of a planet at the end of a given number of solar years. This is obtained from the proportion :

Sun's revolutions in *yuga* : planet's revolutions in *yuga* :: given solar years : ?.

See also *SŚe*, ii. 39 and *SŚi*, I. i. e. 9.

revolutions), gives (in signs, degrees and minutes) the mean longitude of the planet at the end of the previous year and is known as *Graha-dhruva.*

[Calculation of Laghvahargaṇa]

31. The *tithis* elapsed since the beginning of the light half in the month of *Caitra,* lessened by the *śuddhi* should be put down in two places. At the first place it should be multiplied by

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31. This gives the method to find the *laghvahargaṇa* or civil days from the beginning of the current solar year to a day within the year. Let A be the solar years since the epoch. First to find the civil days till the end of the last year :

A solar years = 12 A solar months

= (12 A + intercalary months + corresponding residue or *adhiśeṣa*) lunar months

= (360 A + 30 intercalary months + 30 corresponding residue or *adhiśeṣa*) lunar days

= (360 A + 30 intercalary months + 30 corresponding residue or *adhiśeṣa*—omitted *tithis* or *avamadina*—residue or *avamaśeṣa*) in civil measurement. (1)

Let this number of omitted *tithis* be *avamadina*-1.

Now, the civil days since the epoch till the sunrise of the given day in the current year (verses 13-15)

= 360 A + *tithis* since the end of *amāvasya* in *Caitra* of current year + 30 intercalary months—omitted *tithis* or *avamadina* corresponding to lunar days elapsed till the sunrise of the given day or *avamadina*-2. (2)

Thus the difference between (2) and (1) is the required number of civil days,

= *tithis*—30 *adhiśeṣa* + *avamadina*-1 + *avamaśeṣa*—*avamadina*-2

= *tithis*—(30 *adhiśeṣa*—*avamaśeṣa*)—(*avamadina*-2—*avamadina*-1

= *tithis*—*śuddhi*—(*avamadina*-2—*avamadina*-1)

11, increased by the *avamáseṣa* corresponding to the beginning of the mean solar year and divided by 703. The result should be subtracted from the number at the second place. The remainder is the number of civil days (elapsed since the Sun was at the beginning of Aries, that is, the number of civil days in the current solar year).

[Lord of the day]

32. To this *ahargaṇa* should be added the days and hours (*ghaṭikās*) of the *śuddhi*. The sum which is the *ahargaṇa* from the beginning of *Caitra*, should be divided by 7. The remainder counted from this day gives the lord of the day. The longitudes of the planets, etc. calculated from the *ahargaṇa* (verse 31) will be for the sunrise on the current day.

[Sun etc. from Laghvahargaṇa]

33. The *ahargaṇa* diminished by its seventieth part and

Now, *avamadina*-2— *avamadina*-1

= *avamadina* or omitted *tithis* corresponding to lunar days between the end of the last solar year and the sunrise of the given day plus the residue or *avamaśeṣa* at the end of the last year. The above lunar days are *tithis* elapsed *minus śuddhi*. In 703 lunar days there are 11 omitted *tithis*. So, the omitted *tithis* $= \frac{11}{703}$ (*tithis*—*śuddhi*)

and the *avamaśeṣa* in civil units $= \frac{avamaśeṣa}{703}$.

Hence *laghvahargaṇa* or civil days in the current year = (*tithis*—*śuddhi*)—$\left(11\frac{(tithis—śuddhi)}{703}+\frac{avamaśeṣa}{703}\right)$.

Śrīpati gives exactly the same method in *SŚe*, ii. 40-41 ab. Brahmagupta deals with the subject in *BSS*, i. 42-44 and Bhāskara II in *SŚi*, I, i. e. 12 cd-13. Bhāskara I deals with *śuddhi* and *laghvahargaṇa* in *MB*, i. 22-30.

33. To find the Sun's mean longitude during the current year. In a *yuga* of 43,20,000 solar years or of 1,57,79,17,500 civil days]

then by the number of seconds obtained from twice the *ahargaṇa* divided by 5, gives the mean longitude of the Sun in degrees, etc. (during the current year).

34. To the mean longitude of the Sun add the number of degrees equivalent to 12 times the *tithis* (elapsed since the last *amāvāsyā*). Add to the sum the minutes equivalent to the *avamaśeṣa* increased by its $\frac{1}{25}$ part. The result is the mean longitude of the Moon.

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the Sun travels 43,20,000 × 360 degrees or 1,55,52,00,000 degrees.

$$1,55,52,00,000 = 1,57,79,17,500 - 2,27,17,500$$

Thus, during 1,57,79,17,500 civil days 2,27,17,500 degrees must be subtracted to get the Sun's motion.

Now, if A is the number of civil days elapsed since the beginning of the year till the day when the longitude is required, the required longitude

$$= A - \frac{2,27,17,500}{1,57,79,17,500} \cdot A \text{ degrees}$$

$$= A - \frac{A}{70} + \left(\frac{1}{70} - \frac{2,27,17,500}{1,57,79,17,500}\right) A \text{ degrees}$$

$$= \left(A - \frac{A}{70}\right) \text{degrees} - \frac{2}{5} A \text{ seconds, nearly.}$$

This formula is given in *MB*, i. 31 ; *BSS*, i. 45 ; and *SŚe*, ii. 43.

34. To find the longitude of the mean Moon when the mean longitude of the Sun is known.

On the day of the *amavasya* the Sun and Moon coincide. So, at sunrise on any given day the longitude of the Moon is equal to that of the Sun *plus* the degrees gained by the Moon during the *tithis* between the end of the last *amāvasyā* and the sunrise of the given day.

Now, these *tithis* consist of a complete number of *tithis* *plus* the fraction of a *tithi* between the end of the last *tithi* and the given sunrise and is called *avamaśeṣa* or residue of omitted *tithis*. It is calculated in the following manner :

35. Divide the *ahargaṇa* by 2. Increase the result by $\frac{1}{21}$ of itself. Add the sum in degrees to the dhruva of Mars (at the end of the previous year). Then add the minutes obtained from dividing the *ahargaṇa* by 80. The result is the mean longitude of Mars in the current year.

In 703 lunar days there are 11 omitted *tithis.* So, in the above *tithis,* (say A), there are $\frac{11}{703}$ A omitted *tithis.*

$$\frac{11}{703} \text{ A omitted } tithis = \text{omitted } tithis + \frac{\text{residue } (avamaśeṣa)}{703}$$

$$\text{So, } tithis = \text{complete number of } tithis + \frac{avamśeṣa}{703} \text{ (civil units)}$$

$$= tithis + \frac{64}{63} \cdot \frac{avamśeṣa}{703} \text{ (lunar units)}$$

$$\text{So, mean Moon} = \text{Sun} + 12 \left(tithis + \frac{64}{63} \cdot \frac{avamśeṣa}{703} \right) \text{ degrees}$$

$$= \text{S} + (12 \ tithis) \text{ degrees} + \frac{26}{25} \ avamaśeṣa \text{ minutes, nearly.}$$

The form in *BSS,* i. 46 and in *ŚSe,* ii. 44 can be derived from the above.

35. To find the mean longitude of Mars. Let A be the *laghvahargaṇa* or civil days from the beginning of the current year.

Then, the longitude of Mars = *Dhruvaka* of Mars + motion in A days

$$\text{Motion in A days} = \frac{22,96,824}{1,57,79,17,500} \cdot 360 \text{ A degrees}$$

$$= \frac{A}{2} - \left(\frac{A}{2} - \frac{22,96,824}{1,57,79,17,500} \cdot 360 \text{ A} \right) \text{degrees}$$

$$= \frac{A}{2} + \frac{A}{42} - \left(\frac{A}{42} - \frac{37,89,789}{1,57,79,17,500} \text{ A} \right) \text{degrees}$$

$$= \left(\frac{A}{2} + \frac{1}{21} \cdot \frac{A}{2} \right) \text{degrees} + \frac{A}{80} \text{ minutes, nearly.}$$

According to Brahmagupta it is $\frac{11}{21}$ A degrees + $\frac{11}{875}$ A minutes (*BSS,* i. 47 ab).

36. (To obtain) the *śīghrocca* of Mercury add $4+\frac{6}{65}$ degrees daily to its *dhruva* (at the end of the previous year).

(The mean longitude of Jupiter is obtained) by adding to its *dhruva* (at the end of the previous year), the minutes equivalent to 5 times the *ahargaṇa* diminished by its $\frac{5}{354}$ part.

37 The *ahargaṇa* multiplied by 8 and the product divided severally by 5 and 62, gives results, respectively, in degrees and minutes.

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36 ab. To find the longitude of the *śīghrocca* of Mercury. Let A be the *laghvahargaṇa* or civil days during the current year.

So, the required longitude $\doteq$ *Dhruvaka* + motion in A days

$$\text{Motion in A days} = \frac{1,79,37,020}{1,57,79,17,500} \times 360 \times \text{A degrees}$$

$$= 4\text{A} + \frac{6 \times 2,42,762}{1,57,79,175} \times \text{A degrees}$$

$$= 4\text{A} + \frac{6}{65}.\ \text{A degrees, nearly.}$$

Brahmagupta gives the same formula in *BSS*, i. 47 cd.

36 cd. To find the mean longitude of Jupiter. Let A be the *laghvahargaṇa* or civil days during the current year.

Then, the longitude of Jupiter = *dhruvaka* + motion in A days

$$\text{Motion in A days} = \frac{3,64,224}{1,57,79,17,500} \times 360 \times 60.\ \text{A minutes}$$

$$= \left(5 - \frac{5 \times 2,23,491}{5 \times 1,57,79,175}\right) \text{A minutes}$$

$$= \left(5 - \frac{5}{353}\right) \text{A minutes, nearly.}$$

Lalla gives 354. Brahmagupta gives the same form in *BSS*, i. 48 ab ; only he has 354 instead of 353.

37. To find the longitude of the *śīghrocca* of Venus. Let A be the *laghvahargaṇa* or civil days in the current year.

Then, the required longitude = *dhruvaka* + motion in A days

Add these to the *dhruva* (at the end of the previous year) of the *śīghrocca* of Venus. The result is its mean longitude.

38-39. To the *dhruva* of Saturn (at the end of the previous year) add the minutes obtained by increasing twice the *ahargaṇa* by $\frac{1}{315}$ of the product. (The result is the mean Saturn).

To the *dhruva* of the Moon's apogee (at the end of the previous year) add the degrees and minutes obtained from dividing the *ahargaṇa* by 9 and 61, respectively. Add 3 signs to the result. (The sum is the Moon's apogee).

$$\text{Motion in A days} = \frac{70{,}22{,}388}{1{,}57{,}79{,}17{,}500} \times 360 \times A \text{ degrees}$$

$$= \tfrac{8}{5} A \text{ degrees} + \frac{12{,}71{,}880 \times 8}{7{,}88{,}95{,}875} \times A \text{ minutes}$$

$$= \tfrac{8}{5} A \text{ degrees} + \tfrac{8}{62} A \text{ minutes, nearly.}$$

Brahmagupta gives the same formula in *BSS*, i. 48 cd.

38-39. Formulas for Saturn, Moon's apogee and node.

Let A be the *laghvahargaṇa* or civil days in the current year.

Then, Saturn's required longitude = *dhruvaka* + motion in A days

$$\text{Motion in A days} = \frac{1{,}46{,}564}{1{,}57{,}79{,}17{,}500} \times 360 \times 60 \text{ A minutes}$$

$$= 2A + \frac{2 \times 49{,}737}{1{,}57{,}79{,}175} \cdot \text{ A minutes}$$

$$= 2A + \frac{2A}{317} \text{ minutes, nearly.}$$

The formula gives 315. Brahmagupta also gives 315. (*BSS*, i. 49 ab).

Now the Moon's apogee. Let A be the *laghvahargaṇa* or civil days in the current year.

Then, Moon's apogee = *dhruvaka* + motion in A days + 3 signs (since it was 3 signs at the epoch)

$$= \frac{4{,}88{,}219}{1{,}57{,}79{,}17{,}500} \times 360 \times A \text{ degrees}$$

To the *dhruva* of the Moon's node (at the end of the previous year) add the degrees and minutes obtained from dividing the *ahargaṇa* by 19 and 48, respectively. Add 6 signs to the result. Then subtract the sum from 12 signs. (The remainder is the Moon's node.)

[Daily motions of Sun etc.]

40-41. The daily mean motions of the Sun, (Moon, Mars, *Śīghrocca* of Mercury, Jupiter, *Śīghrocca* of Venus, Saturn, and Moon's apogee and node) are, respectively, 59′ 8″, 790′ 35″, 31′ 26″, 245′ 32″, 5′ 0″, 96′ 8″, 2′ 0″, 6′ 41″ and 3′ 11″.

The mean longitudes of planets, (as calculated above), when diminished by half of their respective daily motions, give the longitudes at the previous sunset.

$$=\frac{A}{9}-\left(\frac{A}{9}-\frac{4{,}88{,}219\times 360\times A}{1{,}57{,}79{,}17{,}500}\right)\text{ degrees}$$

$$=\frac{A}{9}\text{ degrees}+\frac{A}{61}\text{ minutes, nearly.}$$

Brahmagupta gives the same ; only instead of 61 it is 67 (*BSS*, i. 49 cd).

Now the Moon's node. Let A be the *laghvahargaṇa* or civil days in the current year.

Then, Moon's node $=$ *dhruvaka* $+$ motion in A days $+$ 6 signs (since at the epoch it was 6 signs).

Since the motion is in the opposite direction, the final result should be deducted from 12 signs.

$$\text{Motion in A days}=\frac{2{,}32{,}226}{1{,}57{,}79{,}17{,}500}\times 360\times A\text{ degrees}$$

$$=\frac{A}{19}-\left(\frac{A}{19}-\frac{2{,}32{,}226\times 360\times A}{1{,}57{,}79{,}17{,}500}\right)\text{ degrees}$$

$$=\frac{A}{19}\text{ degrees}+\frac{A}{48}\text{ minutes, nearly.}$$

Brahmagupta gives 45 instead of 48 (*BSS*, i. 50).

Bhāskara in *MB*, i. 31-40 gives formulas to calculate the positions

[Difference in Terrestrial longitude]

42. The mean longitudes (of the planets calculated above) are for places on the meridian line passing through Laṅkā, Ujjayinī and Himālaya. When to these longitudes are applied corrections for difference in terrestrial longitudes, the results are longitudes for places east or west (of the meridian line).

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of planets during the current year. The forms of these formulas, excepting that of the Sun, are different from those of Lalla. The principle, however, is the same. Śrīpati and Bhāskara II both give formulas to find the longitudes of the planets in the current year. Their formulas, though based on the same principle as that of Lalla, are given in different forms. (*SŚe*, ii. 45-50 ; *SŚi*, I. i. e. 15-21).

42. According to the Indian astronomers the meridian line passing through Laṅkā, Ujjayinī and Himālaya was considered as the prime meridian. Laṅkā is an imaginary place on the intersecting point of the terrestrial equator and the Indian prime meridian. Bhāskara I in *MB*, ii. 1-2, names the following cities on the prime meridian from Laṅkā towards the north : Kharanagara, Sitorugeha, Pāṇāṭa, Misitapurī, Ātaparṇī, Sitavara (mountain), Vātsyagulma, Vananagarī, Avantī, Sthāneśa, and then Meru (north pole).

In *LB*, Bhāskara I does not give all these places but his commentator Śaṅkaranārāyaṇa gives the following names in the commentary on i. 23 : Laṅkā, Kharanagara, Nisitapurī-parvata, Vātsyagulma-nagara, Ujjayinī, Sthāneśvara and Meru.

Āmarāja in his commentary on *Khaṇḍakhādyaka* (*KK*), i. 13, says : 'Lalla gives in his commentary, *Khaṇḍakhādyapaddhati*, (the following places) : Laṅkā, Kumārī, Kāñcī, Pārṇāṭā, Kṛṣṇā, Śvetaśaila, Vātsyagulma, Ujjayinī, Gargarāṭa, Āśraya, Mālavanagara, Āyuśiva, Rohītaka, Kurukṣetra, Himavat and Meru (BMJ, *i.e.*, Babua Misra's ed., p. 27). These names occur in *verses* in some of the MSS of *KK* of the longer recension (*KK*, with Bhaṭṭotpala's commentary, Ed. Bina Chatterji, Volume II, p. 9). Both Bhaṭṭotpala and Āmarāja, in their respective commentaries on *KK*, have quoted Puliśācārya as saying : 'Ujjayinī, Rohītaka, Kuru, Yamunā, Himālaya and Meru are on the prime meridian of Laṅkā'.

43. The (mean) circumference of the earth is 33C0 yojanas. Its diameter is 1050 *yojanas*. The (mean) circumference multiplied by 12 and divided by the hypotenuse of the equinoctial shadow (*palakarṇa*) of a place, gives the corrected circumference of the earth (at that place).

(*KK*, BMJ's ed., p. 26 : *KK*, Bhaṭṭotpala's commentary, Volume II, p. 8).

Śrīpati in *SŚe* has given the following names : Laṅkā, Kumārī, Kāñcī, Pannāṭa, Sitādri, Ṣaḍāsya, Vatsagulma, Māhiṣmatī, Ujjayinī, Āśrama, Paṭṭaśiva, Gargarāṭ, Rohita, Sthāṇvīśvara, Himālaya and Meru (*SŚe*, ii. 95-96).

Bhāskara II in his commentary on the above verse has named the following places : 'Laṅkā, Kumārī, Vatsagulma, Māhiṣmatī, Ujjayinī, Gargarāṭ, Kurukṣetra and Himācala'.

Mallikārjuna in his commentary has given the following names : Laṅkā, Kāñcī, Kumārī, Vatsagulma, Māhiṣmatī, Ujjayinī, Gargarāṭ, Kurukṣetra, Himavat, etc. (text, p. 26).

According to *SS*, i. 62, Laṅkā, Rohītaka, Avantī, Kurukṣetra and Meru are on the meridian.

43. If the circumference of the earth is 3300 yojanas and the diameter is 1050 *yojanas*, $\pi = \frac{3300}{1050} = \frac{22}{7}$.

Āryabhaṭa gives the diameter of the earth in *ĀBh*, *Daśagītikā*, 7 (Sun 4410, Moon 315). Bhāskara I takes $(3299 - \frac{8}{25})$ *yojanas* as the earth's circumference in *MB*, ii. 3-4, but 3299 in *LB*, i. 24. Brahmagupta gives 5000 *yojanas* as the circumference (*BSS*, i. 37). If so, the diameter will be 1581 *yojanas*, taking $\pi = \sqrt{10}$ according to the rule *vyāsavargād daśaguṇāt padaṃ bhūparidhir bhavet*, 'the circumference of the earth is the square root of 10 times the square of the diameter.' In *KK*, Pūrvabhāga, Brahmagupta he gives the circumference as 4800 *yojanas* (*KP*, i. 15) but in *KK*, Uttarabhāga, he gives it as 5000 (*KU*, i. 6. Brahmagupta criticises Āryabhaṭa's measurement of the diameter, but does not give any reason (*BSS*, xi. 15).

According to Śrīpati, the circumference is 5000 *yojanas* and the diameter 1581, (*SŚe*, ii. 94). Bhāskara II says that the circumference is 4967 *yojanas* and the diameter 1581, (*SŚi*, I. i. g. 1 ab).

So $\pi = 3{\cdot}1416$, nearly.

44-45. Ascertain the shortest distance in *yojanas* between a place on the meridian line (passing through Laṅkā) and the observer's station. Multiply the degrees in the difference of latitudes of these two places by 3300 and divide by 360.

Square the result. Subtract it from the square of the *yojanas*

The second part of the verse gives correction to the mean circumference. This follows from the proportion :

Radius : mean circumference :: R sin colatitude (of the place) : ?

or hypotenuse of noon equinoctial shadow : mean circumference :: 12 : ?

This follows from similar triangles U, Z enunciated in the Introductory Notes to Ch. IV, below.

The first correction is given by Bhāskara I in *MB*, ii. 10 cd, and *LB*, i. 24, by Brahmagupta in *KU*, i. 6 and by *SS* in i. 60. Both the corrections are given by Bhāskara II in *SŚi*, I, i. g. 2, and by Śrīpati in *SŚe*, ii. 101.

(found above). Find the square root. This is the distance (in *yojanas*) between the meridian lines passing through the two places.

Multiply it by the mean daily motion of a planet and divide by the corrected circumference of the earth. The result in minutes should

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44-45. Formula for calculating the terrestrial longitudinal correction.

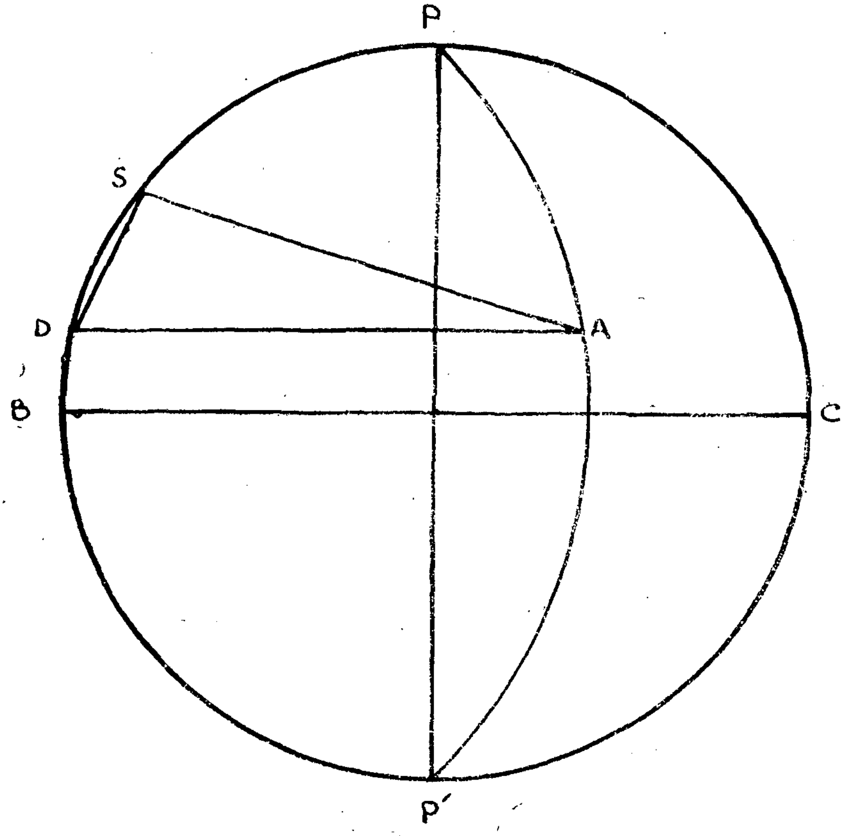

FIG 1

Let PAP′ be the prime meridian, PSP′ the meridian through S, the observer's station, BC the equator, and A, a place on the prime meridian. Let AD be parallel to BC. AD is required. AS is known.

arc SD = arc SB — arc BD

So SD = *yojanas* corresponding to latitude at S — *yojanas* corresponding

be added (to the mean longitude of a planet), for places to the west of the meridian line of Laṅkā and subtracted for places to the east.

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to the latitude at $D = \frac{\text{circumference of earth}}{360} \times$ (latitude at S—latitude at D). Thus SD is known.

The plane triangle ASD is right-angled at D.

So, $AD^2 = AS^2 - SD^2$

and $AD = \sqrt{AS^2 - SD^2}$, which is the correction.

This is known as *Deśāntarayojanas.*

Mallikārjuna in his commentary on these verse says, "By tradition one should know the distance in *yojanas* between the place on the prime meridian and the observer's station" (text, p. 27).

Then, from the proportion :

Corrected circumference of earth in *yojanas*

: daily motion of planet :: AD : ?,

the motion of the planet corresponding to the longitudinal correction is found. This is called *Deśāntaraphala* or simply *Deśāntara.*

The *ahargaṇa* calculated above is for sunrise at Laṅkā. So, the mean longitudes of the planets calculated from this *ahargaṇa* are for sunrise at Laṅkā. When these longitudes are corrected by the respective *Deśāntara* of the planets, the results are their mean places at sunrise on any other meridian line, east or west of the prime meridian line, but at the same latitude as that of Laṅkā. The *Deśāntara* is subtractive in the case of places to the east of the prime meridian and additive to the west of the prime meridian.

This method is given by Bhāskara I in *MB*, ii. 3-4, and *LB*, i. 25-26, 33, by Brahmagupta in *BSS*, i. 37-38, by Śrīpati in *SŚe*, ii. 99-100 and by Bhāskara II in *SSi*, I. i. g. 3. Bhāskara I (*MB*, ii. 5; *LB*, i. 27) and Śrīpati (*SŚe*, ii. 104) have, however, pointed out the incorrectness of this method because of the sphericity of the earth and

because of the probable inaccuracy in determining the distance between two places.

The method to determine the longitudinal correction by means of a lunar eclipse is more trustworthy. The difference between the calculated and the observed time will be the *Deśāntaraghaṭī*. This also is given by Bhāskara I in *MB*, ii. 9, and *LB*, i. 29-30, by Brahmagupta in *BSS*, xvi. 27-28, by Śrīpati in *SŚe*, ii. 105, by Bhāskara II in *SSi*, I, i.g. 4-5, and *SS*, i. 63-65. Bhaṭṭotpala, in his commentary on *KP*, i. 15, quotes the above two verses of Brahmagupta and states that this method of calculation is more accurate (*KK*, II, p. 10).

Thus ends Chapter One

on

MEAN PLANETS

CHAPTER II

TRUE SUN AND MOON

[R sines and Versed sines]

1-4. 225, 449, 671, 890, 1105, 1315, 1520, 1719, 1910, 2093, 2267, 2431, 2585, 2728, 2859, 2978, 3084, 3177, 3256, 3321, 3372, 3409, 3431 and 3438 minutes are, respectively, the R sines (or *jyās* of arcs of 225', 450', 675' etc.) as given by Āryabhaṭa.

5-8. 7, 29, 66, 117, 182, 261, 354, 460, 579, 710, 853, 1007, 1171,

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MATHEMATICAL NOTES

1-8. These verses give the tables of R sines and R versed sines.

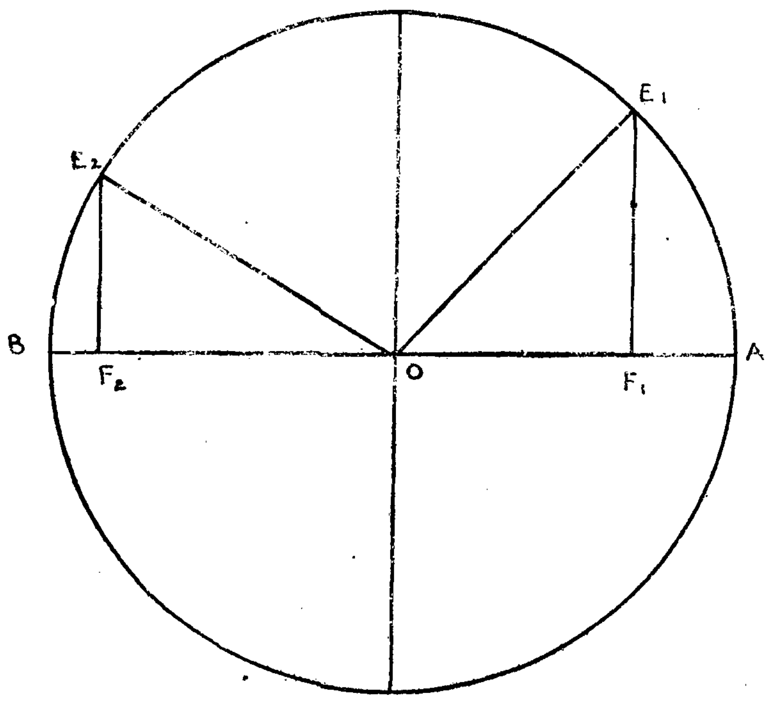

FIG. 2

1345, 1528, 1719, 1918, 2123, 2333, 2548, 2767, 2989, 3213 and 3438 minutes are, respectively, the R versed sines (or *utkramajyās* of 225', 450', etc.).

It is said that 3438, the R sine (*jyā*) of an arc of 90°, is also the radius of the circle whose circumference is 360° or 21600'.

9. It is given that the square of the R sine of an arc of 90°, (=3438), is 1,18,19,844. The R sine of an arc of 24° is 1397. The Sun's apogee is 78°.

[**Sine and Cosine**]

10. When the apogee (*mandocca* or apex of slow motion) of

According to Indian conception, $E_1 F_1$, $O F_1$ and $A F_1$ in the above figure are, respectively, the *jyā, koṭijyā* and *utkramajyā* of arc AE_1. Similarly, $E_2 F_2$, $O F_2$ and $B F_2$ are, respectively, the *jyā, koṭijyā* and *utkramajyā* of arc $\overline{AE_2}$. And so on. The radius or *jyā* 90° is *trijyā*. *Jyā* is, therefore, radius multiplied by modern sine, or R sine ; *koṭijyā* is R cosine ; and *utkramajyā* is R versed sine.

As Lalla himself acknowledges, the table of R sines is the same as that given by Āryabhaṭa in *ĀBh, Daśagītikā*, 12. Bhāskara I has also used this table. In *MB*, vii. 16, he says, *jīvā makhyādayo matāḥ*, 'the R sin differences are *makhi* (225), etc.'

Both Āryabhaṭa and Bhāskara II (*SŚi*, I, ii. 3-9), take 3438 units as the radius of the circle with a circumference of 360°. This will be so when $\pi = 3.1416$. *SS* uses the same table (ii. 17-27) Brahmagupta takes 3270 units as the radius (*BSS*, ii. 5) and Śrīpati 3415 units (*SŚe*, ii. 6) when the circumference is 360°.

For a detailed analysis of Āryabhaṭa's table of R sines see Appendix VI in volume I of *KK* with Bhaṭṭotpala's commentary (pp. 176-221).

9 cd. See Table III, below.

10-11. The concentric is divided into 4 anomalistic quadrants

a planet is deducted from its longitude, the remainder is called its mean anomaly (*mandakendra*, after *mandocca*). When the longitude of a planet is deducted from iis *śīghrocca* (or apex of quick motion) the remainder is called *śīghrakendra*, (after *śīghrocca*). Each anomalistic quadrant consists of three signs. The odd and even quadrants are, respectively, called *bhuja* and *koṭi*.

11. When an arc is greater than 90° (but less than 180°), subtract it from 180°. When it is greater than 180° (but less than 270°), subtract 180° from it. When it is greater than 270°, subtract it from 360°. The remainder in each case is known as the *bhuja* of the arc. So say the (wise astronomers), who know.

12. (To find the R sine of a given arc) express the arc in minutes. Divide it by 225. (The quotient gives the number of arcs of 225′) already passed. Multiply the remainder by the difference of the last R sine passed and the next to be passed and divide the product by 225. To the result add the R sines of the arcs passed. (The sum is the required R sine).

13. (To find the arc corresponding to a given R sine), subtract from it as many R sines (of arcs of 225′) as possible. Multiply the remainder by 225 and divide by the difference between the R sine last subtracted and the next R sine. Add the result to the product of 225 and the number of R sines subtracted. The sum is the required arc.

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or *pāda-s* of 90° each and into 12 anomalistic signs of 30° each. The signs are given the same names as the signs of the zodiac, namely, Aries, Taurus, etc. The same definitions of *bhuja* and *koṭi* are given in other astronomical treatises also.

12-13. These give rules for the determination of the R sine of a given arc and vice versa by means of simple ratio. These are the general methods.

Brahmagupta, however, is the *first mathematician in the world* to use the method of interpolation up to the second order (*KU*, i. 4 ; see edn., vol. I, p. 73, pp. 137-39 ; vol. II, pp. 177-81). The same

[True Moon]

14. Multiply the R sines of the mean anomalies (of the Sun and the Moon) by 3 and 7, respectively, and divide each product by 80. The results (called *mandaphalas* or corrections to be given to the mean longitudes) are in minutes. Add or subtract the results (to or from the mean longitudes of the Sun and Moon) according as their mean anomalies are within 6 signs beginning from Libra or within 6 signs beginning from Aries, (that is, greater or less than 180°). Thus are obtained their true longitudes.

[True motion of Sun and Moon]

15. Divide the *bhogyakhaṇḍa* of the Sun by 101 and multiply that of the Moon by 10 and divide by 33. (The results called *manda-gatiphalas* or corrections to be given to the mean motions) should be applied to their respective mean motions negatively, if the respective mean anomalies are within six signs beginning with Capricorn, and positively if they are within six signs beginning with Cancer ; (that is, negatively, positively, positively or negatively, according as their mean anomalies are in the first, second, third or fourth quadrant). Thus are obtained their true motions.

[Correction for longitude and precession]

16. The mean daily motion of a planet multiplied by the Sun's equation of the centre in minutes (*mandaphala*) and divided by 21,600,

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verse occurs in the *Dhyānagraphopadeśādhyāyā* of *BSS*, xxv. 17. Bhāskara II follows this remarkable method and proves it in his on *SŚi*, I. ii. 16.

14-15. The methods to find the true longitudes of the Sun and Moon and their true motions are explained in Appendix III, below.

16. The formulas in the first chapter give the mean longitudes of the Sun, Moon and planets at the time when the mean Sun rises. So, these longitudes must be corrected to obtain those at the time when the True Sun rises. This correction is known as *Bhujāntara* or correction for the equation of time due to the eccentricity of the ecliptic. The

gives in minutes, etc. the correction for the difference between the Sun's mean and true longitudes or *bhujāntaraphala.* It should be added to or subtracted from the longitude of the planet, according as the equation of the centre is added to or subtracted from the Sun's longitude.

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difference between the True Sun and the Mean Sun is the *mandaphala* or equation of centre. So, one must obtain the motion of planets during the time taken by the celestial sphere to turn through an arc equal to the Sun's *mandaphala.* This follows from the proportion :

360·60 : planet's motion :: *mandaphala* : ?

The longitude of the planet corrected by this result is the longitude at the time of true sunrise.

This rule is given in *MB,* iv. 7, *LB,* ii. 4 cd and 22; *BSS,* ii. 29 ab; *SŚe,* iii. 46 ab; and in *SS,* ii. 46. Bhāskara II's rule (*SŚi,* I, ii. 61) is more correct.

While commenting on verse 16, Mallikārjuna has given a rule for determining *ayanāṃśa* or precession of the equinox. The verse is his own. The rule is as follows : The total number of years from the beginning of the *Kaliyuga,* multiplied by 3 and divided by 200, gives a result in degrees. This should again be divided by 54 and the remainder, say x, is subtracted from 54. The result is, say, y. Of x and y, the smaller amount is the *ayanāṃśa.*

$$\frac{3}{200}=\frac{54}{60\times 60}\text{degrees.}$$

Thus, Mallikārjuna takes 54″ as the annual rate of precession. Then, according to the belief of one school that the equinox has a pendular movement, 27° to the west and back and then again 27° to the east and back, when $\frac{3}{200}$ is multiplied by the solar years and divided by 54 the remainder is the precession. If 54 *minus* this remainder is less than the remainder, then this difference is the precession.

Mallikārjuna then adds that the precession is additive in the even quadrants and subtractive in the odd. This can be determined by observation. If the midday shadow when the Sun is on the first point

17. The R sine of the true longitude of the Sun multiplied by the R sine of 24° and divided by the radius gives the R sine of the declination. This multiplied by the equinoctical midday shadow and divided by 12, gives the earth-sine or *kṣitijyā*.

[Ascensional difference]

18. The square root of the difference of the squares of the R sine of the declination and of the radius is the radius of the day-circle. The product of the earth-sine and radius divided by the radius of the day-circle, gives R sine of the ascensional difference.

19. The corresponding arc is the ascensional difference in *asus* (*caradala*). The daily motion of a planet multiplied by the ascensional difference (at the observer's station) and divided by the number of minutes in 360° (=21,600′), gives in minutes (the planet's *caraphala* or

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of Aries is longer than that when the Sun is on the first point of Libra, the precession is additive ; otherwise subtractive. Mallikārjuna quotes this very verse in his commentary on *SS*, ii. 28. There he adds another verse :

meṣasaṅkrama-madhyāhnacchāyto'bhyadhikā yadi |
tulāmadhyandinachhāyā calāṃśāḥ syur dhanaṃ tathā. ||

17-19. These verses give the method to find the ascensional difference or half the variation of the day from 30 *ghaṭikās* or *caradala*.

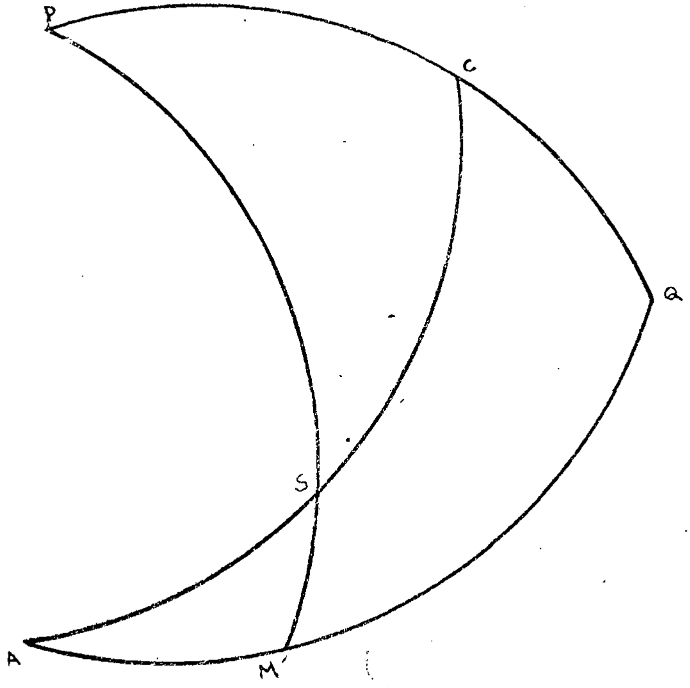

FIG. 3

Let AC be the ecliptic and AQ the celestial equator. Let P be the pole and S the Sun. AS is the Sun's longitude. Let PSM be a secondary through P and S, and PCQ a secondary through P and C, so that arc AC is 90°. Then SM is the Sun's declination, CQ the obliquity of the ecliptic or 24°, and PC 65°.

$$\text{Then, R sin declination} = \frac{\text{R sin AS} \times \text{R sin CQ}}{\text{R sin AC}}$$

$$= \frac{\text{R sin longitude} \times \text{R sin } 24^\circ}{\text{R}}.$$

The triangle with sides R sin declination, 'earth-sine' and R sin amplitude is similar to the triangle with sides 12 gnomon, equinoctial midday shadow and hypotenuse of equinoctial shadow.

or correction for ascensional difference). It should be added to the longitude of the planet at sunset and subtracted at sunrise, if the Sun is in the half of the ecliptic beginning with Aries. If it is in the half beginning with Libra, the process must be reversed.

$$\text{So, earth-sine} = \frac{\text{R sin declination} \times \text{equinoctial shadow}}{12}$$

$$\text{Now, day-radius} = \sqrt{\text{R}^2 - (\text{R sin declination})^2}$$

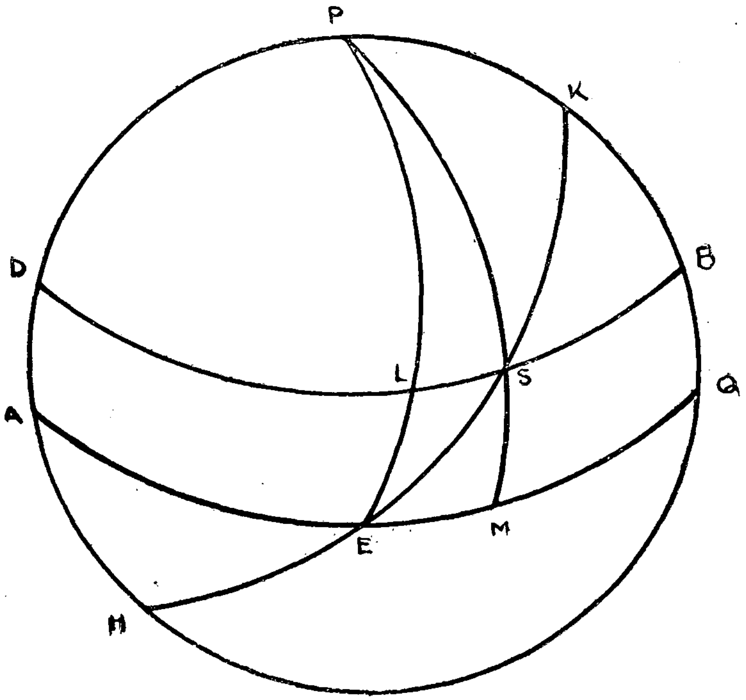

Fig. 4

Let AEMQ be the celestial equator and P its pole. Let DSB be the day-circle of the Sun at S. Let H E S K be the horizon, P L E the six o'clock circle, and PSM the secondary to the equator through S.

[Let R be the radius of the celestial sphere and R' the radius of the Sun's day-circle.]

[Measure of day and night]

20-21. (The ascensional difference) in *asus* divided by 6

Then R' sin arc LS is earth-sine and R sin arc EM is the R sin ascensional difference.

$$\text{Now, } \frac{\text{R sin arc EM}}{\text{R' sin arc LS}} = \frac{\text{R sin arc PM}}{\text{R sin arc PS}}$$

$$\text{So, } \text{R sin (ascensional difference)} = \frac{\text{R. earth-sine}}{\text{radius day-circle}} \cdot$$

Hence the ascensional difference in *asus*.

Then from the proportion :

21,600 *asus* (=60 *ghaṭikās*) : planet's daily motion=ascensional difference : ?,

the motion of the planet during this time is obtained. This is known as *caraphala*.

The longitude of a planet at sunrise or sunset at the latitude of Laṅkā, when corrected by its *caraphala*, gives the same at sunrise or sunset at the observer's latitude.

For the proof of the method of application see, below, *ch.* xvi, *vv.* 9-10.

The same method for calculating and applying *caraphala* is given in *MB*, iii. 6-7 and iv. 26-27, in *LB*, ii. 16-20, in *BSS*, ii. 55-59, in *SŚe*, iii. 63-67, 69, in *SŚi*, I. ii. 47-49 ab, 53 and in *SS*, ii. 28, 61-63.

In the commentaries on the above verses, both Mallikārjuna (text, p. 35) and Bhāskara II have said that the Sun's longitude should be first corrected for the precession of equinox and then used for further calculations.

20-21. To find the length of a day and night at any latitude by using the ascensional difference. This is the method followed in other

becomes *vinādī* ; and (further) divided by 60, it becomes *ghaṭikā*. When the ascensional difference in *ghaṭikā* is added to and subtracted from 15 *ghaṭikās*, and the results doubled, the lengths of the night and day are obtained (at the observer's station), provided the Sun is in the southern hemisphere beginning with Libra. The same give the lengths of the day and night, if the Sun is in the northern hemisphere beginning with Aries.

[Computation of Tithi]

22. When the true longitude of the Moon is lessened by that of the Sun, and the remainder, expressed in minutes, is divided by 720, the quotient gives the number of *tithis* passed. (The remainder, which is the portion of the current *tithi*) passed, when divided by the difference of the true motions of the Sun and Moon, gives the day (since the *tithi* began).

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works. Bhāskara II in his commentary on the above verses has defined *prāṇa* or *asu*. He says that the time of the breath of a healthy man is *prāṇa*.

22. This is the usual formula for calculating the *tithis*, based on the definition of a *tithi*. During this period the Moon gains 12° over then Sun. If T denotes *tithis*, then,

$$T = \frac{(M-S)}{12^\circ\ (=720')} \text{ minutes.}$$

The remainder gives the portion of the current *tithi* passed, and 720′ *minus* remainder gives the portion to elapse. Both these portions divided by the difference of the motions of the Sun and Moon, would give the times.

This is the usual procedure and is given in all astronomical treatises. In addition to this method, Brahmagupta has given another in *BSS*, xiii. 23-25, which is followed by Śrīpati in *SŚe*, iii. 72-74.

[Computation of Yoga]

23. The true longitude of a planet expressed in minutes, and divided by 800, gives the number of *nakṣatras* (beginning with *Aśvinī*, passed by the planet). When the portions of the current *nakṣatra* passed and to be passed are divided by the true motion of the planet, the results are days (elapsed since the planet entered the *nakṣatra* and to elapse till the planet completes it).

When the sum of the true longitudes of the Sun and Moon, expressed in minutes, is divided by 800, the quotient gives the number of *yogas* passed. (When the portions of the current *yoga* passed and to be passed) are divided by the sum of the true motions of the Sun and Moon, (the results are, respectively, the number of days elapsed since the current *yoga* began and to elapse till it lasts).

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23 ab. This formula is based on the general assumption that the zodiac is equally divided among 27 *nakṣatras* or asterisms, each occupying 800′. Thus, the longitude of a planet in minutes divided by 800′ gives the number of *nakṣatras* passed, and the remainder gives the portion of the current *nakṣatra*. That subtracted from 800′ gives the portion of the *nakṣatra* to be passed. Hence the days can be calculated by dividing the portions by the motion of the planet. This is the usual rule followed by the astronomers.

Brahmagupta has also given another method in *BSS*, xiv. 46-54, where the zodiac is unequally divided among 28 *nakṣatras*, the addition being *Abhijit*. The extension of 15 each is the same as the motion of the Moon, 790′ 35″; the extension of 6 each is $\frac{1}{2}$ of 790′ 35″ ; and the extension of another 6 is $1\frac{1}{2}$ of 790′ 35″ each ; and the remaining portion of the zodiac is that of Abhijit. Brahmagupta adds that this division is given in *Pauliśa*, *Romaka*, *Vāsiṣṭha*, *Saura* and *Paitāmaha Siddhāntas* and also in the *Gargasaṃhitā* and other Saṃhitās. This division is also given by Brahmagupta in *KU*, i. 7-11 (vol. II, pp. 183-186).

In 51 cd-52 ab, Brahmagupta gives another method for

[Computation of Karaṇa]

24-25. Subtract the true longitude of the Sun from that of the Moon and divide the remainder, expressed in minutes, by 360. Subtract 1 from the quotient and divide the remainder by 7. (The remainder gives) the *karaṇa* (beginning with *Bava*). Remember that the method (to find the time etc.) is the same as that in the case of *tithis*.

Abhijit. Śrīpati gives both these methods in *SŚe*, iii. 78-80, 81 (nod. 77) and Bhāskara II only the first in *SŚi*, I, ii. 71-75. For proof Bhāskara says that it is the traditional belief.

23 cd. The formula in respect of *yoga* is based on the definition of a *yoga*. This is the period during which the Sun and Moon together complete 800′ of the zodiac. Hence $yoga = \frac{(S+M)}{800'}$ minutes. The quotient gives the number of *yogas* passed and the remainder the portion of the current *yoga* passed. That subtracted from 800′ gives the portion to be passed. These portions, divided by the sum of the motions of the Sun and Moon give the corresponding times. This is the usual method followed by astronomers.

There are 27 *yogas*. They are *Viṣkambha, Prīti, Āyuṣmān, Saubhāgya, Śobhana, Atigaṇḍa, Sukarmā, Dhṛti, Śūla, Gaṇḍa, Vṛddhi, Dhruva, Vyāghāta, Harṣaṇa, Vajra, Siddhi, Vyatīpāta, Varīyān, Parigha, Śiva, Siddha, Sādhya, Śubha, Śukla, Brahmā, Indra* and *Vaidhṛta*.

Lalla has given this list in his *Ratnakośa* in the Section *Yogaprakaraṇa*.

24-25. A *karaṇa* is half of a *tithi*. There are 60 half *tithis* in a lunar month and each *karaṇa* is of the duration during which the Moon gains 6° over the Sun. 4 *karaṇas*, as given in the verse, are

(In the dark half of the lunar month) when the Moon decreases, the second half of the fourteenth *tithi* is always called *Śakuni*. Following *Śakuni* come *Catuṣpada* and *Nāga*, the first and second half, respectively, of the *amāvasyā*. *Kiṃstughna* is the first half of the next *tithi* *Pratipad*. So (the wise) say.

[New and Full Moon]

26-27. The true motions of the Sun and Moon should be multiplied by the *ghaṭikās* passed since the end of the fifteenth *tithi* and divided by 60. One should subtract the results (in minutes) from the respective minutes (in their true longitudes). The longitudes then have the same signs, degrees, minutes and seconds, if the fifteenth *tithi* is *amāvasyā* (New Moon) and the same degrees, minutes and seconds, if it is *pārṇimā* (Full Moon).

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fixed. Out of the remaining 7 movable ones, each occurs 8 times in a lunar month. Hence the formula.

As regards the four fixed *karaṇas*, Bhāskara II says in his commentary, 'these *karaṇas* are so placed according to tradition. There does not appear to be any specifie reason'. This is in general the method for determining the *karaṇas*.

The movable *karaṇas* are *Bava*, *Bālava*, *Kaulava*, *Taitila*, *Gara*, *Vaṇij* and *Viṣṭi*.

26-27. On the day of *amāvasyā* or new Moon, the Sun and Moon coincide, and on the day of *pūrṇimā* or Full Moon, their distance is 180°. At the end of a *tithi* their longitudes differ by multiples of 12°.

Hence follow the rules to calculate their true longitudes at these times.

(The results obtained in the same manner from the) *ghaṭikās* to elapse till the fifteenth *tithi*, should be added.

(The same procedure should be followed for any other *tithi*). At the end of every other *tithi*, the Sun and Moon are equal in respect of minutes and seconds.

Thus ends Chapter Two

on

TRUE SUN AND MOON

CHAPTER III

TRUE PLANETS

[Degrees of Apogees and Anomalies of the planets]

1. The apogees of Mars, Mercury, Jupiter, Venus and Saturn are, respectively, 118°, 210°, 180°, 90° and 236°.

The (circumferences of their *manda* epicycles divided by $4\frac{1}{2}$) or *mandaguṇakas* are 14°, 7°, 7°, 4° and 9°, respectively.

The (circumference of their *śīghra* epicycles divided by $4\frac{1}{2}$) or *śīghraguṇakas* are, respectively, 53°, 31°, 16°, 59° and 9°.

2. Multiply the R sines of the anomalies (*mandakendras*) of Mars, Mercury, Jupiter, Venus and Saturn, respectively, by 4, 2, 1, 2 and 4 and divide each product by the radius (3438). The results added to the *mandaguṇakas* (see above) of Mars, Jupiter and Saturn and subtracted from those of Mercury and Venus, make them (*viz.*, *mandaguṇakas*) more correct.

MATHEMATICAL NOTES

1-2. Āryabhaṭa gives the same apogees in *ĀBh, Daśagītikā,* 9 cd. (See Table III, below).

While commenting on the apogees, Mallikārjuna says that there is an alternate reading according to which the apogees of Mars, etc. are 128°, 220°, 172°, 80° and 238°, respectively. The sentence quoted by Mallikarjuna occurs in Va also. But there is no commentary on this reading by Bhāskara II. Bhāskara II in his commentary on these two verses says that as the motion of the apogees is very little during a comparatively long period, the Ācārya reads them as if they were fixed.

Multiply the R sines of the *śighrakendras* of (the above planets), respectively, by 2, 2, 1, 2 and 1 and divide each product by the radius. The results subtracted from their respective *śīghraguṇakas* (see above), make them more correct.

According to Āryabhaṭa, the *mandaparidhis* or *manda* epicycles of Mars, Mercury, Jupiter, Venus and Saturn, each divided by 4½, are, respectively, 14,° 7°, 7°, 4°, and 9° in the first and third quadrants and 18°, 5°, 8°, 2° and 13°, respectively, in the second and fourth quardants (*ĀBh*, *Daśagītikā*, 10-11 ; see Table IV, below).

Lalla gives the first series as *mandaguṇakas*, as he calls them. Then he gives the method to find the change in the circumference for every *mandakendra* or mean anomaly. It follows from the proportion :

Radius : Āryabhaṭa's difference between the odd and even quadrants
: : R sin mean anomaly : ?

The result added to or subtracted from the given *mandaguṇaka* gives the corrected *mandaguṇaka* or *sphuṭa-mandaguṇaka*. It is added when the *mandaparidhis* in the first and third quadrants are less than those in the second and fourth ; otherwise it is subtracted.

Again, according to Āryabhaṭa, the circumferences of the *śīghra* epicycles or *śīghraparidhis* of Mars, Mercury, Jupiter, Venus and Saturn, each divided by 4½, are, respectively, 53°, 31°, 16°, 59° and 9° in the first and third quadrants and 51°, 29°, 15°, 57° and 8°, respectively, in the second and fourth quadrants (*ĀBh*, *Daśagītikā*, 10-11 ; See Table V, below).

Lalla gives the first series as *śīghraguṇakas*, as he calls them. Then he gives the proportion from which one finds the correction for every *śīghrakendra*. It is :

Radius : Āryabhaṭa's difference between the odd and even quadrants :: R sin *śīghrakendra* : ?

[Manda and Śīghra hypotenuse]

3. The square root of the difference of the squares of the R sine of 90° and R sine of an arc, is the R cosine (*koṭijyā*) of the arc. Or, the R sine of the difference of 90° and an arc is the R cosine (*koṭijyā*) of the arc.

When the R sine and R cosine of the *mandakendra* and the *śīghrakendra* of a planet are multiplied by the planet's corrected *mandaguṇaka* and *śīghraguṇaka*, respectively, and divided by 80, the results are, respectively, called *doḥphala* and *koṭiphala*. From these the hypotenuse or *hṛti* (or *karṇa*) should be calculated.

[Computation of True planets]

4-7. Subtract the longitude in degrees of the apogee of a planet from its mean longitude. (The remainder is the mean anomaly. It should be reduced to the) first quadrant. Find its R sine and hence the *doḥphala* (as given above). Find the corresponding arc, (which is the *mandaphala*). Add or subtract half of it to or from the mean longitude of the planet, according as the mean anomaly is (greater or less than 180°). (The result is the longitude of the planet after the first correction) or *anaṣṭa*.

Subtract (*anaṣṭa*) from the longitude of the planet's *śīghrocca*. The remainder is called *śīghrakendra*. Find its R sine and R cosine and hence the *doḥphala* and *koṭiphala* (as above). Add or subtract the *koṭiphala* to or from the radius, according as the *śīghrakendra*

This result subtracted from the *śīghraguṇaka* gives the corrected *śīghra-guṇaka* or *sphuṭa-śīghraguṇaka*.

The rule for correcting the circumference of *manda* and *śīghra* epicycles is given in *MB*, iv. 38-39 ab, *LB*, ii. 31-32, *BSS*, ii. 13, *SŚe*, iii. 22 and in *SS*, ii. 38. Bhāskara II has discussed this point in *ŚSi*, I, ii. 23-25.

3-8. See Appendix IV, below.

is in the first and fourth or second and third quadrants. (The result is called *sphuṭakoṭi*). The hypotenuse or *karṇa* is the square root of the sum of the squares of the *doḥphala* and *sphuṭakoṭi*.

Multiply the *doḥphala* by the radius and divide by the hypotenuse. The arc corresponding to the result as R sine is called *śīghraphala*. Add or subtract half of the *śīghraphala* to or from the once-corrected longitude of the planet (*anaṣṭa*), according as the *śīghrakendra* is in the first and second quadrants or in the third and fourth. (The result is the longitude of planet after the second correction).

From the twice-corrected longitude calculate the *mandaphala* as before and apply the whole of it to the *mean* longitude. (From the longitude of the planet thus corrected) calculate the *śīghraphala* as before and apply the whole of it to the (thrice-corrected longitude of) the planet. The result is the true longitude of the planet.

8. Or, (first calculate the *śīghraphala* from the planet), and apply half of it to the longitude of the planet. Then (calculate the *mandphala* from the corrected longitude) and apply half of it to the corrected longitude. Then, as before, calculate and apply the whole of the *mandaphala* and *śīghraphala*. Thus corrected, the planet's true longitude is obtained.

9. Some say that the true longitudes of Mercury and Venus (should be obtained in the following manner) : The *mandaphala* (calculated) from the difference between the mean planet and its

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9. This verse gives a method for Mercury and Venus only. This is as follows : First calculate the *mandaphala* from the difference in the longitude of the mean planet and that of its apogee. Apply the whole of it to the longitude of the planet according to the rule given before. Then calculate the *śīghraphala* from the difference in the longitude of the *mean planet* and that of its *śīghrocca*. Apply the whole of it to the corrected planet.

In his commentary Mallkārjuna does not say that the method should be *repeated*, as given in Bhaskara II's commentary on this verse.

apogee, should be first applied to the planet. Then the *śīghraphala* (calculated) from the difference between the planet and its *śīghrocca*, should be applied to the corrected planet. (The results are correct Mercury and Venus).

10. When the true longitude of Venus (as calculated above) is diminished by half the maximum *mandaphala* of the Sun, the longitude is correct.

Subtract the Sun's apogee from the *śīghrocca* of Mercury. From the remainder calculate the *mandaphala* using the Sun's equations. Apply it to the (twice-corrected) Mercury in the same manner as it would be applied to the Sun. (The result is more correct).

[Manda and Śīghra motion]

11-14. The mean motion of a planet multiplied by the *bhogyakhaṇḍa* resulting from its mean-anomaly and also by the

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A similar rule is given by Brahmagupta in *BSS*, ii. 35, not only for Mercury and Venus but also for Jupiter and Saturn. The process there is repeated. While commenting on it, Pṛthūdaka says that the once-corrected planet is called '*madhyama*' by Brahmagupta but we shall call it '*mandasphuṭa*'. This *mandasphuṭa* should be subtracted from the *śīghrocca* of the planet. (*BSS*, R.S. Sharma's edition, p. 204). Bhāskara II gives the rule as interpreted by Pṛthūdaka in *SSi*, I, ii. 34 cd-35 ab. In this connection, see the paper, '*The Pañcasiddhāntikā* etc.', *Gaṇita*, June, 1973.

10. The tenth verse gives further corrections to Mercury and Venus.

In the very beginning of the chapter Mallikārjuna says (text, p. 41) that various methods for calculating the true longitude of a planet had to be prescribed from time to time so that there is accord between the calculated and observed results. *Āryabhaṭa*, therefore, *prescribed several methods*. Mallikārjuna, however, neither mentions these methods

corrected *mandaguṇaka* and divided by 225 and 80 gives in minutes the correction to motion or *mandagatiphala.* It is to be applied to the mean motion as explained before. (The result is the motion corrected once).

Subtract it (*viz.*, corrected motion) from the motion of the *śīghrocca* of the planet. The remainder multiplied by the *bhogyakhaṇḍa* resulting from the *śighrakendra,* the radius and the corrected *śīghraguṇaka* and divided by 225, 80 and the hypotenuse, gives the second correction of *śīghragatiphala.*

When it is applied to the once-corrected motion, (i) positively, (ii) negatively, (iii) negatively and (iv) positively, according as the *śīghrakendra* is in the first, second, third or fourth *pāda* (see the next verse), the result is the true motion.

When the result is negative, the motion is said to be retrograde.

The first *pāda* extends from 0° to 90° *plus* 45°, 31°, 20°, 50° and 6° for Mars, Mercury, Jupiter, Venus and Saturn, respectively. The third *pāda* extends from 180° to 270° *minus* 45°, 31°, 20°, 50° and 6°, respectively, for these planets. This definiton of *pāda* should be kept in view while applying the *śīghragatiphala* positively or negatively to the motion.

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nor names the works wherein these are mentioned. They are not in *ĀBh.*

While commenting on the tenth verse, Mallikārjuna states that these special methods were given by some of the pupils of Āryabhaṭa. But Bhāskara II in his commentary on the above two verses does not say anything. See also the article refered to in verse 9.

11-13. See Appendix V, below.

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14. The following table illustrates the rule :

Planets	*Śīghrakendra* (in degrees)	*Pāda*	*Śīghragatiphala*
Mars	0—135	1	+ ve
	135—180	2	— ve
	180—225	3	— ve
	225—360	4	+ ve
Mercury	0—121	1	+ ve
	121—180	2	— ve
	180—239	3	— ve
	239—360	4	+ ve
Jupiter	0—110	1	+ ve
	110—180	2	— ve
	180—250	3	— ve
	250—360	4	+ ve
Venus	0—140	1	+ ve
	140—180	2	— ve
	180—220	3	— ve
	220—360	4	+ ve
Saturn	0— 96	1	+ ve
	96—180	2	— ve
	180—264	3	— ve
	264—360	4	+ ve

Mallikārjuna in his commentary explains that the correction to the motion should follow the same pattern as that to the longitude of a planet. Accordingly, at first, half of the *mandagatiphala* should be applied, then half of the *śīghragatiphala* ; then both again completely, in the same order.

[Motion of the Moon and other planets]

15. If the mean longitude of a planet is greater than its true longitude, add half of their difference to the planet's *śīghrocca* ; otherwise subtract. From this *śīghrocca* subtract the true longitude. If the remainder is within 4, 6 and 8 signs, the motion of the planet is known as *vakra* (beginning of regression), *ativakra* (maximum regression) and *kuṭila* (end of regression), respectively.

16. The pupils of Āryabhaṭa calculated the *mandagatiphala* or correction to the Moon's motion by dividing the motion of its mean anomaly by 225. But this method gives the true motion for the previous day. It should not be used to calculate the motion for the present day.

17. In the case of the Sun and Moon, multiply their *mandaguṇakas* by their respective *manda* hypotenuse and divide each product by the radius. The results are corrected *mandaguṇakas*. Hence

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Mallikārjuna notes here that on the last days of the first and fourth *pādas*, the *śīghragatiphala* is positive and maximum, and on the last days of the second and third *pādas*, the *śīghragatiphala* is negative and maximum. On these days, the *mandasphuṭabhukti* becomes the true motion.

15. This verse defines *Vakra*, *Ativakra* and *Kuṭila* motions of a planet. The same definition is given in *MB*, iv. 56-57, *BSS*, ii. 50-50, and in *ŚSe*, iii. 59. According to *SS*, ii. 12 there are 8 kinds of motion of planets. They are retrograde, maximum retrograde, end of retrograde, slow, slower, mean, fast and faster. The last five are direct and the first three are indirect or retrograde.

16. While commenting on this verse, Bhāskara II says that Lalla here criticises the methods given by Prabhākara and others.

17-18. She Appendix IV. below.

find their *doḥphalas* and *koṭiphalas* and hence their *manda* hypotenuse. Repeat the process till the *mandaguṇakas* and the hypotenuse are fixed.

18-19. When the mean daily motion of a planet is multiplied by the radius and divided by the *manda* hypotenuse, the result is the motion once-corrected.

Subtract it from the motion the *śīghrocca*. (The remainder is the motion of the *śīghrakendra*). Multiply it by the radius and the *bhogyakhaṇḍa* resulting from the *śīghraphala* and divide by the *śīghra* hypotenuse and 225, the R sine of the first arc. Subtract the result from the motion of the *śīghrocca*. The remainder is always the true motion of the planet (by another method).

If the above quotient is greater than the motion of the *śīghrocca*, the planet is said to have a retrograde daily motion.

While commenting on these verses Mallikārjuna says (text, pp. 49-53), that these methods for calculating the true motions of the Sun, Moon and the five planets give more accurate results. He continues that the methods for calculating the true longitude are different in different *Siddhāntas*, *Tantras* and *Karaṇas*, simply because *the teachers wanted to synchronize the calculated and the observed results.* There again, the methods given in the *Somasiddhānta* are respected by many. The methods given in the *Sūryasiddhānta* for calculating the mean planets are agreed to by all. As regards the *Khaṇḍakhādyakaraṇa*, here the positions of the mean planets as in the *Sūryasiddhānta* have been used, after giving some corrections, for obtaining their true longitudes in accordance with the *Somasiddhānta*.

Then Mallikārjuna gives twelve verses composed by him setting out the rules to find the true longitudes and motions of the planets which would agree with those in the *Somasiddhānta*. His constants are as follows : The apogees given by him are : Mars 127°, Mercury 220°, Jupiter 170°, Venus 80° and Saturn 240°.

[Retrograde motion]

20. The retrograde motion of the planets beginning with Mars (*viz.*, Mars, Mercury, Jupiter, Venus and Saturn) commences when their *śīghrakendras* are, respectively, 163°, 145°, 125°, 165° and 113°. When the *śīghrakendras* are, respectively, 360° *minus* each of these values, their retrograde motion ceases.

21. It is said that the retrograde motion of Mars, (Mercury, Jupiter, Venus and Saturn) lasts for 66, 21, 112, 52 and 134 days, respectively.

22-25. When the *śīghrakendras* (of Mars, Mercury, Jupiter, Venus and Saturn) are, respectiively, 28°, 205°, 14°, 183° and 20°, they rise in the east. When their *śīghrakendras* are 360° *minus* these values, respectively, they set in the west.

When their *śīghrakendras* are, respectively, 51° and 23°, Mercury and Venus (when direct) rise in the west. When they are 360° *minus* these values, respectively, they set in the east.

Mercury and Venus, once set, rise again 32 and 71 days, respectively, after setting.

For 120, 16, 30, 8 and 36 days, respectively, Mars, etc. remain invisible in the west. They set 660, 37, 372, 251 and 342 days, respectively, after rising.

He calculates *mandaphala* as equal to $\frac{\text{R sin mean anomaly} \times 10}{\textit{manda} \text{ divisor}}$.
These divisors are Sun 263, Moon 114, Mars 51, Mercury 128, Jupiter 112, Venus 263 and Saturn 75.

He calculates the *śīghraphala* as given in the text. His *sīghraguṇakas* are, however, Mars 51° 55′, Mercury 31° 15′, Jupiter 16° 3′, Venus 57° 57′ and Saturn 8° 50′.

20-25. See Table VI, below.

[Heliacal rising and setting of planets]

26. The difference in minutes between the (last) *śīghrakendra* of a planet (obtained while calculating its true longitude), and that for its heliacal rising, setting or retrograde motion, when divided by the difference in minutes between the motion of the *śīghrocca* and the planet's motion after correction or *mandasphuṭagati*, gives the days passed or to be bassed in respect of any of the above phenomena.

Thus ends Chapter Three

on

TRUE PLANETS

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26. This follows from : (Given *śīghrakendra* for any phenomena ~ *śīghrakendra* on any day) ÷ motion of *śīghrakendra*. The result is days passed or to pass.

CHAPTER IV

THREE PROBLEMS RELATING TO DIURNAL MOTION

[Direction from the 12-digit gnomon shadow]

1. When a gnomon 12 *aṅgulas* high and cylindrical in shape is placed vertically at the centre of a circle drawn on a smooth surface, the points (on the circumference of the circle) where the shadow of the gnomon comes inside the circle and goes outside it, are, respectively, known as the west and the east points.

MATHEMATICAL NOTES

Introductory

Before explaining the formulas, certain similar right-angled triangles might be described.

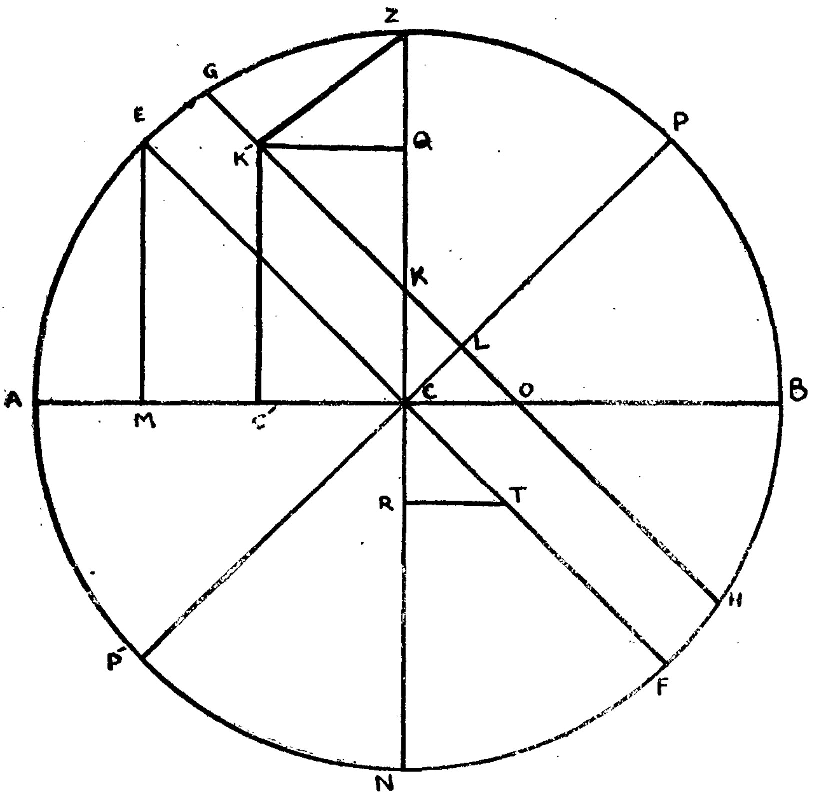

FIG. 5

Let ZANB be the meridian with Z as zenith. Let ACB be the diameter of the horizon, ECF that of the celestial equator, with P and P′ as the poles. Let PCP′ represent the six O'clock circle. Let GH be the diameter of a diurnal circle of the Sun meeting the diameter of the prime vertical ZN at K, PCP′ at L, and ACB at O. Let EM be perpendicular on ACB.

Now, EZ would measure the latitude of the place or *akṣāṃśa*, and AE the colatitude or *lambāṃśa*.

Thus, CM is R sin latitude, EM is R sin colatitude and CE is R.

CME is then a right-angled triangle, with R sin colatitude, R sin latitude and R as sides. Let this triangle be called U.

CL is the distance between the celestial equator and the diurnal circle of the Sun. It is, therefore, R sin declination. L is the point of intersection of the diurnal circle and the six O'clock circle and LO is the perpendicular drawn from that point on the line of intersection of the diurnal circle and the horizon. Thus LO is earth-sine or *kujyā*. And CO, which lies on the horizon, is the distance between the east-west line and the rising and setting line. This is called the R sin Sun's amplitude or *agrajyā*. Thus, the right-angled triangle CLO having R sin declination, earth-sine and R sin amplitude as its sides is similar to U. Let it be called V.

When the Sun is at K, it is on the prime vertical. KC is then the R sin prime vertical altitude or *samaśaṅku*, CO is R sin amplitude and KO is the prime vertical hypotenuse or *taddhṛti*. This right-angled triangle KCO, having R sin prime vertical altitude, R sin amplitude and prime vertical hypotenuse as sides, is similar to U and V. Let it be called W.

When the Sun is at K′, K′C′ is R sin altitude at any time or *śaṅku*. C′O is the base or *śaṅkutala* and K′O the hypotenuse at that time or *iṣṭahṛti*. The right-angled triangle K′C′O, with these three as sides, is similar to U, V and W. Let it be called X.

KL=KO—LO

=*taddhṛti*—*kujyā* or *kujyona-taddhṛti*.

The right-angled triangle KLC having sides *kujyona-taddhṛti*, R sin declination and R sin prime vertical altitude as sides, is similar to U, V, W, X. Let it be called Y.

Now, suppose the Sun is at E, the equinoctial point. Let CR be the gnomon of 12 *aṅgulas*. Then RT is the midday equinoctial shadow or *palabhā* and CT the corresponding hypotenuse or *palakarṇa*. The right-angled triangle CRT, with gnomon, shadow and the hypotenuse as sides, is similar to U, V, W, X and Y. Let it be called Z.

Thus the triangles U, V, W, X, Y and Z are six right-angled similar triangles.

2-3. (Mark) the extremities of three shadows of the gnomon (on any day, two being the two above and the third, the extremity of the midday shadow). Draw two fish-figures (by means of intersecting arcs of circles drawn with the first and second points as centres and with

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They are as follows :

1. With sides *lambajyā*, *akṣajyā* and *trijyā* (U).
2. With sides *krāntijyā*, *kujyā* and *agrajyā* (V).
3. With sides *samaśaṅku*, *agrajyā* and *taddhṛti* (W).
4. With sides *śaṅku*, *śaṅkutala* and *iṣṭahṛti* (X).
5. With sides *kujyona-taddhṛti*, *krāntijyā* and *samaśaṅku* (Y).
6. With sides *śaṅku*, *palabhā* and *palakarṇa* (Z).

These triangles are generally used in all the astronomical works. Triangle nos. 1-3, 5 and 6 are given by Bhāskara II in *SŚi*, I. iii. 13-15 ; see also *BSS*, iii. 7-12, and *SSi*, II. vii. 42-48.

1-3. These verses give the method to determine the directions and the locus of the extremities of the shadow of the gnomon.

On any level surface draw a circle with any point as the centre and any length as the radius. Place the vertical gnomon of 12 *aṅgulas* at the centre. This circle is the dial.

Mallikārjuna gives the following procedure in his commentary (Text, pp. 58-59) : Let the radius be 24 *aṅgulas* in length and let the cylindrical gnomon be 12 *aṅgulas* in height.

Now, in the forenoon, mark the point on the circumference of this circle when the extremity of the shadow of the gnomon just touches it. This is the west point. Then, in the afternoon, mark the point on the circumference of this circle where the extremity of the shadow again just touches it. This is the east point. These are the east and west points. Here, the declination of the Sun is assumed to remain fixed during the interval. The inaccuracy thus caused is negligible for practical purposes.

a radius of any length and again with the second and third points as centres and a radius of the same length as before). Draw the two straight lines (passing through the points of intersection of these arcs) known as mouths and tails of the fish-figures. Their point of intersection, the Sun being in the northern hemisphere, is the southern direction, the extremity of the midday shadow being to the north, that is, in the direction of the polar star. The point is the northern direction (the Sun being in the southern hemisphere and the midday shadow falling to the south. The line joining the north or south point to the foot of the gnomon is the north-south line). Then, the straight line drawn at right angles to the north-south lines by means of a fish-figure

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The line passing through the foot of the gnomon and parallel to the line joining the east and west points is the east-west line in the dial. The line at right angles to this line and passing through the foot of the gnomon will be the north-south line. The points where these lines cut the circumference of the dial are the four cardinal points.

Pṛthūdaka, while explaining Brahmagupta's relevant verse, considers the change in the declination and says that the real east point will not be the point where the shadow leaves the dial, but a little away from it. The correction would be :

$$\frac{\text{hypotenuse shadow}}{\text{R sin colatitude}} \times \text{R sin difference of declinations,}$$

when the shadow enters the dial and when it leaves it.

In his commentary (Text, p. 59), Mallikārjuna takes the three shadows as the shadow when it enters the circle, the midday shadow and the shadow when it leaves the circle. He draws three circles with each point as centre and radius equal to half the distance between the east and west points. Then the point of intersection of the lines joining the points of intersection of the above arcs is the south point in respect of the gnomon, if the Sun is in the northern hemisphere and the north point in the southern hemisphere. The end of the midday shadow is north in the first case and south in the second case.

(through the foot of the gnomon) is the east-west line.

Describe the circle with the above point of intersection as the centre and passing through the three extremities of the shadows. The shadow (of the gnomon) never falls outside this circle, just as a woman of a noble family never abandons her family tradition.

[Equinoctial shadow]

4. One-half of the sum of the lengths of the two midday shadows of the gnomon, when the Sun is at the first points of Aries and Libra, is the *palabhā* or 'equinoctial midday shadow'. The square root of the sum of the square of 12 (gnomon) and the shadow is the *palakarṇa* or the hypotenuse of the equinoctial midday shadow. The square root of the difference of the squares of the hypotenuse and 12 is the equinoctial midday shadow.

[Local latitude and colatitude]

5. The radius severally multiplied by the equinoctial midday shadow and 12 (gnomon), and divided by the hypotenuse of the

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Generally, according to Indian astronomers, the locus of the extremity of the shadow of the gnomon on any day is determined by marking the extremities of the shadows at three different times on the day and drawing a circle to pass through them. An arc of this circle is the locus.

For determining the directions and the locus of the extremity of the shadow the astronomers follow more or less the same methods.

The change in the declination has been hinted at by Brahmagupta and has been specifically mentioned by Śrīpati and Bhāskara II.

See *MB*, iii. 1-3 ; *LB*, iii. 1 ; *BSS*, iii. 1-3 ; *SŚe*, iv. 1-6 ab ; *SŚi*, iii. 8-9 ; and *SS*, iii. 1-4.

4. The second rule follows from the right-angled triangle Z.

5. These results follow from the similar right-angled triangles U and Z.

shadow, gives, respectively, the R sine of the latitude of the place and the R sine of the colatitude. Their arcs, divided by 60, gives, respectively, in degrees (the latitude and the colatitude). The latitude is always to the south. The direction of the declination of the Sun depends on its (*i.e.*, the Sun's) position.

[Samamaṇḍala-Śaṅku and Hṛti]

6. When the hypotenuse of the equinoctial midday shadow is multiplied by the R sine of the declination of the Sun and the product divided severally by the gnomon (of 12 *aṅgulas*) and by the equinoctial midday shadow, the results are, respectively, the R sine of the amplitude of the Sun or R sine of the *agrā* or *agrajyā*, and the R sine altitude of the Sun when on the prime vertical or *samaśaṅku*. The square root of the sum of the squares of the R sine *agrā* and *samaśaṅku* is the hypotenuse or *taddhṛti*.

[Ascensional differences]

7. Taking the Sun's longitude as 1, 2 and 3 signs (that is, assuming it at the end of Aries or Taurus or Gemini), calculate the three corresponding ascensional differences (at the observer's station). The first, the difference between the first two, and the difference between the third and second, are, respectively, the ascensional differences corresponding to the first, second and third signs at the observer's station.

[Times of the Rsing of the signs at Laṅkā]

8. Multiply the R sines of 30°, 60° and 90° by R sine of 66° and divide each product by the corresponding radius of the day circle.

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6. The first two formulas follow from the similar right-angled triangles V and Z, and Y and Z. The third follows from the right-angled triangle W.

7. The calculation of ascensional difference has already been explained earlier in ch. iii, *vv.* 17-19.

Find the arcs corresponding to the three quotients as R sines. The first quotient gives, in *asus*, the time of the rising of Aries in Laṅka ; the second diminished by the first gives that of Taurus ; and the third diminished by the second that of Gemini. So say the wise.

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8. This gives the method to find the time of rising of a sign of the zodiac or *rāśi* at the equator (Laṅkā).

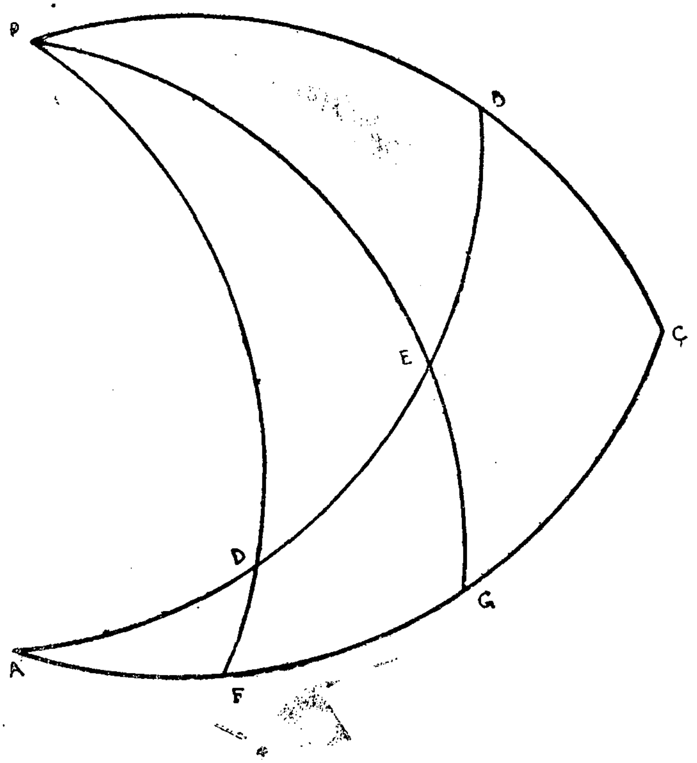

Fig. 6

Let AB and AC be the quadrants of the ecliptic and the equator, respectively. Let P be the pole. Let D and E be the ends of the first and second signs of the zodiac, respectively. Then B is the end of the third sign. Let PDF, PEG and PBC be secondaries to the equator, cutting AC in F, G and C, respectively. Then, AF represents the time, measured in *asus*, taken by the first sign AD to rise above the horizon at the equator ; that is, AF is the equivalent of the first sign in right ascension in the right sphere.

9. These three times are, respectively, 1670, 1795 and 1935 *asus*. When these are diminished, respectively, by the three corresponding ascensional differences at the observer's station, the remainders are the times of rising of the first three signs of the zodiac at the same place.

When the ascensional differences at the observer's station are written in the reverse order and are added to the three times (at Laṅkā), also written in the reverse order, the results are the times of rising of the next three signs at the same place.

These six times written in the reverse order are the times of rising of the last six signs.

$$\text{Now, } \frac{\text{R sine AF}}{\text{R sin AD}} = \frac{\text{R sin PB}}{\text{R sin PD}}$$

$$\text{So, R sin AF} = \frac{\text{R sin PB} \times \text{R sin AD}}{\text{R sin PD}}$$

$$= \frac{\text{R sine } 66^\circ \times \text{R sin } 30^\circ}{\text{radius of day-circle}}$$

when the longitude is 30°. So, AF is the corresponding arc.

Similarly, AG and AC may be found and hence FG and GC.

The same rule is given in *MB*, iii. 9 ; *LB*, iii. 4 ; *BSS*, iii. 15 ; *SŚe*, iv. 15 ab ; *SŚi*, I. ii. 57 ; and *SS*, iii. 41-42 ab.

9. This gives the method to find the time of rising of a sign of the zodiac at any other latitude or its equivalent in oblique ascension. From the above, the times will be approximately 1670, 1795 and 1935 *asus* at the equator. By applying the local ascensional differences, the corresponding times at that latitude are obtained. The same times and the same methods of application are given in *MB*, iii. 10 ; *LB*, iii. 5-6 ; *SŚe*, iv. 17, 15 cd ; and *SS*, iii. 42 cd-44. *BSS*, iii. 17 gives the application for local latitudes. *SŚi*, I. ii. 58-59 ab gives times and application. Here, the times are 1670, 1793 and 1937 *asus*.

[Rising point of the ecliptic]

10. Whatever portion of a sign of the zodiac has arisen in the east above the horizon, the same portion of the sign (seventh from it), has set in the west.

When the orient ecliptic point or *lagna* for any time in the day is calculated, the true longitude of the Sun at that time is considered. But when the orient ecliptic point at any time in the night is calculated, the true longitude of the Sun is increased by 6 signs.

[Rising point of the ecliptic at any time]

11-12. Multiply the remaining degrees etc. of the sign of the zodiac in which the Sun is by the time of rising (in *asus*) of that sign and divide by 30. Subtract the quotient in *asus* from the given time, also in *asus*. Add the remaining degrees etc. of the sign to the longitude of the Sun. (Call the result l_1).

From the remaining *asus* subtract as many times of rising (*asus*) of the signs as possible (beginning with the one next to the Sun), and add the same number of signs to l_1. (Call the sum l_2).

Multiply the remaining time (in *asus*) by 30 and divide by the time of rising (in *asus*) of the sign next to the one subtracted. Add the quotient in degrees etc. to l_2.

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10. The same rule occurs in *MB*, iii. 33 cd ; and *SŚi*. I, ii. 59 cd.

11-12. These indicate the method to calculate the longitude of the orient ecliptic point or *lagna*.

Let AB and AC represent the ecliptic and the equator, respectively. Let D, E, B be the ends of the first three signs, respectively. Let S be the position of the Sun at *t asus* after sunrise. Let KLMN be the position of the horizon at that time, meeting AB in L and AC in M. Then, L is the position of the orient ecliptic point or *lagna* at the time at which the longitude is required.

The final result is thus the longitude of the orient ecliptic point or rising ecliptic point.

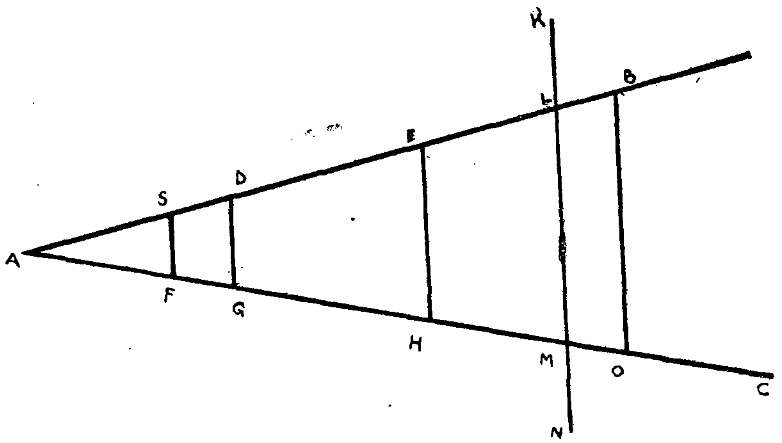

Fig. 7

Let F, G, H and O be the points on the equator which rise at the same times as S, D, E and B, respectively. Then AG, G H and HO are the equivalents of the first three signs in oblique ascension or the times of rising of the first three signs at the observer's latitude. FM represents *t asus* after sunrise.

$$\text{Longitude of L} = \text{arc AS} + \text{arc SL}$$
$$= \text{Sun's longitude} + \text{arc SD} + \text{arc DE} + \text{arc EL}$$
$$= \text{Sun's longitude} + (30^\circ - \text{Sun's longitude}) + \text{second sign} + \text{arc EL}$$

$$\text{Arc FG} = \frac{\text{arc SD} \times \text{arc AG}}{\text{arc AD}}$$
$$= \frac{(30^\circ - \text{Sun's longitude}) \times \text{time of rising of first sign}}{30}$$

$$\text{Arc HM} = \text{arc FM} - \text{arc FG} - \text{arc GH}$$
$$= t\ asus - \text{arc FG} - \text{time of rising of second sign.}$$

$$\text{Now, arc EL} = \frac{\text{arc EB} \times \text{arc HM}}{\text{arc HO}}$$
$$= \frac{30^\circ \times \text{arc HM}}{\text{time of rising of third sign}}.$$

So, longitude of L = Sun's longitude + (30° − Sun's longitude) + second sign + EL (known).

This is the general rule to find the position of the orient ecliptic point or the point of the ecliptic on the eastern horizon.

The same rule is in *MB*, iii. 30-32 ; *LB*, iii. 17-19 ; *BSS*, iii. 18-20 ; *SŚe*, iv. 18-20 ab ; and *SŚi*, I. iii. 2-4. See also *SS*, iii. 45-47.

Bhāskara II and *SS* have, in this connection, mentioned the correction due to the precession of equinox. So has Mallikārjuna in his commentary.

The calculation of the longitude of the orient ecliptic point is an important topic in the Indian astronomy. The following is a brief note on the subject.

Verses 11-12 give the method of calculation when the time is *after* sunrise. But when it is *before* sunrise the method as given below should be followed.

The true longitude of the Sun at that time with all the necessary corrections as given before should be found first. Then the portion of the sign of the zodiac covered by the Sun expressed in minutes should be multiplied by the time of rising of this sign and divided by 1800. This time should be subtracted from the given time. The elapsed portion of the sign should be subtracted from the longitude of the Sun. Let l_1 be the result. Then, from the remaining time the times of rising of as many signs as possible, beginning from the one next to that in which the Sun is, and going in the reverse direction, should be subtracted, and the number of signs thus sbutracted should be subtracted from l_1. The remainder is, say, l_2. The remaining time should be multiplied by 30 and divided by the time of rising of the sign next to the one (in the reverse order) last subtracted. The quotient should be subtracted from l_2. The result is the longitude of the orient ecliptic point at the given time *before* sunrise.

[Time elapsed from Sunrise]

13. When 180° is added to the longitude of the orient ecliptic point, the result is the longitude of the setting point of the ecliptic or *asta-lagna*. So they say.

Multiply the number of degrees etc. passed by the orient ecliptic point in the sign of the zodiac by the local time of rising (in *asus*) of the sign and divide by 30. The quotient is the time in *asus*. Then,

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If the given time is *after sunset but before midnight*, the true longitude of the Sun should be increased by 6 signs. Then the method for the time after sunrise should be followed.

If the given time after sunrise is shorter than the calculated time which is to be subtracted, then as the rule in 14 ab says, the given time should be multiplied by 30 and divided by the time of rising of the sign in which the Sun is and the result should be added to the longitude of the Sun. The result is the longitude of the ecliptic point.

13. The setting point of the ecliptic is the point of the ecliptic on the western horizon. See *MB* iii. 33 ab.

The second part of the verse gives the method to find the time elapsed since sunrise, when the longitudes of the Sun and the orient ecliptic point are known.

In the above diagram (Fig. 7, p. 69), arc AS and arc AL are given. To find arc FM.

$$\text{Arc FM} = \text{arc FG} + \text{arc GH} + \text{arc HM}$$

Arc FG is found as before. Arc GH is known. Arc EL is the given longitude of the orient ecliptic point omitting the signs.

$$\text{Arc HM} = \frac{\text{arc HO} \times \text{arc EL}}{\text{arc EB}}$$

$$= \frac{\text{time of rising of third sign} \times \text{arc EL}}{30}$$

find (as before) the time of rising of the remaining portion of the sign in which the Sun is. Add both these times and to the sum add the local times of rising (in *asus*) of all the signs intervening between the orient ecliptic point and the Sun. The sum is the required time (in *asus*, calculated) from the given orient ecliptic point.

14. If the given time (when the longitude of the orient ecliptic point is required) is less than the time of rising of the remaining portion of the sign of the zodiac in which the Sun is, as calculated above, multiply the given time by 30 and divide by the time of rising of this sign. When the quotient in degrees etc. is added to the true longitude of the Sun, the result is the longitude of the orient ecliptic point.

If the Sun and the orient ecliptic point are in the same sign of the zodiac, multiply the difference in their longitudes by the time of rising of this sign and divide by 30. The quotient gives the required time (in *asus*).

Thus the time is known. This rule is given in *MB*, iii. 34-36 ; and *LB*, iii. 20. See also *BSS*, iii. 21-23 ; *SŚe*, iv. 20 cd-28 ; *SŚi*, I. iii. 5-7 ; and *SS*, iii. 49.

14. In the case when the Sun and the orient ecliptic point are in the same sign of the zodiac, their difference in degrees should be multiplied by the time of rising of the sign in which they are and divided by 30. The result would be the time required. This rule is explained in the second part of verse 14.

In case the longitude of the Sun at midnight is given and the time is required, then the time should be calculated from the midnight Sun as given above. The longitude of the Sun for that time should be found. Hence again the time. The process should be repeated till the time is fixed.

Bhaṭṭotpala in his commentary on *KK*. iii. 5 has given the details in a lucid manner (vol. II, pp. 91-94). See also *SŚe*, iv. 18-20.

[Meridian zenith distance]

15. The sum of the declination of the Sun and the colatitude of the observer's station is the meridian altitude or *unnatāṃśa* of the Sun when it is in the northern hemisphere and on the meridian. The difference is taken when it is in the southern hemisphere. When the altitude is subtracted from 90°, the remainder is the meridian zenith distance or *natāṃśa*.

16. Or, the sum of the declination of the Sun and the latitude (of the observer's station) is the meridian zenith distance or *natāṃśa* of

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15-16. These verses explain how to find the Sun's meridian zenith distance.

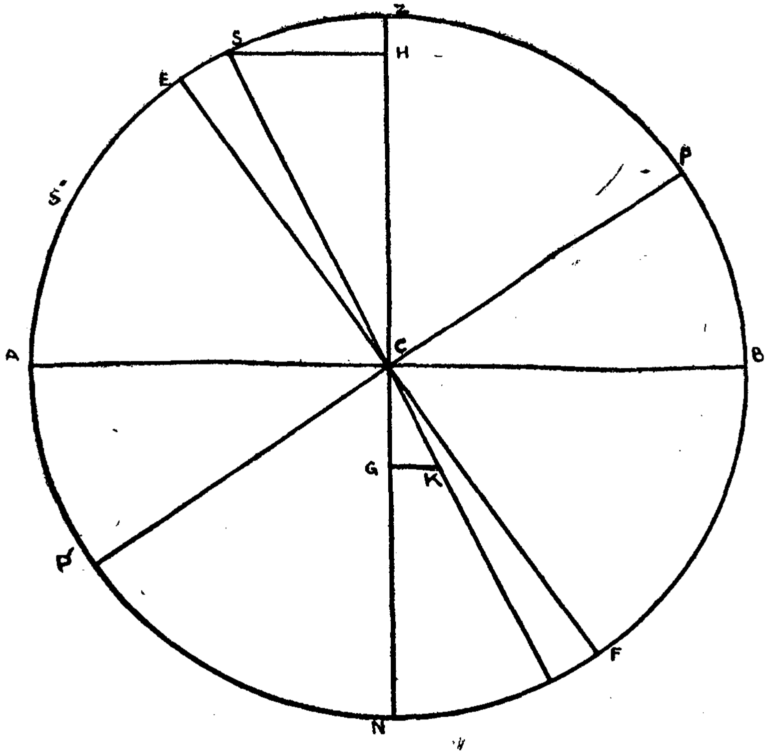

FIG. 8

Let Z A N B be the observer's meridian and A C B the horizon. Let ECF be the celestial equator and P and P′ the north and south

the Sun when it is in the southern hemisphere. But their difference is the meridian zenith distance when the Sun is in the northern hemisphere. When the zenith distance is subtracted from 90°, the remainder is the altitude or *unnatāṃśa*.

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poles, respectively. Let S be the position of the Sun to the north of the equator and S′ to the south.

EZ denotes the latitude of the place and EA the colatitude. SE is the Sun's declination in the northern hemisphere and S′E in the southern hemisphere.

Then, when the Sun is in the northern hemisphere,

SA = SE + EA = declination + colatitude

= *unnatāṃśa* or meridian altitude

and SZ = 90° — (SE + EA) = *natāṃśa* or meridian zenith distance. (1)

In the southern hemisphere,

S′A = AE — S′E = colatitude — declination

= *unnatāṃśa* or meridian altitude

and S′Z = 90° — (EA — S′E) = *natāṃśa* or meridian zenith distance (2)

(1) can written as :

Meridian zenith distance or *natāṃśa*

= EZ — SE = latitude — declination

and (2) can be written as :

Natāṃśa = EZ + S′E = latitude + declination.

The rule in verse 16 is given in *MB*, iii. 11, *BSŚ*, iii. 13, 47 ; and in *SS*, iii. 20. Both the rules are in *SŚe*, iv. 42-43 ; and in *SŚ i*, I. iii. 32.

While commenting on these verses, Mallikārjuna observes that the equinoctial midday shadow to the north of the Vindhyas is $5\frac{1}{20}$ *aṅgulas*. He further states : South declination + latitude = meridian

[Midday shadow and hypotenuse]

17. When the Sun is on the meridian, multiply the R sine of its zenith distance and the R sine of 90°, each by 12 and divide each product by the R sine of its meridian altitude. The quotients are, respectively, the midday shadow (of the gnomon) and the corresponding hypotenuse.

18. The radius of the day-circle of the Sun increased or diminished by the earth-sine and the radius increased or diminished by the R sine of the ascensional difference, according as the midday Sun is

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zenith distance (to the south of zenith) ; north declination ~ latitude =meridian zenith distance, which is to the south of the zenith if the declination is less than the latitude and to the north if the declination is greater than the latitude.

17. This explains how to find the midday shadow and the hypotenuse.

In the above diagram (see p. 73), let CG be the gnomon of 12 *aṅgulas* and GK the perpendicular to ZN meeting SC produced in K. Then, GK is the midday shadow and CK the hypotenuse. Let SH be perpendicular to ZN. Then, SZ is the meridian zenith distance or *natāṃśa* or z, SH is its R sine or *natāṃśajyā*. SA is the altitude or *unnatāṃśa* or a, CH its R sine or *unnatāṃśajyā*.

Then, $GH = \dfrac{SH \times CG}{CH}$

So, midday shadow $= \dfrac{R \sin z \times 12}{R \operatorname{Sin} a}$

and $CK = \dfrac{SC \times CG}{CH}$

or hypotenuse $= \dfrac{R \times 12}{R \sin a}$

These formulas are in *BSS*, iii. 28 ab and 29 ab ; *SŚe*, iv. 45 ; *SS*, iii. 21 ; and *SŚi*, I, iii. 40 ab.

18. This defines *antyā* and *antyakā*.

in the northern or southern hemisphere, give, respectively, the midday *antyā* and the *antyakā*.

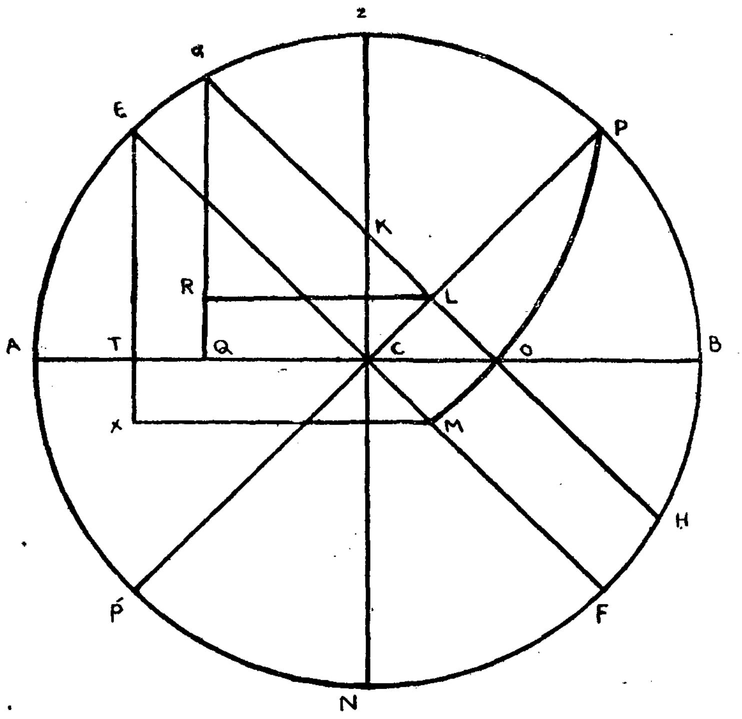

Fig. 9

Let Z A N B be the observer's meridian, A C B the diameter of the horizon, Z the zenith, ECF the diameter of the equator, PCP′ the diameter of the six O′ clock circle, G K L O H the diameter of a day-circle of the Sun, and POM a vertical circle through O.

Then, GL is the radius of the day-circle or *dyujyā*, CM is the R sin ascensional difference or *carajayā*, and LO earth-sine or *kujyā*. Let the Sun be at G.

Then, GO = GL + LO

= radius of diurnal circle + earth-sine

= *antyā* or *cheda*,

and EM = EC + CM = R + R sin ascensional difference

= *antyakā* or *hāraka*.

19. They are also called *cheda* and *hāraka*, respectively. When the *hāraka* is multiplied by the radius of the day-circle of the Sun and divided by the radius, the result is *cheda*. By the reverse process, (*cheda* is converted into *hāraka*).

[R sine of the midday altitude]

20. *Cheda* multiplied severally by the R sine of the colatitude,

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Brahmagupta calls GO *'dyudalāntyā'* and EM, *'antyā'* (*BSS.* iii. 34). Śrīpati calls GO, *'antyakā'* and EM, *'antyā'* (*SŚe*, iv. 46 ab). In *SS* (iii. 34 ab), EM, is *'antyā'*. Bhāskara II calls GO, *'hṛti'* and EM *'antyā'* (*SŚi.* I. iii. 34 cd). In his commentary on the above verse, however, he keeps the names *antyā* and *antyakā*. *Cf.* : 'When the Sun is in the southern hemisphere, *antyā* = radius of the day-circle — earth-sine, and *antyakā* = R — R sin ascensional difference'.

19. This explains how to express *cheda* in terms of *hāraka* and vice versa.

In the above diagram, (fig. 9), let GQ be perpendicular to AB. Draw LR parallel to AB meeting GQ in R. Draw ET perpendicular to AB and produce it to meet MX parallel to AB at X. Then, GQ is R sin altitude or *śaṅku*, QO is *śaṅkutala* or base, and GO is *antyā* or *cheda*. It is evident that the triangle GQO is similar to the triangles U, V, W, X, Y and Z. Let it be S.

From the four similar triangles EMX, ECT, GLR and GOQ, it follows that

$$\frac{EM}{GO} = \frac{EC}{GL}$$

$$\text{or, } \frac{\textit{antyakā} \text{ or } \textit{hāraka}}{\textit{antyā} \text{ or } \textit{cheda}} = \frac{R}{\text{radius day-circle}}$$

$$\text{or } \textit{cheda} = \frac{\textit{hāraka} \times \text{radius day-circle}}{R}.$$

This formula is in *ŚSi*, I. iii. 35.

20. This gives four different ways of finding the Sun's R sin midday altitude.

12, R sine of the Sun's altitude when it is on the prime vertical, and by the R sine of declination, and divided respectively by the radius, the hypotenuse of the equinoctial midday shadow, *taddhṛti* and by the R sine of the Sun's amplitude, gives in each case the R sine of the Sun's midday altitude.

[Midday shadow and hypotenuse : Alternate method]

21. When the altitude of the Sun at midday or *unnatāṃśa* is subtracted from 90°, the R sine of the remaining arc is called *dṛgjyā* or R sine meridian zenith distance. This multiplied by 12 and divided by the R sine meridian altitude, gives the midday shadow.

22. When the Sun is on the meridian, divide the R sine meridian altitude by 12 and then divide the R sine of 90° by this result. The final result is the hypotenuse of the midday shadow in *aṅgulas*. (So the wise) say.

$$\text{R sin midday altitude} = \frac{\textit{cheda} \times \textit{sin colatitude}}{\text{R}} \qquad (1)$$

$$\text{R sin midday altitude} = \frac{\textit{cheda} \times 12}{\text{hypotenuse of the equinoctial shadow}} \qquad (2)$$

$$\text{R sin midday altitude} = \frac{\textit{cheda} \times \text{R sin prime vertical altitude}}{\textit{taddhṛti}} \qquad (3)$$

$$\text{R sin midday altitude} = \frac{\textit{cheda} \times \text{R sin declination}}{\text{R sin amplitude}} \qquad (4)$$

(1) follows from triangles U and S ;

(2) follows from triangles Z and S ;

(3) follows from triangles W and S ;

(4) follows from triangles V and S ;

(1) is noticed also in *SS*. iii. 34 cd-35 ab and *ŚSe*, iv. 34 ab, (2) in *BSS*, iii. 27 ab and *ŚSe*. iv 32 cd, (3) in *ŚSe* iv. 33 cd., and (4) in *ŚSe*, iv. 31 ab.

21-22. The formulas are the same as those given earlier in verse 17 of this chapter. *Dṛgjyā* is another name for 'R sin meridian

[**Hour angle**]

23. That part of the sky which is to the east (of the observer's meridian) is the eastern hemisphere and the part to the west is the western hemisphere. So the wise say.

When the Sun is in the eastern hemisphere, the time passed since its rising is called *unnatakāla*, and the time to pass (till it reaches the meridian) is called *natakāla* or hour angle.

(When the Sun is) in the western hemisphere, the time to pass before it sets is called *unnatakāla*, and the time passed (since it crossed the meridian) is called *natakāla* or hour angle.

[**Shadow at any time**]

24-25. When the Sun is in the northern hemisphere, subtract the local ascensional differnce from the time elapsed since sunrise or to elapse till the Sun sets and add when the Sun is in the southern hemisphere. Find the R sine (of the result). Multiply it by the radius of the day-circle of the Sun and divide by the radius. The result increased or decreased by the earth-sine, (according as the Sun is in the northern or southern hemisphere), is the divisor or *cheda* at the given time or *iṣṭaccheda*. From this one can find the R sine altitude, shadow and the hypotenuse at any time, by the method given above.

The *dṛgjyā* or R sine zenith distance (at any time) is the square root of the difference of the squares of the radius and the R sine altitude.

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zenith distance' or *natāṃśajyā*. *Dṛgjyā* or *Dṛg-guṇa* is also used for R sin zenith distance.

24-25. These explain how to find *cheda*[1] at any time or *iṣṭahṛti*

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1. The *cheda* or *iṣṭahṛti* is the distance of a heavenly body from its rising-setting line. —*KSS*

(*iṣṭa-cheda*) and hence the shadow at a given time.

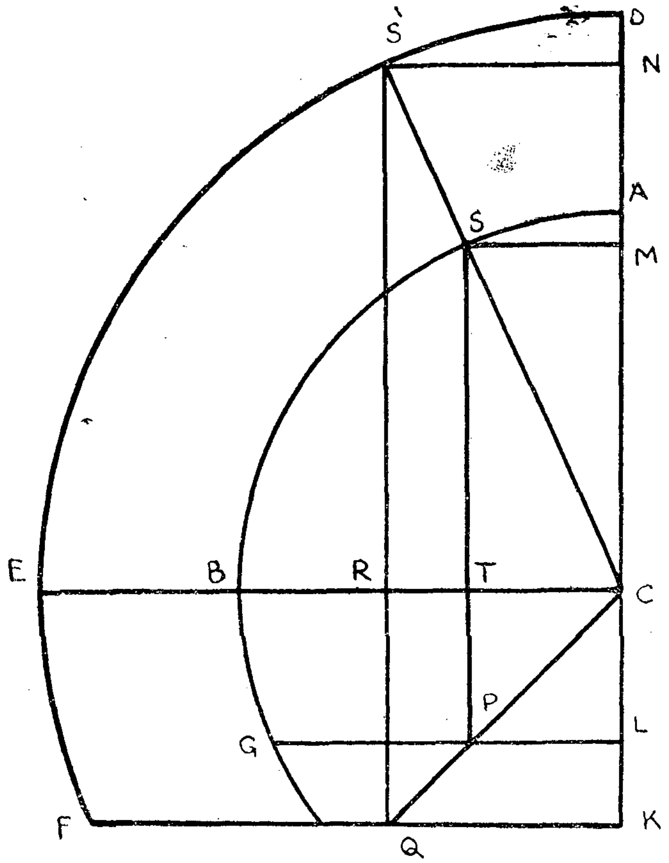

FIG. 10

(Diagram borrowed from *SS*, translated by Burgess, p. 133)

Let CDEF be a portion of [the plane of] the equator, C the centre and CD its line of intersection with [the plane of] the meridian. Let CABG be the corresponding portion of [the plane of] of the day-circle, C the centre and CA its line of intesection with [the plane of] the meridian. Then CD=CE=R, and CA=CB=radius day-circle. Let CK be R sin ascensional difference, and arc FE the ascensional difference. Let CL be earth-sine. Let S be the Sun and CS extended to meet the equator in S′.

26. Or, find the R versed sine of the *natakāla* or hour-angle (at the given time). Multiply it by the radius of the day-circle of the

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Then, arc S′D denotes the hour-angle or *natakāla*, and FD is half the day when the Sun is in the northern hemisphere and FS′ is the day passed or *unnatakāla*.

Let SM, S′N be perpendiculars on CD, and let SP and S′Q be perpendiculars on LG and KF, respectively.

Now, time passed—ascensional difference$=$FS′—FE$=$ES′

R sin ES′$=$S′R

From the similar triangles CST, CS′R,

$$ST=\frac{CS\times S'R}{CS'}=\frac{\text{radius day-circle}\times S'R}{R}$$

Now, ST$+$earth-sine$=$ST$+$TP$=$SP$=$*cheda* at any time or *iṣṭacheda.*

This is the hypotenuse of the triangle S (p. 77). Then, from the triangles U and S, the altitude can be found.

From the right-angled triangle SCH (fig. 8, p. 73),

$$SH^2=SC^2—CH^2$$

So, SH $=\sqrt{SC^2-CH^2}$

So, *Dṛgjyā* $=\sqrt{R^2-R^2\sin^2\text{altitude}}$

or R sin meridian zenith distance

$$=\sqrt{R^2—R^2\sin^2\text{ altitude}}$$

This relation will obviously hold good even if the Sun is not on the meridian. When the Sun is in the southern hemisphere, the ascensional difference should be added and the earth-sine should be subtracted.

This rule is given in *MB*, iii. 18-20 ; *LB*, iii. 7-10 ; and *BSS*, iii. 25-26. See also *SŚe*, iv. 32, 34, and *SŚi*, I. iii. 53-54.

26. Another method to find the *cheda* at any time. See the

Sun and divide by the radius. Subtract the quotient from the midday *cheda*. The result is the *cheda* at the given time or *iṣṭaccheda*.

27. Find the R sine of the time passed since sunrise or of the time to pass till sunset, increased, or diminished by the local ascensional difference, according as the Sun is in the southern or northern hemisphere. Increase or decrease it by the R sine of the ascensional difference, (according as the Sun is in the northern or southern hemisphere). The result is the *hāraka* at the given time. This should be used to find other values as said above, (that is, *cheda*, etc.).

28. Or, subtract the R versed sine of the hour-angle expressed in *asus* from the *hāraka* at midday. The remainder is called *hāraka*

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fig. 9, p. 76. S′D is the hour-angle or *natakāla*. S′N is its R sine and CN is its R cosine.

DN = CD — CN = R — R cos hour-angle

= R versed sin hour-angle or *natakālotkramajyā*

Then, from CS′ : DN : : CS : ?, or R : DN : : radius day-circle : ?, AM is obtained. Thus :

$$AM = \frac{\text{R versed sin hour-angle} \times \text{radius day-circle}}{R}$$

Now, AL = CA + CL

= radius day-circle + earth-sine

= *cheda* at midday (*verse* 18, above)

So, AL — AM = *cheda* at midday — AM = SP

= *cheda* at any time.

The altitude, shadow, etc. may be obtained as before.

27-28. These explain how to find *hāraka* at a given time. In fig. 10, F′S is the time passed since sunrise or *unnatakāla* or *dinagata*.

FS′ — FE = time passed — ascensional difference

= ES′

at the given time. So they (the astronomers) say. Other values can be obtained as before.

29. If the given time in *asus* is less than the Sun's ascensional difference, subtract the former from the latter when the Sun is in the northern hemisphere. Find the R sine of the remainder. The R sine of the ascensional difference diminished by it is called *hāraka.*

The same multiplied by the radius of the day-circle of the Sun and divided by the radius, is the *cheda* at the given time.

R sin ES′ = S′R

S′R + RQ = S′R + R sin ascensional difference

= S′Q = *hāraka* at a given time.

If the Sun is in the western hemisphere, the time to pass till sunset or *dinaśeṣa* is used instead of *dinagata.*

This rule is in *MB*, iii. 23 ab ; *BSS*, iii. 29 cd ; and in *ŚSe*, iv. 37.

Coming to the next verse, *v*. 28, *hāraka* at midday = R + R sin ascensional difference, when the Sun is in the northern hemisphere (*v*. 18). Thus *DK* is *hāraka*.

So, *hāraka* at any time = S′Q = NK = DK — DN

= *hāraka* at midday — R versed sine hour-angle.

29. The case when the Sun's ascensional difference is greater than the given time is treated here. The time should first be subtracted from the ascensional difference and then the R sine should be found. This should be subtracted from the R sine ascensional difference. The result would be the *hāraka.* Then from *v*. 19,

$$\textit{Cheda} \text{ at given time} = \frac{\textit{hāraka} \times \text{radius day-circle}}{\text{radius}}$$

This rule is in *MB*, iii. 25 ab ; *LB*, iii. 11 ab ; *BSS*, iii. 33; and *ŚSe*, iv. 41.

30. Multiply the *cheda* and the *hāraka*, both at midday, severally, by the hypotenuse of the shadow at midday. Divide the first product by the *cheda* and the second product by the *hāraka*, both at

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30. This gives the formula to find the hypotenuse of the shadow cast by the gnomon at any time, and hence the shadow, from the Sun's zenith distance.

It is evident that the triangle with its sides as gnomon of 12 *aṅgulas*, its shadow at any time and the corresponding hypotenuse is similar to the triangle with sides as Sun's R sin altitude, R sin zenith distance at that time and R.

So, hypotenuse or *chāyākarṇa* at given time

$$= \frac{R \times 12}{R \sin \text{altitude}}$$

$$= \frac{R \times 12}{\dfrac{12 \times \textit{cheda} \text{ at given time}}{\text{hyp. equi. shadow}}}$$

$$= \frac{R \times 12 \times \text{hypotenuse equinoctial shadow}}{12 \times \textit{cheda} \text{ at given time}} \times \frac{R \sin \text{midday altitude}}{R \sin \text{midday altitude}}$$

$$= \frac{R \times 12}{R \sin \text{midday altitude}} \times \frac{\text{hypotenuse} \times R \sin \text{midday altitude}}{12} \times \frac{1}{\textit{cheda} \text{ at given time}}$$

$$= \text{hypotenuse at midday } (v.\ 17) \times \textit{cheda} \text{ (triangles Z and S)} \times \frac{1}{\textit{cheda} \text{ at given time}}$$

$$= \frac{\textit{cheda} \times \text{hypotenuse at midday}}{\textit{cheda} \text{ at given time}}$$

This is one formula. Now, the other formula :

the given time. Each result gives the hypotenuse of the shadow at the given time.

[Time from Shadow]

31. Multiply the square of the radius by the hypotenuse of the equinoctial midday shadow and divide by the product of the hypotenuse of the shadow at the time required and the radius of the Sun's day-circle. Increase or diminish the quotient by the R sine of the ascensional difference, according as the Sun is in the southern or northern hemisphere. Find the corresponding arc. This arc, increased or diminished by the ascensional differnce, (according as the Sun is in the northern or southern hemisphere), is the number of *asus* in the day passed since sunrise, (if it is befere midday), or that to pass till sunset, (if it is after midday).

32. (If the above quotient is less than the R sine of the ascensional difference), and so the latter cannot be subtracted, subtract

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Hypotenuse or *chāyākarṇa* (from above)

$$=\text{hypotenuse at midday} \times \textit{cheda} \times \frac{1}{\textit{cheda} \text{ at given time}}$$

$$=\text{hypotenuse at midday} \times \frac{\textit{hāraka} \times \text{radius diurnal circle}}{\text{R}}$$

$$(v.\ 19) \times \frac{\text{R}}{\text{radius diurnal circle} \times \textit{hāraka} \text{ at given time}}$$

$$=\frac{\text{hypotenuse at midday} \times \textit{hāraka}}{\textit{hāraka} \text{ at given time}}$$

31-32. These give the method to find the time when the shadow is known. From the previous verse, hypotenuse of the shadow at any time

$$=\frac{\text{R} \times \text{hypotenuse equinoctial midday shadow}}{\textit{cheda} \text{ at given time}}$$

So at given time,

$$\textit{cheda} = \frac{\text{R} \times \text{hypotenuse of equinoctial shadow}}{\text{hypotenuse at given time}}$$

the former from the latter. Find the arc of the remainder and subtract it from the ascensional difference. The remainder is the day passed since sunrise, (if it is before midday), or that to pass till sunset, (if it is after midday).

From fig. 10, p. 80,

SP = *cheda* at given time

$$= \frac{R \times \text{hypotenuse of equinoctial shadow}}{\text{hypotenuse}}$$

Again, from the similar triangles CS′Q, CSP, S′Q

= *hāraka* at given time

$$= \frac{SP \times CS'}{CS}$$

$$= \frac{R \times \text{hypotenuse of equinoctial shadow} \times R}{\text{hypotenuse} \times \text{radius diurnal circle}}$$

$$= \frac{R^2 \text{ hypotenuse equinoctial shadow}}{\text{hypotenuse} \times \text{radius diurnal circle}}$$

Now, S′Q—RQ = *hāraka* at given time—R sin ascensional difference

= S′R (*sūtra*)

When the Sun is in the southern hemisphere, *sūtra* is *hāraka* at the given time + R sin ascensional difference. The arc corresponding to S′R as R sin is S′E. So, the time passed since sunrise (*dinagata*)

= S′F = S′E + EF (ascensional difference)

If it is after midday, the result will be the time to pass till sunset. If the Sun is in the southern hemisphere, the ascensional difference should be subtracted.

Verse 32 indicates that when *hāraka* at the given time is less than R sin ascensional difference, the former should be subtracted from the latter and the corresponding arc should be subtracted from the ascen-

33. Or, multiply the *hāraka* by the difference between the hypotenuse of the shadow at midday and the hypotenuse of the shadow at the time required and divide the product by the latter. Find the arc corresponding to the quotient as the R versed sine. The result is the hour-angle in *asus*.

[**R sine altitude in an intermediate direction**]

34-35. When the Sun is in the southern hemisphere and on a *koṇavṛtta* or a vertical circle in an intermediate direction, add its R sine amplitude or *agrajyā* to the assumed length of the *śaṅkutala* or the

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sional difference. See *LB*, iii. 12-15 ; *BSS*, iii. 41-43 ; and *SŚe*, iv. 52-54.

33. This gives a method alternative to that in verse 31. From verse 30, hypotenuse at the given time

$$= \frac{\text{hypotenuse at midday} \times \textit{hāraka}}{\textit{hāraka} \text{ at the given time}}$$

So, hypotenuse at the given time $\times$ *hāraka* at the given time

$= \text{hypotenuse at midday} \times \textit{hāraka}$

So, hypotenuse at the given time (*hāraka* at midday—R versed sin hour-angle)

$= \text{hypotenuse at midday} \times \textit{hāraka}$ (*v.* 28)

$$\text{So, } \frac{\textit{hāraka}\,(\text{hypotenuse at given time} - \text{hypotenuse at midday})}{\text{hypotenuse at given time}}$$

$= \text{R versed sine of hour-angle.}$

So, the hour-angle is the arc corresponding to the left hand side as R versed sine. This rule is to be found also in *BSS*, iii. 44-45 ; and *SŚe*, iv. 55.

34-35. These give the formula to find the *koṇaśaṅku* or R sin altitude of the Sun on a vertical circle in an intermediate direction when its longitude is known.

base of the altitude. Square the result, double it and subtract from the square of the radius. The square root of the remainder is the R sine altitude at that time or *koṇaśaṅku*. Multiply it by the equinoctial midday shadow (at the observer's station) and divide by 12. The result is a more correct (base). Add it to the R sine amplitude or

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Consider fig. 5, on p. 60. Suppose the Sun is at K′. Then, K′C′ is R sin altitude and C′O is the base or *śaṅkutala*.

$$C'O = C'C + CO.$$

So, base = R sin of the Sun's distance from the prime vertical + R sin amplitude = *bhuja* + R sin amplitude.

So, Sun's distance = base − R sin amplitude.

Similarly, when the Sun is to the south of the equator,

Sun's distance = base + R sin amplitude.

Thus, Sun's distance = base ± R sin amplitude.

When the Sun is on any of the vertical circles in an intermediate direction the same relation holds good. When the Sun is on any of these circles, it is equidistant from the meridian and the prime vertical. So, R sine of its distance from the former is the same as the R sine of its distance from the latter. That is, *bhuja* also denotes the R sine of its distance from the meridian.

Now, these two R sines are at right-angles and the hypotenuse of this right-angled triangle is the R sine of the meridian zenith distance or *dṛgjyā*.

So, $dṛgjyā^2 = bhuja^2 + bhuja^2 = 2bhuja^2$

Now, R sin meridian zenith distance, R sin meridian altitude and R make a right-angled triangle. So, in this case, when the Sun is on the vertical circle in an intermediate direction,

$$(\text{R sin meridian zenith distance})^2 = R^2 - (\text{R sin altitude})^2$$

Or, $(\text{R sin altitude})^2 = R^2 - 2\ bhuja^2$

agrajyā and repeat the process till the R sine altitude is fixed. When the Sun is in the northern hemisphere, R sine amplitude should be diminished by the assumed (base). This is the rule.

$$=R^2-2\ (\text{base}\pm R\ \sin\ \text{amplitude})^2 \qquad (1)$$

. (1) is a quadratic equation in terms of *koṇaśaṅku* which, when solved, will give the required length.

Now, the method that Lalla follows to calculate the length of the *koṇaśaṅku* when the longitude of the Sun and the latitude of the place are given but the length of the base is not known, is as follows :

First find the Sun's declination from its longitude. Then, from the similar triangles U and V, find the Sun's R sin amplitude. Then assume some length as the base of the *koṇaśaṅku*. From (1) find *koṇaśaṅku* in terms of this assumed base. Let it be k_1. Then, from the similar triangles X, Z,

$$\text{base}=\frac{\text{equinoctial shadow}\times k_1}{12}$$

Use this value of the base to find again the *koṇaśaṅku*. Repeat the process till the length is fixed.

Śrīpati gives Lalla's method in *SŚe*, iv. 72-73.

Mallikārjuna's commentary explains the verse in the following manner : Instead of assuming some length of the base in the first instance, he takes it as zero. Then from (1)

$$R\ \sin\ \text{altitude}=\sqrt{R^2-2\ (R\ \sin\ \text{amplitude})^2}$$

$$=k_1, \text{ say.}$$

$$\text{So, 1st base}=\frac{\text{equinoctial shadow}\times k_1}{12}$$

$$\text{2nd base}=\frac{\text{equinoctial shadow}\times k_2}{12}$$

For the third step, Mallikārjuna takes the base equal to the half the sum of the first base and the second base.

[Shadow of R sine altitude in an intermediate direction]

36. When the Sun's declination is to the north and is less than the latitude (of the observer's station), the Sun then enters the prime vertical. From the then R sine altitude and from the R sine altitude when the Sun is on the *koṇavṛtta* or a vertical circle in an intermediate direction, the shadows and their hypotenuse can be calculated as above.

The rest is as in the text.

Bhāskara II follows Mallikārjuna's method, omitting the special method in the third step, in *SŚi*, I. iii. 30 (see commentary).

While explaining these verses, Mallikārjuna gives an alternate method which is the same as that given in *SS*, iii. 28-32 ab ; *BSS*, iii. 54-56 and also in *SŚe*, iv. 74-75.

This method is as follows :

$$\frac{144 \times \left(\frac{R^2}{2} - R^2 \sin^2 \text{amplitude}\right)}{(72 + \text{equinoctial shadow}^2)} = karaṇī$$

$$\frac{(12 \times \text{equinoctial shadow}) \times R \sin \text{amplitude}}{(72 + \text{equinoctial shadow}^2)} = phala$$

$$\sqrt{phala^2 + karaṇī} = x, \text{ say.}$$

$x \pm phala$ is *koṇaśaṅku.*

(For proof see the translation of *SS*, iii. 28-32, by Burgess).

While commenting on these verses, Mallikārjuna remarks that when the Sun is in the northern hemisphere, in the land to the north of the Vindhyas, wherever the latitude is more than 24°, there will be only the SE and SW *koṇaśaṅkus*. And, when the Sun is in the northern hemisphere and when the north declination is less than the latitude of the place, there will be SE and SW *koṇaśaṅkus* in the regions to the south of and in the Vindhyas. When the north declination is greater than the latitude, there will be NE *koṇaśaṅku* in the morning

[True Sun from midday shadow]

37-38. When the Sun is on the meridian, assume the shadow of the gnomon at that time as the equinoctial midday shadow. Hence find the latitude by the method given above. The difference between this latitude and the actual latitude of the place is the declination of the Sun. Their sum, however, would be the declination, if the shadow is to the south of the gnomon. The R sine of 90° multiplied by the R sine of the declination and divided by the R sine of the maximum declination gives the R sine of the longitude of the Sun. The corresponding arc is the longitude of the Sun.

Note that if it is in the first quadrant, that is, in the first quarter of the year, it is the longitude ; if the arc is in the second quadrant, the remainder after subtracting it from six signs, is the longitude ;

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and NW *koṇaśaṅku* in the afternoon. In the country to the south of and in the Vindhyas, when the Sun is in the southern hemisphere, there will be SE and SW *koṇaśaṅkus*.

Bhāskara II's remarks can also be noted in this connection. He says (vide the above reference) that the place where the R sin amplitude is greater than 2431 (R sin 45°), there is no *koṇaśaṅku* when the Sun is in the southern hemisphere. When in the northern, all the four *koṇaśaṅkus* occur, provided that the equinoctial midday shadow is greater than 17 *aṅgulas*.

37-38. These give the formula to find the longitude of the Sun from the midday shadow. First observe the midday shadow. Assume it to be the midday equinoctial shadow. Then, using the similar triangles U and Z, calculate the latitude. When the Sun is on the equinox, this would be the meridian zenith distance. Then the difference of these two latitudes would be the declination of the Sun on the given day. Hence, as before, find the longitude of the Sun. The second part follows from ch. ii, verse 11, above. Also note the rules in *SS*, iii. 18-19, and *BSS*, iii. 61-62.

if in the third quadrant, 6 signs should be added to it and the sum would be the longitude ; and, if in the fourth, the remainder after subtracting it from 12 signs, is the longitude of the Sun.

[Declination from shadow]

39. Find the distance between the extremity of the shadow at any time and the east-west line. Subtract it from the equinoctial midday shadow when the Sun is in the northen hemisphere and add when it is in the southern hemisphere. The result multiplied by the R sine of the colatitude (of the observer's station) and divided by the hypotenuse of the shadow, gives the R sine of the declination of the Sun.

Mallikārjuna, while commenting on this verse, says, that the longitude of the Sun obtained from the shadow is called *chāyārka.* If this is greater than the longitude of the Sun otherwise calculated, the difference is the *ayanāṃśa* or movement of equinox. If less, the difference is subtractive *ayanāṃśa*. But this will be so only 7200 years after the beginning of the *Kaliyuga,* according to tradition. The same remark he makes also in his commentary on *SS,* iii. 11. According to *SS,* iii. 11, if *chāyārka* is greater than the calculated longitude, the ecliptic moves to the east ; if less, it moves to the west.

39. This gives the method to find the declination of the Sun from the shadow at any time.

Consider fig. 5, on page 60. Let the Sun be at K′. Then K′C′ is R sin altitude of the Sun, C′O is base, C′C is *bhuja* and CO is R sin amplitude or *agrajyā.*

CO=C′O−C′C, or, R sin amplitude=base—*bhuja.*

When the Sun is in the southern hemisphere, R sin amplitude

=base+*bhuja*. So, R sin amplitude=base±*bhuja*

Now, here R sin amplitude, base and *bhuja* are all in a circle, the radius of which is R. They are to be reduced to the circle, the radius of which is the shadow of the gnomon placed at the centre of

the face of the dial at the given time. This radius is called *chāyākarṇa.* The *agrajyā* will be the *agrajyā* in the new circle and is known as *chāyākarṇagolīyāgrajyā.* The *bhuja,* which is the R sin of the Sun's distance from the prime vertical, will be the distance of the extremity of the shadow from the east-west line, when reduced to the smaller circle.

Now, to find the *śaṅkutala* or base in the new circle : From verse 30,

$$K'C' \text{ or R sin altitude} = \frac{12 \times R}{\text{hypotenuse } (\textit{chāyākarṇa})} \qquad (1)$$

The base in the smaller circle will be found from the proportion :
R : base : : hypotenuse : ?. Let it be x.

$$\text{Then, } x = \frac{\text{base} \times \text{hypotenuse}}{R}$$

$$= \frac{\text{R sin altitude} \times \text{equinoctial shadow}}{12} \times \frac{\text{hypotenuse}}{R}$$

(*cf.*, similar triangles X, Z)

$$= \frac{12 \times R}{\text{hypotenuse}} \times \frac{\text{equinoctial shadow}}{12} \times \frac{\text{hypotenuse}}{R}$$

(from 1)

=*equinoctial shadow.*

So, new *agrajyā*=new base—new *bhuja*

=equinoctial shadow—distance of the shadow from east-west line.

The first part of the verse finds the value of the reduced *agrajyā*. Then, from the proportion, hypotenuse : new *agrajyā* : : R : ?, *agrajyā* in the big circle is found.

$$\textit{agrajyā} = \frac{\textit{agrajyā} \text{ in the new circle} \times R}{\text{hypotenuse}}$$

Then, from similar triangles U, V,

$$\text{R sin declination} = \frac{\textit{agrajyā} \times \text{R sin colatitude}}{R}$$

[True Sun from shadow in an intermediate direction]

40. Find the shadow from the R sine of the altitude when the Sun is on the north-east or north-west vertical circle (*koṇaśaṅku*), square it and halve the result. Find its square root. If the *śaṅkutala* or the base is to the north, add the result to the equinoctial midday shadow (at the observer's station) and subtract, if to the south. The result multiplied by the R sine of the colatitude (of the observer's station) and divided by the hypotenuse of the shadow gives the R sine of the Sun's declination. This multiplied by the radius and divided by the R sine of 24°, gives the R sine of the Sun's longitude.

$$=\frac{\textit{agrajyā}\text{ in the new circle}\times R}{\text{hypotenuse}}\times\frac{R\text{ sin colatitude}}{R}$$

$$=\frac{\textit{agrajyā}\text{ in the new circle}\times R\text{ sin colatitude}}{\text{hypotenuse}}$$

and hence the Sun's longitude.

Śrīpati gives this rule in *SŚe*, iv. 65. He repeats the process for a better result.

40. This gives the method to find the longitude of the Sun when on an intermediary vertical circle or *koṇavṛtta*, if the length of the shadow of the gnomon at that time is known.

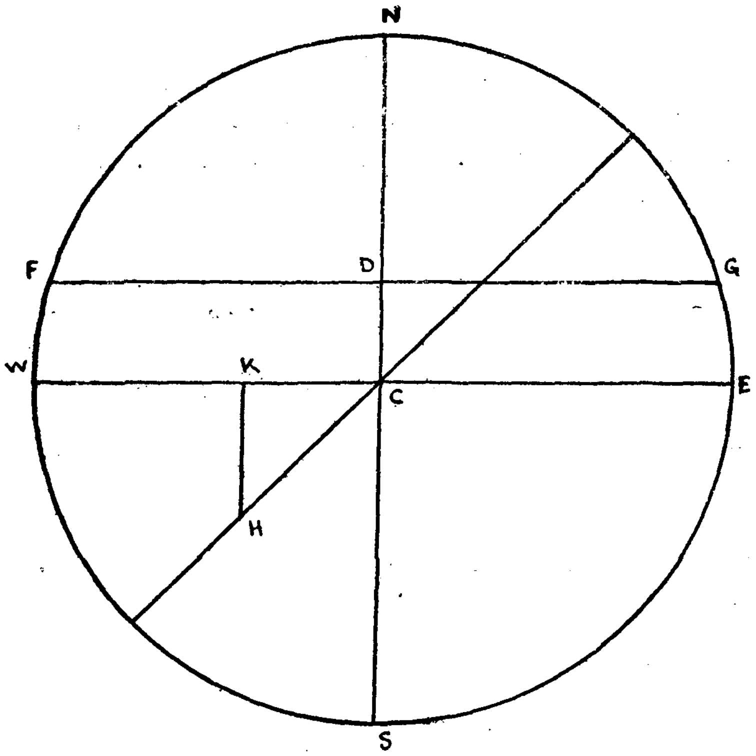

Fig. 11

Let NWSE represent the plane of the dial. Let NS be the north-south and EW the east-west lines. Let C be the base of the gnomon. Let CD be the equinoctial shadow. Let FDG be parallel to WCE. Let the Sun be on the north-east circle. Then CH, the shadow of the gnomon, coincides with the north-east line. Let HK be perpendicular to EW.

Then, $HK^2+CK^2=CH^2$

or $2\ HK^2=CH^2$

So $HK^2=\dfrac{CH^2}{2}$

HK is the distance of the extremity of the shadow from the

[True Sun from midday shadow : Alt. method]

41. The radius multiplied by 12 and by the R sine of the latitude (of the observer's station) and divided by the hypotenuse of the shadow when the Sun is on the prime vertical, and also by the R sine of 24°, gives the R sine of the longitude of the Sun. The corresponding arc is its longitude.

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east-west line or *bhuja* reduced to the smaller circle. (See the previous note). The remaining part of the rule is the same as that in the previous verse. Śrīpati gives this rule in *SŚe*, iv. 64.

41. This gives the method to find the longitude of the Sun when the shadow of the gnomon at the time when the Sun is on the prime vertical is known. When the Sun is on the prime vertical, the distance of the extremity of the shadow from east-west line is nil. So *bhuja* is nil.

So, reduced *agrajyā*=equinoctial shadow.

Now from iii. 39,

$$\text{R sin declination} = \frac{\text{reduced } agrajy\bar{a} \times \text{R sin colatitude}}{\text{hypotenuse}}$$

$$= \frac{\text{equinoctial shadow} \times \text{R sin colatitude}}{\text{hypotenuse}}$$

$$= \frac{12 \times \text{R sin latitude}}{\text{hypotenuse}} \quad (cf.\text{, similar triangles U, Z}).$$

So, R sin longitude of the Sun

$$= \frac{\text{R sin declination} \times \text{R}}{\text{R sin } 24^\circ}$$

$$= \frac{12 \times \text{R sin latitude} \times \text{R}}{\text{hypotenuse} \times \text{R sin } 24^\circ}$$

Thus, the corresponding arc is the longitude.

Śrīpati gives this rule in *SŚe*, iv. 66. See also *BSS*, iii. 57-62.

While commenting on this verse, Mallikārjuna again mentions that the difference between the longitude of the Sun calculated in the

42. Multiply the R sine of the Sun's declination by the hypotenuse of the shadow at any time and divide by the R sine of the colatitude (of the observer's station). This quotient increased or diminished by the equinoctial midday shadow, according as the Sun is in the southern or northern hemisphere, is called *bāhu* or *bhuja*. The square root of the difference of the squares of the shadow and the *bhuja* is called *koṭi* or perpendicular. This line is the east-west line in the circle, the centre of which is the point of intersection of the lines indicating the directions (in the plane of the dial), and the radius is the shadow (of the gnomon at the centre).

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usual manner and that calculated from the shadow is the movement of the equinox.

Thus, verses 37-41 give four methods for calculating the longitude of the Sun from four kinds of shadows : (1) midday shadow ; (2) shadow at any time ; (3) shadow when the Sun is on an intermediary vertical circle ; and (4) shadow when the Sun is on the prime vertical.

42. From verse 41 above, R sin declination

$$= \frac{\textit{agrajyā} \text{ reduced to smaller circle} \times \text{R sin colatitude}}{\text{hypotenuse}}$$

So, reduced $\textit{agrajyā} = \dfrac{\text{R sin declination} \times \text{hypotenuse}}{\text{R sin colatitude}}$

As seen before,

Reduced *agrajyā* = equinoctial shadow $\pm$ distance of the shadow from east-west line.

So, distance of the shadow = reduced *agrajyā* $\pm$ equinoctial shadow.

It is called *bāhu* or *bhuja* and is HK in the diagram on page 96.

The remaining part of the rule, to find CK, follows from the right-angled triangle CHK. Now,

$CK^2 = CH^2 - HK^2$. So, $CK = \sqrt{CH^2 - HK^2}$

[Path of the Shadow]

43-45. Then from the two ends of the shadows drawn through the centre (and meeting the above circle), two *bhujas* must be drawn in directions opposite to their own. Then the midday shadow should be laid from the point of intersection of the directions along the north-south line in a direction opposite to the midday *bhuja.* The points of extremities of these three shadows should be marked, and hence three intersecting arcs resembling two fish-figures. Join the mouth and tail of each figure. With the point of intersection of the two lines thus formed describe a circle passing through those three extremities. That is the path along which the extremity of the shadow moves.

The locus of the foot of the *śaṅku* is a circle drawn in the same way but the *bhuja* and the midday shadow are fixed in a manner contrary to the above.

46. The extremity of the shadow of the gnomon placed at the centre of the dial, that is, at the point where the lines of the directions meet, should not go beyond the above circle, (*viz.*, the locus), while the shadow moves.

The gnomon moves in such a manner that, while moving, the extremity of its shadow always passes through the point where the lines of the directions meet.

This is the east-west line. This is explained also in *SŚe*, iv. 80.

43-46. The *bhuja* and *koṭi* are defined in verse 42. These four verses now describe the locus of the extremities of the shadow of the gnomon placed at the centre of the face of the dial, where the lines of the directions meet and also the locus of the foot of the gnomon.

Śrīpati gives the same method in *SŚe*, iv. 80-82. That the locus of the extremity of the shadow on a given day is a circle is refuted by Bhāskara II in *SŚi*, II, xi. 38.

[Nalaka instrument]

47-48. (In the *chāyāvṛtta* or a circle with the shadow as radius, mark the north-south and east-west lines. From their point of intersection along the east-west line measure off the *koṭi* of the shadow cast by the planet at the time of observation. If the planet is in the eastern hemisphere, the *koṭi* must be marked to the west ; but to the east, if the planet is in the western hemisphere). At the extremity of the *koṭi* draw the *bhuja* in its own direction. Place a bamboo joining the extremity of the *bhuja* to the centre, (to represent the shadow). Connect the point of intersection of the *bhuja* and the shadow to the top of the gnomon by a bamboo. Then, the planet will be visible along this line on the top of the gnomon (after fixing the *nalaka* instrument).

Then, at the end, fix the *nalakayantra* on two bamboo sticks, so that it (*i.e.*, its hollow) is in the same direction as the joining line, and its height is that of the observer's eye. Now, the observer shall look through the instrument and he will see in the sky the eclipse, the rising Moon or the planet. The reverse process must be followed for visibility in water.

49. On any given day, multiply the Sun's R sine altitude or *śaṅku* by the equinoctial midday shadow and divide by 12. The result is the *śaṅkutala* or *śaṅkvagra* or base of the altitude. This is to the south (from the rising and setting line). In the night, because of different spheres, the *śaṅkvagra* is the Sun's north *agrajyā*.

When the end of the shadow is to the south, the degrees in the latitude are to the north.

[R versed sine of the hour angle and its arc]

50. When the hour-angle is greater than 5400 *asus*, find the

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47-48. These give instructions to use a *Nalakayantra* to view a planet. Śrīpati gives the same method in *SŚe*, iv. 84-85, and Bhāskara in *SŚi*, I. iii. 105-108.

50. This is based on the formula R versed sin $(90° + A) = R + R$ sin A,

R sine of their difference. Add to it the R sine of 90°. The result is the R versed sine of the hour-angle.

Again, if the R versed sine of the hour-angle is greater than the R sine of 90°, find the arc corresponding to the excess as the R sine. To the result add 5400 *asus*. The sum is the arc corresponding to the R versed sine of the *natakāla*.

[Shorter methods]

51. (*Akalā* is a number and *Vikalā* is another expressed as its sixtieth part). Square *vikalā* and divide the result by 60. To the quotient add twice the product of *akalā* and *vikalā*. Divide the sum by 60. To the quotient add the square of *akalā*. The result is the square of the sum of *akalā* and *vikalā*.

52. Find the integral portion of the square root of a non-square number as usual. Add 1 to the remainder. Multiply it by 60. Add the *vikalā* to it. Divide the sum by twice the square root increased by 2. The result is the fractional portion of the square root.

[53. In order to obtain the square of seconds only, minutes being absent, multiply the seconds by themselves and then divide (the resulting product) by 60 : the result is the square of the given seconds.

(Conversely), in order to obtain the square root of Seconds only, Minutes being absent, multiply the Seconds by 60 and then take the square root (of the resulting product) ; the result is the square root in terms of seconds.] — *KSS*

51-54. These give simple algebraical formulas.

[53. Let us denote square minutes by Minutes (writing the initial m with the capital letter) and sixtieth parts of square minutes by Seconds (again writing the initials with the capital letter). Then,

$$(b \text{ seconds})^2 = \left(\frac{b}{60}\right)^2 \text{ square minutes}$$

$$= \frac{b^2}{60 \times 60} \text{ Minutes}$$

54. Take a number. Add to it some other number. Multiply the sum by their difference. To the product add the square of the second number. The result is the square of the first number by a short process.

Thus ends chapter Four

on

THREE PROBLEMS
RELATING TO DIURNAL MOTION

$$=\frac{b^2}{60} \text{ Seconds}$$

Conversely,

$$\sqrt{\frac{b^2}{60} \text{ Seconds}} = \sqrt{b^2 \text{ square seconds}}$$

$$= b \text{ seconds.}$$

This is the usual method which has been employed in India.

—*KSS*]

CHAPTER V

LUNAR ECLIPSE

[Time of the eclipse]

1. If one wants to ascertain (the time of) a lunar eclipse, one must find the true longitudes of the Sun, Moon, and its ascending node,

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MATHEMATICAL NOTES

In the beginning of the chapter Mallikārjuna gives ten verses composed by him, (1) to determine the possibility of the occurrence of an eclipse and (2) to ascertain the deity presiding over the several days of the fortnight, the lords of the year, month and *horā* (*i.e.*, 2½ *nāḍīs*). Application of terrestrial longitude correction and the cycle of Jupiter's era are also dealt with.

As regards the deity presiding over an eclipse, he says that the true longitudes of the Sun and Rāhu (Moon's node) should be calculated and (after applying the necessary corrections) should be added together and the sum converted into degrees. This should then be divided by 180. The quotient divided by 7 gives the number of groups of seven deities, Brahmā, Moon, Indra, Kubera, Varuṇa, Agni and Yama, passed, and the remainder, the deity counted from Brahmā.

Then Mallikārjuna draws attention to the *Bṛhatsaṃhitā* of Varāhamihira where the effects of each deity are given (*Cf.*, *BS*, v. 19-23 and the commentary thereon by Bhaṭṭotpala).

The remainder, after dividing by 180, gives the time of the present deity passed, and when subtracted from 180, the result gives the time to be passed. If either of these is less than 14°, there is the possibility of a lunar eclipse on the Full Moon day ; if less than 18°, there is the possibility of a solar eclipse on the New Moon day ; if it is equal

on the fifteenth *tithi* (*i.e.*, Full Moon day) in the light half of the lunar month, at sunset.

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to 14° or greater than 14°, there is no possiblity of a lunar eclipse ; and if it is equal to 18° or greater than 18°, there is no possibility of a solar eclipse.

Now, the lord of the year. Calculate the *ahargaṇa* from the beginning of the *Kaliyuga* and subtract 373 from it. Divide the remainder by 360. Multiply the quotient by 3 and add 1 to it. Divide the sum by 7 and the remainder gives the lord of the year counting from the Sun. The remainder obtained in the division by 360 gives the days passed when the present lord began.

Next, the lord of the month. Subtract as before, 373 from the *ahargaṇa* and divide the remainder by 30. Multiply the quotient by 2 and add 1 to it. Divide the result by 7. The remainder gives the lord of the month counting from the Sun. The remainder obtained in the division by 30 gives the days passed when the present lord began.

Mallikārjuna says that the distance in *yojanas* between a place on the prime meridian and the observer's station divided by 80 gives the difference in *ghaṭikās*. He calls these *deśāntara-ghaṭikās*.

A *horā* is equivalent to 2½ *ghaṭikās*. Thus, there are 24 *horās* in a day of 60 *ghaṭikās* and there will be 24 deities, one presiding over each *horā*. If one wants to find the deity of a particular *hora*, one should first calculate the number of *ghaṭikās* elapsed since sunrise at that particular time. They should be doubled and then divided by 5. The quotient would give the lord of the hour. The lord of the first *horā* is the lord of the day. The lord of the second *horā* would be the sixth from it. And so on. The order is Sun, Venus, Mercury, Moon, Saturn, Jupiter and Mars.

The lords of the year, month and *horā* have been very lucidly explained by Bhaṭṭotpala in his commentary on the *Khaṇḍakhādyaka* towards the end of the first chapter (Edn., Pt. II, pp. 39-41). See also *BSS*, xiii. 41, 43-46 ; *SS*, ii. 87-90.

[Circumference of the sky]

2. Multiply 21,600 by 10 and then by the revolutions of the Moon in a *yuga*. The result is the *yojanas* in the circumference of the sky up to which the rays of the Sun reach.

Mallikārjuna then gives the method to calculate Jupiter's year, *viz.*, the period it takes to pass through one sign of the zodiac.

First find the mean longitude of Jupiter on the particular day and give the necessary corrections. Then,

$$\frac{(\text{Jupiter's revolutions} \times 12 + \text{the signs in the longitude}) - 34}{60}$$

gives the years beginning with *Prabhava,* etc. From the degrees, etc. of the longitude, the months, days, etc. can be calculated. *Prabhava* is the thirtyfifth year. See *SS*, i. 55. There, the years are counted from *Vijaya* and therefore, 34 is not subtracted.

2. According to this formula, the circumference of the sky is 5,77,53,336 × 21600 × 10 = 1,24,74,72,05,76,000 *yojanas.* This is the measurement given by Āryabhaṭa in *ĀBh, Daśagītikā*, 6, and Bhāskara I in *MB*, vii. 20.

According to Brahmagupta, Śrīpati and Bhāskara II, whose epoch starts from the *kalpa*, the circumference of the sky is 18,71,20,69,20,00,-00,000 *yojanas* (*BSS*, xxi. 11 ; *SŚe*, ii. 59 ; and *SŚi*, I. iv. 1 *ab*).

Mallikārjuna says in his commentary that the above measurement is *kalpita* or 'imaginary'. This multiplied by 1000 would be the circumference of the sky when the epoch is *kalpa*. The measurement given by Brahmagupta etc. is not so. The rule, however, is the same, but the result differs because of the difference in revolutions of the Moon in a *kalpa*.

[Distance of the planet from the Earth]

3. (The circumference of the sky in *yojanas*) divided by 72,000 gives the circumference of the orbit of the asterisms. Again, (the circumference of the sky in *yojanas*) divided by the number of revolutions of each planet in a *yuga*, gives the circumference of the planet's orbit

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3. $\frac{1,24,74,72,05,76,000}{72,000} = 17,32,60,008$ is the measurement of the orbit of the stars.

According to the Indian astronomers, a planet's orbit

$$= \frac{\text{circumference of the sky}}{\text{planet's revolutions in a } yuga}$$

$$= \frac{5,77,53,336 \times 21,600 \times 10}{\text{planet's revolutions}}$$

The radius of the planet's orbit is taken as the distance of the orbit from the centre of the earth or the planet's *yojanakarṇa*. The *yojanakarṇa* or radius follows from :

Earth's circumference : earth's radius : :

planet's circumference : ?

or, 3927 : 625 : : planet's circumference : ?

$$\text{So, planet's radius} = \frac{\text{circumference} \times 625}{3927}$$

$\pi = \frac{3927}{1250} = 3.1416$, as given by Lalla and Mallikarjuna in the commentary. This is the value of π in the table of R sines which is borrowed from Āryabhaṭa.

In i. 43, Lalla takes $\pi = \frac{22}{7}$. Śrīpati takes $\pi = \sqrt{10}$ in *SŚe*, v. 4, when he calculates the *yojanakarṇas*. The ancient Indian astronomers generally took $\pi = \sqrt{10}$.

(in *yojanas*). When this circumference is multiplied by 625 and divided by 3,927, the result is the distance of the planet from the earth (in *yojanas*).

[Distance of the Sun and Moon from the Earth]

4. The mean distance of the Sun from the earth's centre is 4,59,585 *yojanas* and that of the Moon is 34,377 *yojanas*.

5. When their mean distances are multiplied by their respective *mandasphuṭa* hypotenuse and divided by the radius, the results are their

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Bhāskara II gives the same method for calculating the *yojanakarṇas* in *SŚi*, I. v, commentary on the third verse.

While commenting on this verse, Mallikārjuna says that in this work 2 *yojanas* are equal to 3 *yojanas* of the *Siddhāntas*. So, the orbits obtained here, when multiplied by 3 and divided by 2, will give results equal to those in the *Siddhāntas*, *SS*, etc.

4. Bhāskara I gives the same distance as Lalla in *MB*, v. 2 ; *LB*, iv. 2. Since, according to Bhāskara II, the Sun's orbit is 43,31,497½ *yojanas* and the Moon's orbit is 3,24,000 *yojanas*, their *yojanakarṇas* are, respectively, 6,89,377 and 51,566. (*SŚi*, I.v. 3). Śrīpati gives the same orbits as Bhāskara, but as he takes $\pi = \sqrt{10}$, according to him the *yojanakarṇas* are, respectively, 6,84,870 and 51,229 (*SŚe*, ii. 63 ; v. 7).

5. This gives the methods to find the correct distances of the Sun and Moon from the earth.

First find their *manda* hypotenuse. Then, from the proportion :

R : *manda* hypotenuse : : distance in *yojanas* : ?,

the more accurate distance is obtained.

The same rule is in *MB*, v. 3 ; *LB*, iv. 3 ; *BSS*, xxi. 31 ; *SŚe*, v. 4 *cd* ; and in *SŚi*, I. v. 5 *ab*.

correct distances. Or, the mean distance multiplied by the mean motion and divided by the true motion, gives the correct distance (of the Sun or Moon) from the earth.

6. The diameter of the Moon is 315 *yojanas* and that of the Sun is 4410. The Sun's distance from the earth multiplied by 5 and divided by 16 gives the height of the cone of the earth's shadow.

The second formula follows from the proportion :

True motion : mean motion = R : *manda* hypotenuse
= distance in *yojanas* : ?.

6. This gives the method to calculate the height of the cone of the earth's shadow.

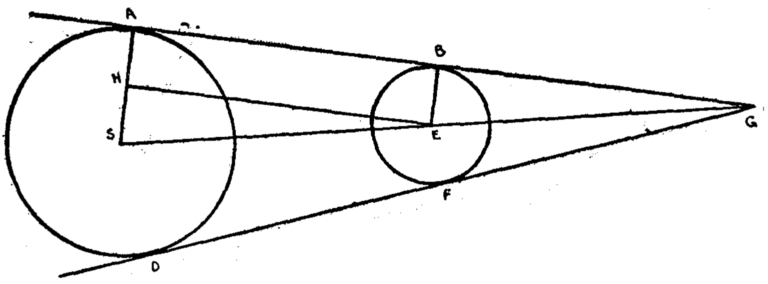

Fig. 12

Let S be the Sun, E the earth, and AB and DF their common tangents meeting in G. EH is parallel to AB. Then, GE is the height of the cone of the earth's shadow or *mahīprabhā*, and $= \dfrac{ES \times BE}{HS}$

$$= \frac{\text{Sun's distance from earth} \times \text{earth's radius}}{\text{Sun's radius} - \text{earth's radius}} \qquad (1)$$

$$= \frac{\text{Sun's distance} \times \frac{1050}{2}}{\frac{4410-1050}{2}} \quad \text{(given by Lalla, i. 43)}$$

$$= \frac{\text{Sun's distance} \times 525}{1680}$$

[Diameter of the Earth's shadow in yojanas]

7. The length of the earth's shadow (as found above) diminished by the correct distance of the Moon from the centre of the earth, then multiplied by 1050 and divided by itself, gives, as result, the diameter of the earth's shadow in *yojanas* (in the Moon's orbit).

$$= \frac{\text{Sun's distance} \times 5}{16} \qquad (2)$$

(1) is in *MB*, v. 71 ; *LB*, iv. 6 ; *BSS*, xxiii. 8 ; and in *SŚe*, v. 8 *ab*. (2) is in *MB*, v. 72.

7. This gives the method to find the diameter of the earth's shadow in *yojanas*.

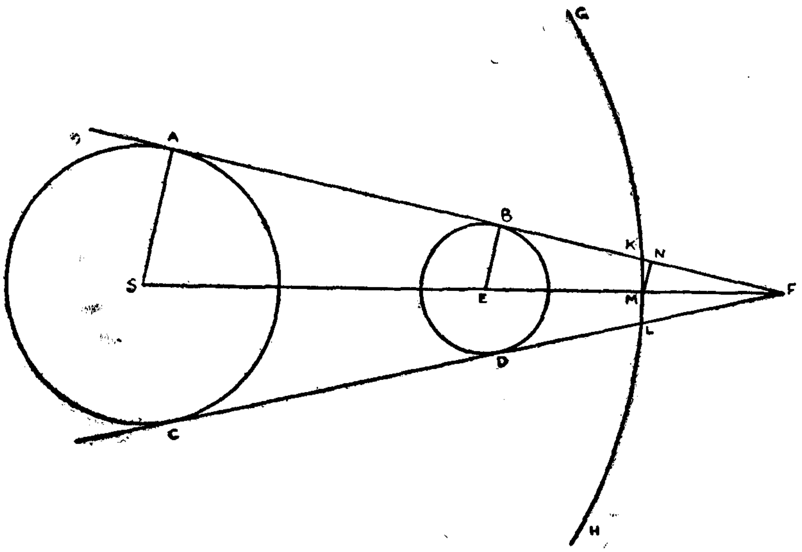

Fig 13

Let S be the Sun, E the earth, AB and CD the common tangents meeting at F, and GH the Moon's orbit meeting the tangents in K and L, and SE produced in M. Then, KML is the path of the Moon which is within the shadow. Let MN be perpendicular to ABF.

[Angular diameter of the Sun etc.]

8. The diameters of the Sun, Moon and the shadow, (each expressed in *yojanas*), and multiplied by the radius and divided, respectively, by the Sun's distance from the earth, the Moon's distance and the Moon's distance, (in *yojanas*), give the respective angular diameters in minutes, whence the arcs corresponding to the quotients as R sines are found. The results can also be treated approximately as diameters (without finding the arcs).

Then the diameter of the earth's shadow within the Moon's path is measured by 2 MN, approximately.

$$\text{Now, } MN = \frac{EB \times FM}{FE}$$

$$= \frac{\text{earth's radius } (FE - ME)}{FE}.$$

$$\text{So, } 2\ MN = \frac{1050\ (\text{earth's shadow} - \text{Moon's distance})}{\text{earth's shadow}}$$

This formula is in *MB*, v. 72 *cd*-73 *ab* ; *LB*, iv. 7 ; *BSS*, xxiii. 9 and in *SŚe*, v. 8 *cd*.

8. This gives the method to convert the diameters of the Sun, etc., expressed in *yojanas*, into minutes. From the proportion :

Sun's distance : Sun's diameter : : R : ?,

the R sine of the Sun's angular diameter is obtained. The corresponding arc in minutes is the angular diameter. Or, the result from the proportion could also be treated as the angular diameter, approximately. Similarly, for the Moon and the earth's shadow.

This rule is given in *MB*, v, 5 ; *LB*, iv. 5 ; *BSS*, xxi. 34 ; *SŚe*, v, 6, and in *SŚi*, I, v. 7,

9. Or, the true motion of the Moon multiplied by 11 and divided by 272 gives its angular diameter in minutes. That of the Sun is obtained by multiplying its true motion by 11 and dividing by 20. The difference between 8 times the true motion of the Moon and 25

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9. This gives the rule to find the angular diameters of the Sun, etc. from their respective true motions.

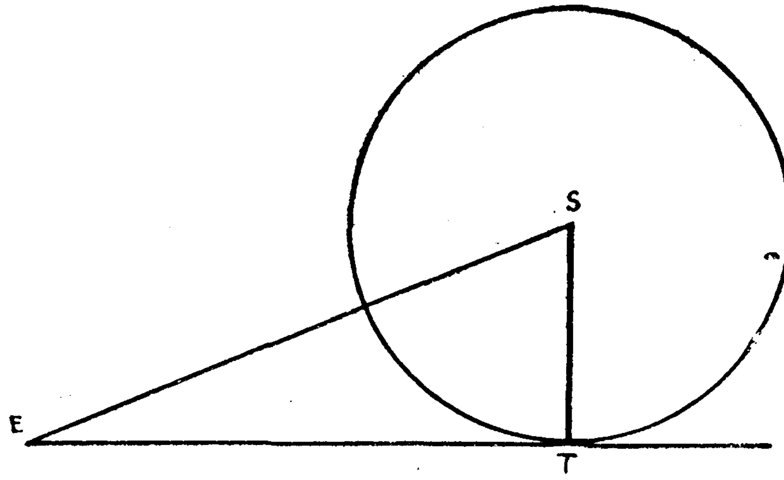

FIG. 14

Let S be the Sun, E the earth and ET the tangent. Then, twice the angle SET is the angular diameter of the Sun.

$$R \sin SET = \frac{ST \times R \sin STE}{SE}$$

$$= \frac{\text{Sun's radius} \times R}{\text{Sun's distance}}$$

$$= \frac{\frac{4410}{2} \times \text{true motion}}{7906 = \frac{1,24,74,72,05,76,000}{1,57,79,17,500} \text{ mean motion}}$$

times that of the Sun, divided by 60, gives the angular diameter of the shadow (in minutes).

[The eclipser and the eclipsed]

10. The shadow is the Moon's obscuring body and the Moon is the Sun's obscuring body. There are total and partial eclipses of the

---

$$=\frac{\text{true motion}\times \frac{4410}{2} \div \frac{4410}{22}}{7906 \div \frac{4410}{22}}$$

$$=\frac{11 \text{ true motion}}{40}, \text{ nearly.}$$

So, 2 SET $=\frac{11}{20}$ true motion, in minutes, nearly.

Similarly, the Moon's angular diameter

$$=\frac{2\times\text{true motion}\times \frac{315}{2} \div \frac{315}{22}}{7906 \div \frac{315}{22}}$$

$$=\frac{11}{276} \text{ true motion, in minutes, nearly,}$$

It would be 272 if the diameter is 320.

Similar rules are in *MB.* v. 6-7 ; *BSS*, iv. 6 *ab* ; *SŚe*, v. 9 *ab*, and *SŚi*, I. v. 8.

The earth's shadow

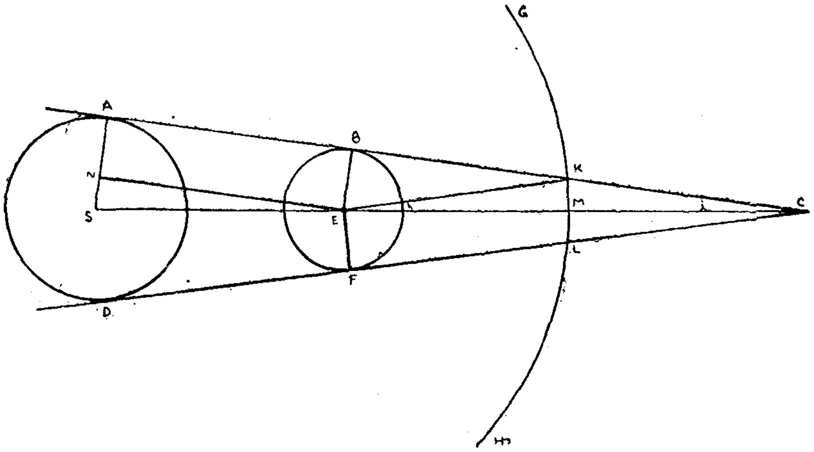

FIG. 15

obscured body caused by the obscuring body. (The eclipse) is named after the (eclipsed portion of the) obscured body.

Let S be the Sun, E the earth, and GKMLH the Moon's orbit cutting the common tangents AB and DF at K and L, respectively. Let EK be joined and EN be parallel to AB. Then, twice KEC is the angular diameter of the shadow.

$$\text{R sin SEN} = \frac{\text{SN} \times \text{R sin SNE}}{\text{SE}}$$

$$= \frac{\text{R (Sun's radius—earth's radius)}}{\text{Sun's distance}}$$

$$= \frac{\text{R} \times \text{Sun's radius}}{\text{Sun's distance}} - \frac{\text{R} \times \text{earth's radius}}{\text{Sun's distance}}$$

$= \frac{1}{2}$ Sun's angular diameter—Sun's horizontal parallax.

So, angle SEN = the same, nearly.

Again, R sin EKB

$$= \frac{\text{EB} \times \text{R sin KBE}}{\text{EK}}$$

$$= \frac{\text{R} \times \text{earth's radius}}{\text{Moon's distance}}$$

= Moon's horizontal parallax,

So, angle EKB = the same, nearly.

So, 2 angle KEC = angular diameter of the earth's shadow

= 2 (angle EKB—angle ECK)

= 2 (angle EKB—angle SEN)

= 2 Moon's horizontal parallax—$2 \times \frac{1}{2}$ Sun's angular diameter

+ 2 Sun's horizontal parallax

$$= 2 \times \frac{\text{Moon's true motion}}{15} - \frac{11}{20}\ \text{Sun's true motion}$$

$+ \frac{2}{15}$ Sun's true motion

[Latitude of the Moon]

11. When the Moon is equal to the Sun in respect of minutes etc., subtract from its longitude, the longitude of its ascending node. Find the R sine of the remaining arc, multiply it by 15 and divide by

---

$$= \frac{8 \text{ Moon's true motion} - 33 \text{ Sun's true motion} + 8 \text{ Sun's true motion}}{60}$$

$$= \frac{8 \text{ Moon's true motion} - 25 \text{ Sun's true motion}}{60}$$

(According to Indian astronomer's the horizontal parallax is $\frac{1}{15}$ of the planet's true motion).

The same formula is given *BSS*, iv. 6 *cd* ; *SŚe*, v. 9 *cd* ; and *ŚSi*, I. v. 9.

11. This gives the method to find the Moon's latitude.

191. The result is the latitude of the Moon and its direction is according to the hemisphere in which the Moon happens to be diminished by its node.

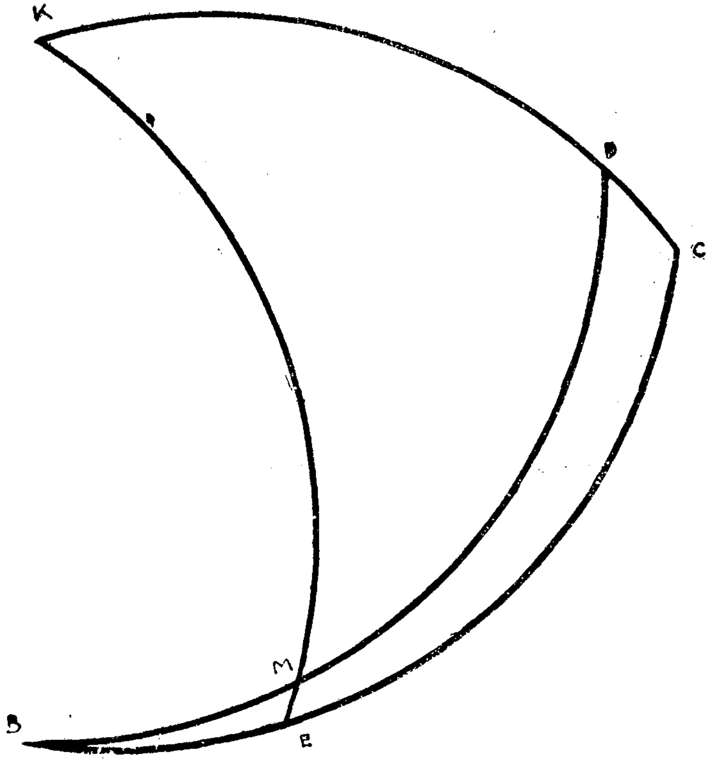

Fig. 16

Let BEC be a quadrant of the ecliptic and K its pole. Let BMD be the Moon's orbit or *vimaṇḍala*, B its node and KME and KDC the verticals. ME is the latitude and DC the maximum latitude or 270′.

$$\text{Then, R sin ME} = \frac{\text{R sin DC} \times \text{R sin BE}}{\text{R sin BC}}$$

$$= \frac{\text{DC (nearly)} \times \text{R sin } d}{\text{R}}$$, where d is the difference between the Moon's and the node's longitudes

$$= \frac{270 \times \text{R sin } d}{3438}$$

[Obscured portion at mid-eclipse]

12. When this latitude is subtracted from the sum of the semi-diameters of the obscuring and the obscured bodies, the remainder is the portion obscured (at the time of mid-eclipse). When the portion is greater than the obscured body, the latter is said to be completely obscured.

$$=\frac{15\times R\sin d}{191}$$

So, $ME=\frac{15}{191}\times R\sin d$ minutes, nearly.

Similar formulas are given in *MB*, v. 30 ; *LB*, iv. 8 ; *BSS*, iv. 5 ; *SŚe*, v. 10 *ab* and *SŚi*, I. v. 10.

12. This gives the method to find the obscured portion at mideclipse.

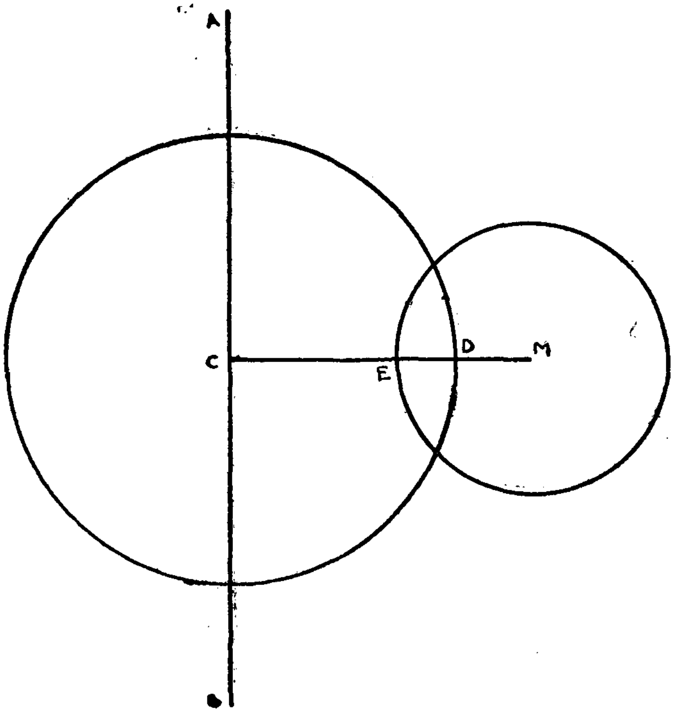

FIG. 17

Let AB be the ecliptic, C the centre ot the earth's shadow and M the Moon. At mid-eclipse the centre C and the Moon M will lie on the same vertical to the ecliptic or the same *kadambaprota*. Thus, CEDM is perpendicular to ACB.

Now, DE=obscured portion

=CD+ME—CM

=sum of the radii—latitude.

The same formula holds good in a solar eclipse as well.

13. Or, subtract half the difference of the diameters of the obscuring and the obscured bodies from the latitude of the Moon. The remainder is the portion not obscured. If there is no remainder, the obscured body is completely obscured.

[Half-duration of the eclipse]

14-15. Take the sum or difference in the semi-diameters of the obscuring and the obscured bodies. Square it and subtract from it the square of the Moon's latitude. Find the square root of the remainder. Multiply it by 60 and divide by the difference of the true motions in minutes of the two bodies. The results are, respectively, the approximate half durations of the eclipse and the total eclipse in *ghaṭikās*.

When these times are severally multiplied by the true motions of the Sun, Moon and its node and each product divided by 60, the results are, respectively, the motions of the Sun, Moon and its node during these times.

16. The Sun's and Moon's motions in minutes should, respectively, be subtracted from their longitudes and the node's motion added to its longitude, if the first half of duration of the eclipse or of the total eclipse is required ; the reverse process is to be followed if the second half is required.

From these longitudes, again calculate the half duration of the

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13. The formula follows from the above diagram, fig. 17. These are the usual methods.

14-16. These indicate how to find the durations of the first half and second half of either a partial or a total eclipse.

eclipse and of the total eclipse. Repeat the process till the times are fixed.

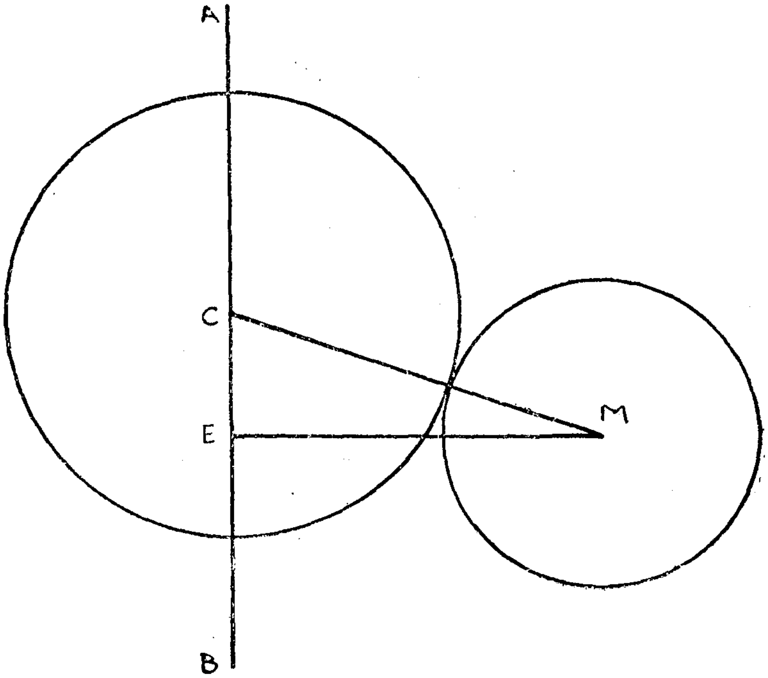

Fig. 18(i)

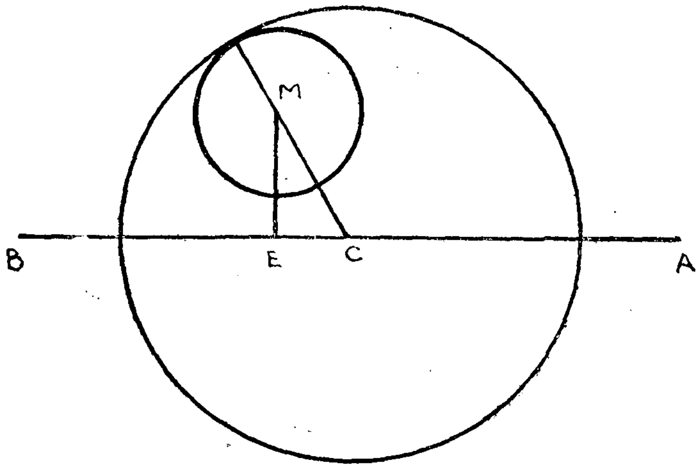

Fig. 18(ii)

17. When the first half of the duration of an eclipse is subtracted from the time of the Full Moon, the remainder gives the time when the eclipse began. When the second half of the duration of an eclipse is added to the time of the Full Moon, the sum gives the time when the eclipse ends. So the wise say.

The sum of the first and second half of the duration of the eclipse is its total duration. The sum of the first and second half of the duration of the total eclipse is the duration of the complete disappearance of the Moon.

Let AB be the ecliptic, C the centre of the earth's shadow, M the Moon touching the earth's shadow externally in fig. 18*a* and internally in fig. 18*b*. Let ME be the Moon's latitude.

Then, assuming the latitude to be fixed, when CE in fig. 18*a* is covered by the Moon, there is mid-eclipse ; and when CE in fig. 18*b* is covered by the Moon, there is mid-total eclipse. CE can be found from the right-angled triangle CEM. Then the corresponding time can be found from the proportion :

Difference in daily motions of the Moon and the Sun : 60 : : CE : ?

Thus the half duration of an eclipse and that of a total eclipse are obtained.

The latitude of the Moon, however, is not constant and, so, the above times are only approximate. In order to get more accurate times, the longitudes of the Moon and its node at the beginning and end of an eclipse or of a total eclipse must be found ; and hence the Moon's latitude. Then the half durations should be calculated. The process is to be repeated till the times are fixed.

The same method is used in the case of the solar eclipse, when the Sun's longitude should also be calculated.

While commenting on verse 15, Mallikārjuna says that in a solar eclipse, when the Moon's latitude is 0, the half duration of a total

18. The end of the Full Moon is the time of mid-eclipse. That time, diminished by the first half of the duration of the total eclipse, gives the time when the Moon is completely obscured. Again, that time added to the second half of the duration of the total eclipse, gives the time when the Moon begins to reappear. So say those who know.

[Obscured portion at any time]

19. When the given time (during an eclipse) is subtracted from the first or second half of the duration of an eclipse, and the remainder is multiplied by the difference in minutes of the motions (of the obscuring and the obscured bodies) and divided by 60, the result is

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eclipse is ¼ of a *ghaṭikā*. This same eclipse, when the Sun's diameter is large, is known as *kaṅkaṇagrahaṇa*.

The above method is in *MB*, v. 75-76 ; *LB*, iv. 10-13 ; *BSS*, iv. 8-9 ; *SŚe*, v. 12-13 ; *SŚi*, v. 12-14 and *SS*, iv. 12-15.

19. This gives the method to find the obscured portion at any given time elapsed after the contact or to elapse before separation.

called *bhuja* or base. The latitude of the Moon at that time is called *koṭi* or perpendicular. The square root of the sum of the squares of the base and the perpendicular is the hypotenuse at that time.

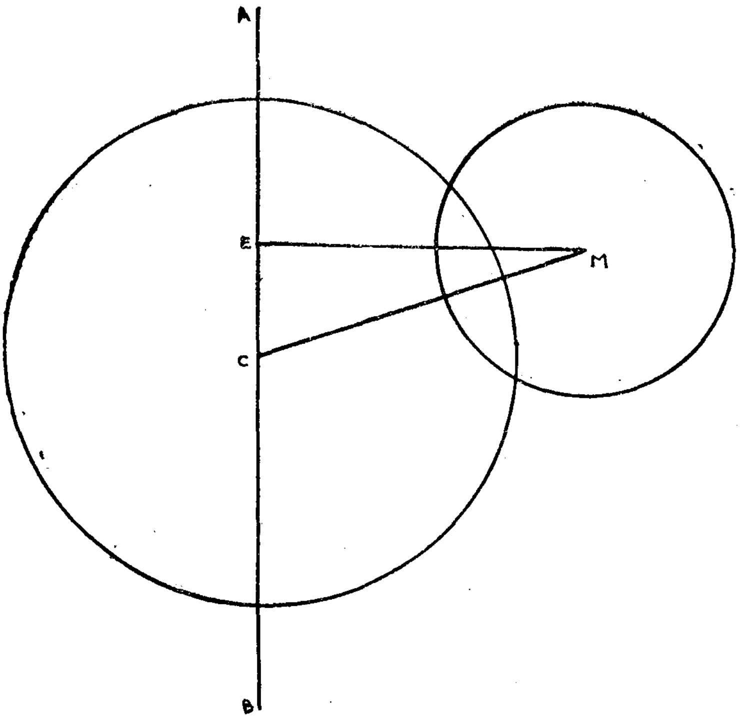

Fig. 19

Let AB be the ecliptic, C the centre of the earth's shadow, M the Moon, and ME its latitude. CE is called *bhuja* or base, and ME the *koṭi* or perpendicular. CE is found from the proportion :

Difference in the motions of Moon and Sun : 60 : : CE : given time.

ME is then calculated for that time. Then CM is calculated from the right-angled triangle CEM thus : $CM = \sqrt{CE^2 + ME^2}$. When CM is subtracted from the sum of the radii of the Moon and the shadow, the

20. In the same way, when the first or second half of a total eclipse is multiplied by the difference of the true motions (of the obscuring and the obscured bodies, and the product is divided by 60, the result is the base). The latitude of the Moon at these times is the perpendicular. (The square root of the sum of the squares of the base and the perpendicular) is the hypotenuse at the beginning of the total eclipse (if the first half is taken), and is the hypotenuse for the end of the total eclipse (if the second half is taken).

(In both cases), when the hypotenuse is subtracted from the sum of the semi-diameters of the two bodies, the remainder is the obscured portion.

[Time from the obscured portion]

21-22. Subtract the given obscured portion from the sum of the semi-diameters of the two bodies. Square the remainder. Subtract from it the square of the Moon's latitude (at the time of mid-eclipse). Find the square root of the remainder. Multiply it by 60 and divide by the difference of the true motions of the two bodies. Subtract the result from the first half of the duration of the eclipse, if the observed portion is between the beginning of the eclipse and the mid-eclipse. But if it is between the mid-eclipse and the end, subtract the quotient

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difference is the obscured portion at the given time. The same method is used also for the solar eclipse.

This rule is given in *MB*, v. 62-63 ; *BSS*, iv. 11-12 ; *SŚe*, v. 14, *SŚi*, I. v. 15-17 *ab* and in *SS*, iv. 18-20.

20. In the same way, in the case of a total eclipse, the hypotenuse in the beginning and end can be calculated.

21-22. These give a process converse to that given above, that is, to find the time when the obscured portion is given. As the Moon's latitude at the time when the portion is obscured is not known, its latitude for the mid-eclipse is used and by the converse of the

from the second half of the duration of the eclipse. The result is approximately the time when the given portion is obscured.

Find the Moon's latitude at this time and repeat the process till the time is fixed. (This then is the correct time).

[Valana : Deflection due to latitude and declination]

23. Multiply the equinoctial midday shadow by the R versed sine or the *utkramajyā* of the hour-angle at the beginning of the eclipse, etc. and divide by the hypotenuse of the equinoctial shadow. Remember that the arc corresponding to this quotient as the R sine, (which is called *ākṣavalana*), is to be taken as north or south according as the obscured body is in the eastern or the western hemisphere.

24. In a lunar eclipse, the Moon, (the obscured body), is said to be in the eastern hemisphere from midday till midnight. From midnight till midday it is said to be in the western hemisphere. In a solar eclipse, (for the Sun, which is the obscured body), the contrary is the case.

25. Increase the number of degrees in the longitude of the obscured body by 3 signs and then find its R versed sine or *utkramajyā*. Hence find, as before, the R sine of the declination. The corresponding arc (*ayanavalana*) has the same denomination as that of the Moon

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method explained above, an approximate time is calculated. Then the longitudes of the Moon and its node are calculated for this time and hence the latitude. Again the time is calculated. The process is repeated till the time is fixed. This then is the required time.

The same method is followed in the case of the solar eclipse also.

This rule is in *BSS*, iv. 13-14 ; *SŚe*, v. 15 ; *SŚi*, I. v. 17 *cd*-18. See also *SS*, iv. 22-23.

23-26. These give the method to find the *valana*. *Valana* is the variation of the eastward direction of the ecliptic from the eastward

increased by 3 signs. The latitude of the Moon has the same denomination as that of the Moon diminished by its node.

26. The *ākṣavalana*, the *āyanavalana* and the Moon's latitude, all in minutes, should be added together if they are of the same denomination but their difference must be taken when of different denominations. Then find the R sine of the sum or difference, as the case may be. The result is the R sine of the *valana* or variation of

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direction of the disc of a planet. According to Indian astronomers it is measured on the arc of a circle which is at a distance of 90° from the place of the planet on the ecliptic. It consists of two arcs, *viz.*, (1) the arc between the east point on the ecliptic and that on the celestial equator, and (2) the arc between the latter point and the east point on the prime vertical. The first is called *āyanavalana* and the second *ākṣavalana*. Thus the *āyanavalana* is also measured by the angle between the secondaries to the ecliptic and the equator, and the *ākṣavalana* by the angle between the secondaries to the equator and the prime vertical.

the eastward direction of the ecliptic from the eastward direction of the disc of the obscured body.

The correct calculation of the *valana* is as follows :

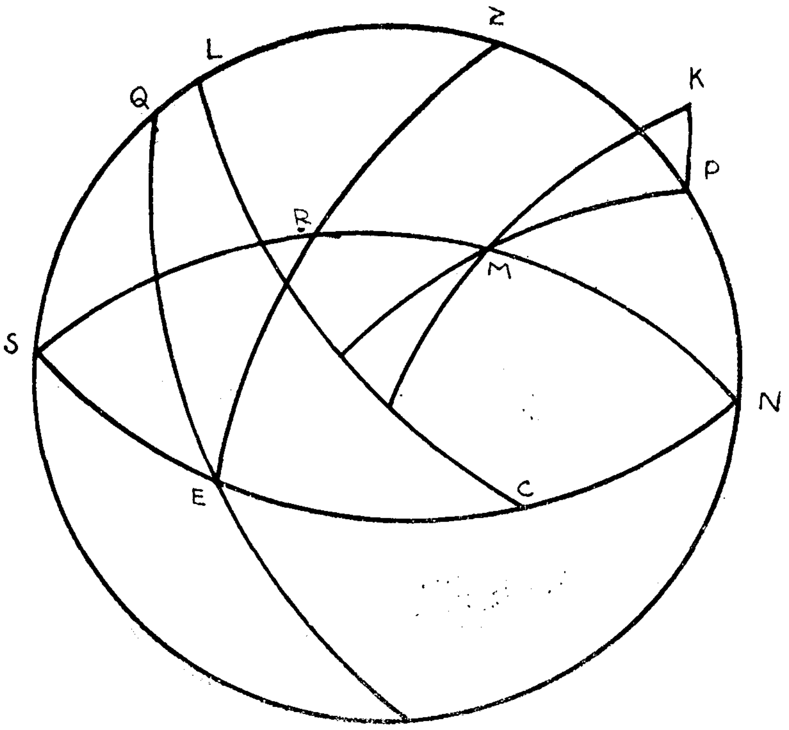

Fig. 20

Let SZPN be the meridian, ZE the prime vertical, EQ the equator and CL the ecliptic. Let Z be the zenith, P the pole of the equator and K the pole of the ecliptic. Let N be the north point, and M the Moon. Through M and N, P and K draw secondaries to the prime vertical, equator and ecliptic. Then, the angle NMP measures the *ākṣavalana* and PMK the *āyanavalana*.

Now, in the spherical triangle PNM,

$$R \sin NMP = \frac{R \sin PNM \times R \sin NP}{R \sin PM}$$

Angle PNM is the Moon's zenith distance on the prime vertical, NP the latitude of the place, and PM the complement of the Moon's declination.

So, R sine NMP or *ākṣavalanajyā*

$$= \frac{\text{R sin Moon's zenith distance} \times \text{R sin latitude}}{\text{radius day-circle } (=\text{R approximately})}$$

$$= \frac{\text{R sin Moon's zenith distance} \times \text{equinoctial shadow}}{\text{equinoctial hypotenuse}}$$

(from the similar triangles U and Z) (1)

So, *ākṣavalana* is the arc corresponding to (1) as *R sine.*

Again, from the spherical triangle PMK,

$$\text{R sin PMK} = \frac{\text{R sin PKM} \times \text{R sin PK}}{\text{R sin PM}}$$

Angle PKM is 90°+Moon's longitude, PK is 24° or measure of obliquity or maximum declination.

Then, R sin PMK or *āyanavālanajyā*

$$= \frac{\text{R sin } (90^\circ + \text{Moon's longitude}) \times \text{R sin } 24^\circ}{\text{radius day-circle } (=\text{R approximately})}$$

$$= \text{R sin declination of the planet increased by } 90^\circ$$

So, *āyanavalana*=declination of the planet increased by 90°.

Brahmagupta gives the above formulas both in *BSS*, iv. 16-17 and *KP*, iv. 7.

According to verse 23, Lalla's formula for *ākṣavalana* is the arc corresponding to

$$= \frac{\text{R versed sin of the hour-angle} \times \text{equinoctial shadow}}{\text{hypotenuse equinoctial shadow}}$$

as R sine. This is wrong, as the above proof shows.

Then, according to verse 25, Lalla's formula for *āyanavālana* is the arc corresponding to

$$= \frac{\text{R sin } 24^\circ \times \text{R versed sine } (90^\circ + \text{longitude})}{\text{R}}$$

as R sine. This also is wrong, as the above proof shows.

Bhāskara I also uses versed sine in *MB*, v. 42-45 and *LB*, iv. 15-17. Pṛthūdaka follows Lalla in the use of hour-angle and R versed sine (see his commentary on the above verses in *BSS*, and his

27. Divide the R sine of the *valana* and 3438, the radius, by 110, to convert them into *aṅgulas*. When the altitude in time (*unnatakāla*) on any day at any given time, is divided by half the duration of the day and 2½ added to the result, the sum is the number of minutes equivalent to 1 *aṅgula*.

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commentary on *KP*, iv. 7. *Cf.* also (*KSG*, pp. 99-100). So does Bhaṭṭotpala (see his commentary on KP, iv. 7 ; *KK* vol. II, pp. 116-19). Āmaśarmā follows the same practice (*KBM*, pp. 134-36). Ś īpati also follows Lalla in *SŚe*, v. 19-20.

Āryabhaṭa does not appear to have used the versed sine. (*Ābh*, *Golapāda*, 45). Readings, however, differ ; according to some it is R sine and according to others it is R versed sine.

Bhā:kara II has given a more accurate method in *SŚi*, I. v. 20-23. He has criticized the use of versed sine (above reference) an also in II, ix. 16-22. Yet while explaining Lalla's verses in his commentary he argues in favour of the versed sine. *SS* uses *natakāla* a changed into degrees but not versed sine (iv. 24-25).

Lalla corrects the total *valana* with the Moon's latitude and thus obtains the corrected *valana*. Śrīpati follows him, as mentioned above (*SŚe*, v. 19-20). Thus Lalla differs from Brahmagupta (iv. 18) and Bhāskara II (above references). See also Bhāskara I's rules (*MB*, v. 46-47 and *LB*, iv. 18-20).

Valana is used for projection of an eclipse. It shows the direction of the ecliptic and hence the position of the obscured body.

27. To convert R sine *valana* and R into *aṅgulas*, Lalla assumes 110 units as equivalent to 1 *aṅgula*.

The second formula relates to the conversion of minutes into *aṅgulas*.

According to Indian astronomers, 2½ minutes are equal to 1 *aṅgula*, when the body is on the horizon, and 3½ minutes are equal to 1 *aṅgula*, when the body is on the merdian. Thus, if the time elapsed after rising is x, the variation is obtained from the proportion : Half day : 1 : : x ; ?

[Conversion of angular deflection to linear deflection]

28. The sum of the radii of the obscuring and the obscured bodies, the latitude of the Moon, the obscured portion, the hypotenuse, the base and the perpendicular, (expressed in minutes) should be divided by the number of minutes in 1 *aṅgulas*.

[Graphical representation of the eclipse]

29. In a lunar eclipse, contact takes place in the eastern portion of the disc of the Moon and separation in the western portion. The contrary is the case in a solar eclipse.

The latitudes of the Moon should always be drawn in a direction contrary to their own (in the projection) of a lunar eclipse. (But in the projection) of a solar eclipse, the latitudes should be drawn in their own direction.

30-33. Draw three (concentric) circles, with radii respectively equal to the radius of the obscured body, the sum of the radii of the obscuring and the obscured bodies, and the radius (*trijyā*). Mark the directions (north, etc.) in these circles. (In the projection) of a lunar eclipse, for contact, the *valana* or deflection should be marked along

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Let it by y.

Then, the variation $y = \dfrac{x}{\text{half day}}$

So, at that time 1 *aṅgula* $= 2\frac{1}{2} + \dfrac{x}{\text{half day}}$ minutes.

Bhāskara I's formula is in *MB*, v. 53 cd and in *LB* iv. 22. Lalla's formula is given by Śrīpati in *SŚ*, v. 23 and by Bhāskara II in *SŚi*, I. v. 24 cd.

28. This conversion is required for drawing the projection of an eclipse.

29-35. These give the construction for the projection of an eclipse, It is clear from the translation.

On the same topic, see *MB*, v. 48-67, 77 ; *LB*, iv. 23-32 ; *BSS*, 14-18 ; v. 25-38 ; *SŚi*, I. v. 26-35 and *SS*, vi.

the third circle, from the east point, in its own direction. For separation, it should be marked from the west point in a direction opposite to its own. In both the cases it should be expressed as an R sine.

In a solar eclipse, for contact, the *valana* should be marked from the west point in a direction opposite to its own ; and for separation, it should be marked from the east point in the same direction as its own. Here again, it should be expressed as an R sine.

In the projection of a solar eclipse, at the time of mid-eclipse, if the Moon's latitude is north, the *valana* or deflection should be marked from the north point eastward, if its own direction is opposite to that of the latitude, and westward, if its own direction is the same. Again, if the Moon's latitude is south, the *valana* or deflection should be marked from the south point eastward, if its own direction is opposite to that of the latitude, and westward, if its own direction is the same. In the projection of a lunar eclipse, the contrary is the case.

(In each case, from the end of the *valana* or deflection thus marked), draw a straight line passing through the centre of the concentric circles. (This is called *valanasūtra*). Mark the point where it cuts the circle with radius equal to the sum of the radii of the obscuring and the obscured bodies. From this point mark along the same circle the latitudes for contact and separation, each expressed as R sine. In a solar eclipse the latitudes should be drawn in their own direction and in a lunar eclipse in the opposite direction.

At the mid-eclipse, the latitude should be marked along the *valanasūtra* from the centre of the (concentric) circles, (towards the *valana*).

In each case, with the extremity of the latitude as the centre and the radius of the obscuring body as radius, describe a circle (cutting the obscured body). Thus are known the points of contact and separation and also the obscured part at the mid-eclipse.

34-35. (Mark) the three extremities of the latitudes, (at the beginning, middle and end of the eclipse). (Draw) two fish-figures, (one passing through the first two points and the other through the

last two points). Draw two lines passing through the mouth and tail of each fish-figure. (With the point of intersection of these two lines as centre) draw a circle passing through the three extremities of the latitudes. This is the path of the obscuring body. Then place the hypotenuse from the centre (of the concentric circles) just touching the path.

(In the projection of a lunar eclipse, when the obscured portion is increasing, that is, between the beginning and middle of the eclipse), the hypotenuse must be drawn eastward. But when the obscured portion is decreasing, (that is, between the middle and end of the eclipse), the hypotenuse must be drawn westward. In the projection of a solar eclipse, the contrary is the case.

With this point of intersection as the centre, draw the obscuring body. Thus is found the obscured portion at any time.

[Colour of the eclipse]

36. At the beginning and end of an eclipse, the Moon is of dense smoky colour. In a partial eclipse, it is always dark as a mass of collyrium. When the obscured portion is greater than half, it is dark red. When it is completely obscured, it is tawny.

The ends Chapter Five

on

LUNAR ECLIPSE

CHAPTER VI

SOLAR ECLIPSE

[Data for computing the Solar eclipse]

1. One intending to ascertain a solar eclipse must first find the true longitudes of the Sun, Moon and its ascending node, on the fifteenth day of the dark half of the lunar month at sunrise.

[Meridian ecliptic point]

2. (To find the meridian ecliptic point or *madhyalagna*), the time when the *amāvāsyā* or New Moon ends, if before midday, should be added to half the duration of the night and the true longitude of the Sun should be increased by 6 signs. The *lagna* calculated from these by means of the times of rising of the signs of the zodiac at Laṅkā, according to the methods given above, is called meridian ecliptic point by the wise. If the time when the *amāvāsyā* ends is after midday, it should be diminished by half the duration of the day and the true longitude of the Sun should be considered without any chage and the *lagna* calculated. (This, again, would be the meridian ecliptic point).

[R sine altitude at the Meridian ecliptic point]

3. The latitude of the observer's station, expressed in degrees, increased or diminished by the declination corresponding to the longitude of the meridian ecliptic point, according as they are in the same or opposite directions, is called *madhya*. Its R sine is called

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MATHEMATICAL NOTES

1-7. These verses explain how to calculate the longitudinal parallax or *lambana*. The first verse prescribes the determination of

madhyajyā. The R sine of 90° minus the *madhya* is called the *śaṅku* or R sine of the altitude of the *madhya*.

[R sine amplitude of the rising point of the ecliptic]

4. Calculate the *lagna* for the time between sunrise and the end of the *amāvāsyā*, using the local times of rising of the signs of the zodiac. Multiply the R sine of the longitude of this *lagna* by the

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the longitudes of the Sun, Moon and its node at sunrise on the New Moon day.

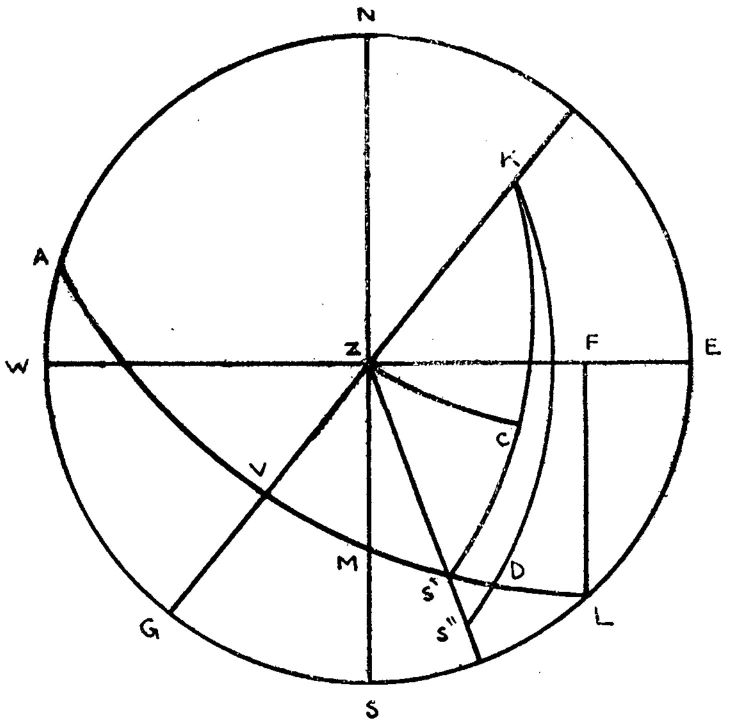

FIG. 21

(*Figure taken from PS, translated by Thibaut*)

Let NWSE be the horizon, NS the meridian, Z the zenith, AL the ecliptic, and K its pole. Let KZV be a secondary cutting AL in V. Thus V is the nonagesimal or *vitribha*. Let AL cut the meridian in M, so that M is the meridian ecliptic point or *madhyalagna*. Let

R sine of 24° and divide by the R sine of the colatitude. The result is called *udayajyā* (or R sine of the amplitude of the rising point of the ecliptic).

[Ecliptic zenith distance]

5. When the *udayajyā* is multiplied by the *madhyajyā* and divided by the radius, the result is called *bāhu* or base. The square

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AL meet the horizon in L ; then L is the orient ecliptic point. Let S′ be the position of the Sun on the ecliptic and S″ its position under parallax. Let KS′, KS″ be two secondaries through S′ and S″. Let ZC be parallel to VS′ meeting KS′ in C.

Now, verse 2 gives the method to calculate the longitude of M which is already explained in iv. 11-12. (see above, pp. 68-71).

Verse 3 gives the method to calculate ZM, the zenith distance of M or *madhya*. From the longitude of M calculate its declination (*cf.*, verse ii. 17ab) and hence its zenith distance (*cf.*, verse iv. 16) and altitude. R sine of this distance is called *madhyajyā* and R cosine is *madhyalagnaśaṅku*.

Verse 4 calculates the longitude of the orient ecliptic point L or *lagna* (*cf.*, above, iv. 11-12) at the time of the conjunction of the Sun and Moon. Then it gives the method to find LF, the perpendicular from L on EW. This is R sin amplitude of L or *agrajyā*. This is called *udayajyā*.

From the similar triangles U and V,

$$\text{R sine LE or LE} = \frac{\text{R sin declination of L} \times \text{R}}{\text{R sin colatitude}}$$

$$= \frac{\text{longitude of L} \times \text{R sin } 24^\circ \times \text{R}}{\text{R. R sin colatitude}} \qquad (cf., \text{ ii. 17ab})$$

$$\text{So, } udayajy\bar{a} = \frac{\text{longitude of L} \times \text{R sin } 24^\circ}{\text{R sin colatitude}}$$

Verse 5 gives the method to find R sin VM or *bāhu*. From fig. 21,

$$\text{R sin VM or } b\bar{a}hu = \frac{\text{R sin GS} \times \text{R sin ZM}}{\text{R sin ZS}}$$

root of the difference of the squares of the *madhyajyā* and the *bāhu* is called *dṛkkṣepajyā* (or R sine of the ecliptic zenith distance).

$$=\frac{\text{R sin LE}\times\text{R sin ZM}}{\text{R}}\qquad(\text{since arc ES}=90°=\text{arc GL})$$

$$=\frac{\textit{udayajyā}\times\textit{madhyajyā}}{\text{R}}$$

Then, considering the triangle ZVM as plane,

$$\text{R sin ZV}=\sqrt{(\text{R sin ZM})^2-(\text{R sin VM})^2}$$

R sine ZV is the R sine zenith distance of the nonagesimal. It is called *dṛkkṣepajyā*.

In verse 6, the author first finds R sin ZC. R sin S′H is calculated from the proportion R sin LV : R sin VG : : R sin S′L : R sin S′H. So, Ris ZS′=R sin (90°—S′H) is found. This is called *dṛgjyā* or the Sun's R sine zenith distance. Then R sin VS′ is obtained from,

$$\text{R sin VS}'=\sqrt{(\text{R sin ZS}')^2-(\text{R sin ZV})^2}$$

and R sin ZC is approximately taken to be equal to R sin VS′. This is called *dṛggatijyā* or simply *dṛggati*.

The second part of this verse and the next verse give the method to find S′D, the longitudinal parallax of the Sun. In the triangle S′ S″ D,

$$\text{R sin S}'\text{D}=\frac{\text{R sin S}'\text{S}''\text{D}\times\text{R sin S}'\text{S}''}{\text{R sin S}'\text{ DS}''}\qquad(1)$$

Now, R sin S'DS″=R sin 90°=R and R sin S' S″=R sin vertical parallax or *dṛglambanajyā*

$$=\frac{\text{R sin maximum parallax}\times\text{R sin ZS}'}{\text{R}}$$

$$=\frac{\text{maximum parallax (nearly)}\times\text{R sin ZS}'}{\text{R}}$$

$$=\frac{\text{radius of earth}\times\text{R}}{\text{Sun's distance}}\times\frac{\text{R sin ZS}'}{\text{R}}\qquad(2)$$

$$=\frac{525\times\text{R sin ZS}'}{\text{Sun's distance}}$$

[Syzygy corrected for Parallax in longitude]

6-7. The difference between the square of the *dṛkkṣepajyā* and that of the R sine of the Sun's zenith distance at the end of the *amāvāsya*, is called the square of the *dṛggatijyā*. Find its square root and multiply it by 525. Divide the product severally by the distances of the Sun and the Moon from the earth.

Again, R sin S'S″ D = R sin (90°—S″S'D)

= R sin (90°—ZS'V)

= R sin ZS'C

From the right angled triangle ZS'C,

$$\text{R sin ZS'C} = \frac{\text{R sin ZC} \times \text{R sin ZCS'}}{\text{R sin ZS'}}$$

$$= \frac{\text{R sin ZC} \times \text{R}}{\text{R sin ZS'}} \qquad (3)$$

So, from (1) $\text{R sin S'D} = \frac{\text{R sin ZC} \times \text{R}}{\text{R sin ZS'}} \times \frac{525 \times \text{R sin ZS'}}{\text{Sun's distance}} \times \frac{1}{\text{R}}$

$$= \frac{\text{R sin ZC} \times 525}{\text{Sun's distance}}$$

So, S'D or Sun's parallax $= \frac{525 \times \textit{ravidṛggati}}{\text{Sun's distance}}$ minutes, nearly.

Similarly, Moon's parallax

$$= \frac{525 \times \textit{candradṛggati}}{\text{Moon's distance}} \text{ minutes, nearly}$$

$$= \frac{525 \times \textit{candradṛggati}}{\text{Moon's distance}} \text{ minutes, nearly.}$$

(Note that at the time of conjunction, the longitudes of the Sun and the Moon are the same.)

So, total parallax

$$= 525 \times \textit{dṛggati} \left(\frac{1}{\text{Moon's dist.}} - \frac{1}{\text{Sun's dist.}} \right) \text{minutes}$$

$$= 525 \times \textit{dṛggati} \left(\frac{1}{\text{M'S}} - \frac{1}{\text{S'S}} \right) \times \frac{60}{\text{difference in motion}} \textit{ ghaṭikās.}$$

A similar method is followed by Bhāskara I in *MB*, v. 8-27, and *LB*, v. 2-10, the difference being that whereas Lalla takes the same

Find the difference of the two quotients. Multiply it by 60 and divide by the difference of the motions of the Sun and the Moon. The result is the parallax in longitude in *ghaṭikās* or *lambana* (at the mid-eclipse). It should be applied positively (or negatively as the case may be), to the calculated time when the *amāvāsya* ends. The parallax should be repeatedly calculated (and applied till the time is fixed).

8. Or, R sine of the hour-angle at the *amāvāsya* multiplied by the R sine altitude of the meridian ecliptic point and divided by 29,54,961, gives the parallax in *ghaṭikās* at the mid-eclipse.

R sines for the Sun and Moon, Bhāskara calculates them separately for the Sun and for the Moon. See also *ĀBh.*, *Golapāda*, 33-34, *PS*, ix. 17-23 and *SS*, v. 3-9.

Brahmagupta criticises Lalla's (Āryabhaṭa's) method in *BSS*, xi. 23-32, and gives a better method in v. 4-5. This method is given by Śrīpati in *ŚSe*, vi. 2-3 and by Bhāskara II in *SŚi*, I. vi. 3-4.

While commenting on these verses, Mallikārjuna has noted that the horizontal parallax of the Sun is 4′ and that of the Moon is 53′.

8. This gives another method for the parallax.

The proof is as given by Bhāskara II in his commentary on this verse. From the proportion :

R sin altitude (=*trijyā*) : maximum parallax

:: given R sin altitude : ?,

the parallax (say x) $= \dfrac{\text{given R sin altitude} \times 4}{\text{R}}$

Then again, from the proportion,

R sin *natakāla* (hour angle) (=*trijyā*) : x=given R sin *natakāla* : ?,

$$\text{parallax required} = \frac{\text{R sin } natak\bar{a}la \times \text{R sin altitude}}{\dfrac{\text{R}^2}{4}}$$

$$= \frac{\text{R sin hour-angle} \times \text{R sin altitude}}{1719^2\ (=29{,}54{,}961)}$$

9. Or, the *dṛggatijyā* divided by 860 gives the parallax in *ghaṭikās*. It should be added to the calculated time when the *amāvasyā* ends, if the Sun is in the western hemisphere, and subtracted, if in the eastern hemisphere. (The result is the time once corrected). The longitudes of the Sun and Moon must be found for this corrected time by adding or subtracting the minutes resulting (from the motions according as the parallax is additive or subtractive) and hence again the parallax. This process must be repeated (till the parallax and the time are fixed).

10. When the Sun is on the meridian, the parallax, if any, should be added to the (calculated time) when the *amāvasyā* ends,

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9. This verse gives another method for the parallax. From (1) above

$$\text{R sin S}'\text{D} = \frac{\text{R sin S}'\text{S}''\text{D} \times \text{R sin S}'\text{S}''}{\text{R sin S}'\ \text{DS}''}$$

$$= \frac{\text{R sin ZC} \times \text{R}}{\text{R sin ZC}'} \times \frac{\text{maximum parallax} \times \text{R sin ZS}'}{\text{R}} \times \frac{1}{\text{R}}$$

[using (2) and (3) above]

So, $$\text{S}'\text{D} = \frac{\text{R } sin \text{ ZC}}{3438} \times \text{maximum parallax, nearly}$$

$$= \frac{d\text{ṛ}ggati}{\frac{3438}{4}}\ ghaṭikās.$$

(According to Indian stronomers the maximum parallax is nearly 4 *ghaṭikās*) $= \frac{d\text{ṛ}ggati}{860}$ *ghaṭikās*, nearly.

Here, for the application for the parallax the meridian is taken as the dividing line. The commentator says that when the Sun's longitude is greater than that of the meridian ecliptic point, the parallax should be subtracted from the time when *amāvāsya* ends ; otherwise added.

10. In verse 9 Lalla takes the meridian as the dividing line for the application of the longitudinal parallax. Here, obviously, he is taking

provided that the Moon is to the south of the Sun. If the Sun is to the south of the Moon, the parallax should be subtracted.

[Parallax in latitude]

11. Multiply the *dṛkkṣepajyā* by 525 and divide severally by the distances of the Sun and Moon from the earth. The difference of the results in minutes is called *nati* or parallax in latitude. Its direction is the same as that of the *madhyajyā*.

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something else as the dividing line. Is he referring to the school of Brahmagupta, according to which the *dṛkkṣepavṛtta*, the great circle passing through the zenith and the nonagesimal, is the dividing line? Brahmagupta's formula for longitudinal parallax is :

$$= \frac{\dfrac{\text{R sin difference of Sun and nonagesimal}}{\left(\dfrac{R}{4}\right)^2}}{\text{R sin altitude nonagesimal}} \quad (BSS,\ \text{v. 4-5}).$$

Here, parallax is nil when the Sun is on the nonagesimal. Hence there can be parallax when the Sun is on the meridian.

Bhāskara II in his commentary on this verse says that *Brahmaguptamatam āśrityācāryeṇa lambanasambhava uktaḥ*, 'this possibility of the occurrence of parallax is according to Brahmagupta's formula'. Again, while commenting on 14cd, he says that according to Brahmagupta parallax is possible when *dṛkkṣepa* and *dṛggati* are equal. His words are : *Brahmaguptamatena dṛkkṣepadṛṅnatisamatve'pi lambanasambhavaḥ*. (Bhāskara II's *dṛṅnati* is the same as Lalla's *dṛggati*.)

While commenting on these verses, Mallikārjuna also mentions that in these cases parallax is possible only when *dṛkkṣepavṛtta* is assumed as the dividing line. (See text, Pt. I, pp. 114-16).

11. This gives the method to find the latitudinal parallax or *nati* of the Sun and the Moon.

In the above diagram, DS″ is the Sun's latitudinal parallax. In the spherical right-angled triangle S′DS″

$$= \text{R sin DS}'' = \frac{\text{R sin S}''\text{S}'\text{D} \times \text{R sin S}'\text{S}''}{\text{R sin S}'\text{DS}''}$$

Or, the *dṛkkṣepajyā* multiplied by the difference in the true motions of the Sun and Moon and divided by 51,570 gives the parallax in latitude.

The latitude of the Moon increased or diminished by this parallax, according as they are in the same or opposite directions, is the corrected latitude.

[Application of the parallaxes]

12. Calculate the first and second half of the duration of the eclipse and of the total eclipse, following the method given for the lunar eclipse. From the corrected time when the *amāvāsyā* ends subtract the first and add to it the second half of the duration of the eclipse. (The results are approximately the times when the eclipse begins and ends, respectively). Then, from these times calculate the parallax at the beginning and end of the eclipse, and apply them (in the manner given in the next two verses) to the approximately calculated half durations.

$$=\frac{\text{R sin ZS}'\text{V} \times \text{R sin S}'\text{S}''}{\text{R}}$$

$$=\frac{\text{R sin ZV} \times \text{R sin ZS}'}{\text{R sin ZS}'} \times \frac{525 \times \text{R sin ZS}'}{\text{Sun's distance}} \times \frac{1}{\text{R}}$$

(from the triangle ZS′V and above)

$$=\frac{\text{R sin ZV} \times \text{R. } 525}{\text{R Sun's distance}}$$

$$=\frac{\textit{dṛkkṣepajyā} \times 525}{\text{Sun's distance}}$$

So, DS″ or *Ravinati* $=\frac{\textit{dṛkkṣepajyā} \times 525}{\text{Sun's distance}}$ minutes, nearly.

Similarly, Moon's parallax $=\frac{\textit{dṛkkṣepajyā} \times 525}{\text{Moon's distance}}$ minutes, nearly.

So, total latitudinal parallax

$$=525 \times \textit{dṛkkṣepajyā} \left(\frac{1}{\text{Moon's distance}} - \frac{1}{\text{Sun's distance}}\right)$$

Now the second formula. From the above

$$\text{DS}''=\frac{\text{R sin S}''\text{S}'\text{D} \times \text{R sin S}'\text{S}''}{\text{R sin S}'\text{DS}''} \text{ minutes, nearly.}$$

Repeat the process till these times are fixed, (which are then the apparent or *sphuṭa* durations of the first and second halves of the eclipse).

13-14. If the parallax for the beginning of the eclipse is greater than that for the middle of the eclipse and both are subtractive, and if the parallax for the end of the eclipse is less than that for the middle of the eclipse and both are subtractive, add their differences to the approximately calculated first and second half of the approximately calculated first and second half of the duration of the eclipse.

$$=\frac{\text{R sin ZV}\times\text{R sin ZVS}'}{\text{R sin ZS}'}\times\frac{\text{maximum parallax}\times\text{R sin ZS}'}{\text{R}}\times\frac{1}{\text{R}}$$

$$=\frac{\textit{dṛkkṣepajyā}\times\text{R}\times\text{maximum parallax}}{\text{R}^2}$$

$$=\frac{\textit{dṛkkṣepajyā}}{\text{R}}\times\frac{1}{15}\text{ Sun's motion.}$$

(According to Indian astronomers, the maximum parallax is $\frac{1}{15}$ of the true motion.)

Similarly, Moon's latitudinal parallax

$$=\frac{\textit{dṛkkṣepajyā}}{\text{R}}\times\frac{1}{15}\times\text{Moon's motion.}$$

So, total latitudinal parallax

$$=\frac{\textit{dṛkkṣepajyā}\text{ (difference of Moon's and Sun's motions)}}{51570\ (=3438\times15)}$$

Then, the latitude of the Moon should be corrected by this parallax.

See also *MB*, v. 28-32 and *LB*, v. 11-12. Here Bhāskara I calculates the Sun's and the Moon's *dṛkkṣepas* separately.

13-14. These give the method of application of the longitudinal parallax or *lambana*. In fig. 21, KZV is the *dṛkkṣepavṛtta*. It is clear from the formula that the longitudinal parallax is nil, when the Sun is on the nonagesimal; then ZC=VS', appoximately, will be 0; it is subtractive, when it is to the east and additive, when it is to the west, It is maximum when the Sun is on the horizon.

If the parallax for the beginning of the eclipse is less than that for the middle of the eclipse and both are additive and if the parallax for the end of the eclipse is greater than that for the middle of the eclipse and both are additive, then also add their differences respectively to the approximately calculated first and second half of the duration of the eclipse.

If the parallax for the beginning of the eclipse is less than that for the middle of the eclipse and both are subtractive, and if the parallax for the end of the eclipse is greater than that for the middle of the eclipse and both are subtractive, then subtract their differences respectively from the first and second half of the duration of the eclipse.

If the parallax for the beginning of the eclipse is greater than that for the middle of the eclipse and both are additive, and if the parallax for the end of the eclipse is less than that for the middle of the eclipse and both are additive, then also subtract their differences respectively from the first and second half of the duration of the eclipse.

If the parallax for the beginning of the eclipse is different in denomination from that for the middle of the eclipse or if the parallax for the end of the eclipse is different in denomination from that for the middle of the eclipse, then always add their sums to the first or second half of the duration of the eclipse, (as the case may be). The result in each case is the apparent (*sphuṭa*) half duration of the eclipse.

If there is parallax when *dṛkkṣepa* and *dṛggati* are equal it should be added to the half duration of the eclipse.

But Lalla does not take the *dṛkkṣepavṛtta* as the dividing line for the parallax (verse 9). He says that parallax is subtractive when the Sun is to the east of the meridian and additive to the west. The method of application is, however, the same as that given by Brahmagupta in *BSS*, v. 14-15. Śrīpati follows this method in *SŚe*, vi. 11-12. The detailed proof of this formula is given in *KK*, I (pp. 127-28).

[Computation of the eclipse]

15. The same rule applies for calculating the first and second half of the duration of the total eclipse. When the apparent duration for the first half and the second half of the eclipse are respectively subtracted or added to the apparent time, when the *amāvāsyā* ends., (the results are the apparent times for the beginning and end of the eclipse, respectively).

Then calculate the Moon's correct latitude at these times in the same manner as the latitude at the mid-eclipse is calculated.

When the obscured portion at any given time is required, calculate the Moon's correct latitude for that time, and this is the *koṭi* or perpendicular.

16. Then calculate the *bāhu* or base (as above). Multiply it by the (approximately calculated first or second) half of the duration of the eclipse, as the case may be, using the corrected latitude, and divide by the apparent duration of the first or second half. The result is the corrected base. The remaining process (to find the obscured portion at any given time) is the same as that in the case of the Moon.

Again when the obscured portion is given, and the corresponding time is required, follow the process as given above. Then multiply this time by the apparent duration of the first or second half of the

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15-16. Verse 15 indicates that the method to calculate the durations of the first and second half of a total eclipse is the same as that given in verses 13-14. While commenting this verse, Mallikārjuna says that generally a total eclipse of the Sun is not possible. If there is, it is only for ¼ *ghaṭikā*.

Verse 16 explains how to calculate the obscured portion at any time and vice versa, which is a peculiarity in a solar eclipse. The translation makes the rule clear.

The same method is given by Brahmagupta in *BSS* v. 18-19, by Śrīpati in *ŚSe*, vi. 14 and by Bhāskara II in *SŚi*, I. v. 18-19.

eclipse, as the case may be, and using the corrected latitude, divide by the approximately calculated duration. The result is the more correct time. This should be subtracted from the half duration. Repeat the process till the time is fixed.

[Invisibility of an eclipse]

17. When $\frac{1}{12}$ of the Sun is obscured, the eclipse cannot be seen because of the Sun's brightness. But owing to the Moon's clearness even $\frac{1}{16}$ part of it when eclipsed can be seen.

Thus ends Chapter Six

on

SOLAR ECLIPSE

17. According to *SS*, vi. 13, owing to clearness even $\frac{1}{12}$ of the Moon when eclipsed can be seen ; but because of its bright rays even 3′ of the Sun when eclipsed cannot be seen.

CHAPTER VII

POSSIBILITY OF AN ECLIPSE

[Possibility of an eclipse at the Sygyzies]

1. Subtract the longitude of the Moon's node from that of the Sun. Find the difference between the remainder and 6 or 12 signs. Divide the result, either greater or less, expressed in minutes, by the sum of the motions of the Sun and the node. The quotient, in days, gives the number of days elapsed (since the possibility of he occurrence of on eclipse, if the above difference is greater than 6 or 12 signs; if less), the quotient gives the number of days to elapse (before the eclipse takes place).

2. If these days happen to be near *amāvasyā*, then there is the possibility of a solar eclipse during day time. If they happen to be near *pūrṇimā*, there is the possibility of a lunar eclipse during

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MATHEMATICAL NOTES

1. In a lunar eclipse, the Moon is at a distance of 6 signs from the Sun. So, if the difference between the longitudes of the Sun and the Moon's node is 6 signs, the Moon must coincide with its node and, consequently, its latitude is 0. So, there is the possibility of a lunar eclipse.

In a solar eclipse, the distance between the Sun's and Moon's places is zero or 12 signs. So, if the difference between the longitudes of the Sun and the Moon's node is 12 signs, the Moon must coincide with its node and, consequently, its latitude is 0. So there is the possibility of a solar eclipse.

See also *BSS*, xvi. 29-30, and *ŚSe*, vii. 1.

2. According to the formulas in *ŚD*, v. 9, above, the sum of the radii of the Moon and the earth's shadow at their mean distance

night. Subtract the longitude of the Moon's node from that of the Moon. If the difference between this remainder and 6 or 12 signs lies within 12°, there is the possibility of a lunar eclipse.

3. This twelve multiplied by 2 gives the latitude of the Moon in *aṅgulas*. The Sun's true motion multiplied by 4 and divided

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is 56′. So there is the possibility of a lunar eclipse, if the latitude of the Moon is less than 56′. This will be so, if the difference between the longitudes of the Moon and its node is within 12°.

The sum of the radii of the Moon and the Sun at their mean distances is 32′. So, there is the possibility of a solar eclipse if the latitude of the Moon is less than 32′. This will be so if the difference between the longitudes of the Moon and its node is within 7°.

See *SŚe*, vii. 2 ab, and *SŚe*. I. iv. 3, commentary.

In the commentary on verse 2, Mallikārjuna says as follows : (1) In places where the equinoctial shadow is 1 *aṅgula*, there will be no possibility of a solar eclipse even if the above difference is 12°. (2) In places where the equinoctial shadow is 5 *aṅgulas*, there will be no possibility of a solar eclipse if the above difference is 15°. (3) In places where the equinoctial shadow is 9 *aṅgulas*, there will be no possibility of a solar eclipse if the above difference is 18°. (4) If the difference is 12° and if the true motion of the Moon is either equal to or less than its mean motion, there is no possibility of a lunar eclipse. (5) If, however, the true motion is greater, there is the possibility of a lunar eclipse. (6) If the difference is 18°, there is no possibility of a lunar eclipse.

3. This gives a rule to convert the Moon's latitude etc. from minutes into *aṅgulas*. The relation taken by Lalla is $2\frac{1}{3}' = 1$ *aṅgula*.

From *ŚD*, v. 9, Sun's angular diameter

$= \frac{11}{20} \times$ true motion in minutes

$= \frac{11}{20} \times \frac{3}{7}$ true motion in *aṅgulas*

by 17 gives its diameter in *aṅgulas*. The Moon's true motion multiplied by 10 and divided by 577, gives its diameter in *aṅgulas*.

[Earth's Shadow]

4. Multiply the Sun's true motion by 5 and divide by 28. Multiply the Moon's true motion by 2 and divide by 35. The difference of the two quotients gives the diameter of the earth's shadow in *aṅgulas*.

[Parallaxes in terms of time]

5-6. The time (in *ghaṭikās*) between the end of the *amāvāsyā* and the midday, when multiplied by 6, gives the product in degrees.

$=\frac{4.33}{4.140}\times$ true motion in *aṅgulas*

$=\frac{4}{17}\times$ true motion in *aṅgulas*.

From *ŚD*, v. 9, Moon's angular diameter

$=\frac{11}{272}\times$ true motion in minutes

$=\frac{11}{272}\times\frac{3}{7}\times$ true motion in *aṅgulas*

$=\frac{10.33}{19040}\times$ true motion in *aṅgulas*

$=\frac{10}{577}\times$ true motion in *aṅgulas*.

[$\frac{4}{17}$ is really the penultimate convergent of $\frac{33}{140}$, and $\frac{10}{577}$ is the penultimate convergent of $\frac{33}{1904}$. *KSS*]

4. From *ŚD*, v. 9, the angular diameter of the earth's shadow $=\frac{8}{60}$ Moon's true motion in minutes—$\frac{25}{60}$ Sun's true motion in minutes

$=\frac{8}{60}\times\frac{3}{7}$ Moon's true motion—$\frac{25}{60}\times\frac{3}{7}$ Sun's true motion in *aṅgulas*

$=\frac{2}{35}$ Moon's true motion—$\frac{5}{28}$ Sun's true motion in *aṅgulas*.

5-7. These give brief and approximate methods to calculate the parallax in longitude.

Subtract it from the then longitude of the Sun if it is in the eastern hemisphere; otherwise add. (The result is) the longitude of the meridian ecliptic point. The declination corresponding to the to sum or difference, when added to or subtracted from the latitude of the place, (if of the same or different denominations, respectively), gives the zenith distance of the meridian ecliptic point. Subtracted from 90°, the remainder is the altitude. Their R sines are the R sine zenith distance and R sine altitude, respectively. The R sine of the zenith distance divided by 860 gives the parallax in longitude in *ghaṭikās*.

[Syzygy corrected for parallax]

7. (As the parallax obtained thus is approximate), multiply it by the R sine of the altitude and divide by the radius. The result

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First find the time between the *amāvāsyā* and the midday. Then this time in *ghaṭikās* should be multiplied by 6 to convert it into degrees. These degrees should be added to the Sun's longitude at the time of *amāvāsyā* if it is after midday ; otherwise, the degrees should be subtracted. The result is the longitude of the meridian ecliptic point. Mallikārjuna says that this is called *'Khārka'*.

Then find the declination of this point. Add it to the local latitude if both are of the same denomination ; otherwise, find the difference. The result is the zenith distance (z) of the meridian ecliptic point. When this is subtracted from 90°, the result is the altitude (a).

Then, from the proportion

R : maximum parallax in *ghaṭikās* :: R sin z : ?,

the parallax in longitude

$$= \frac{4 \times R \sin z}{3438} \textit{ghaṭikās}$$

$$= \frac{R \sin z}{859 \ (= 860 \text{ nearly})} \textit{ghaṭikās}$$

$$= p \ (\text{say}).$$

(According to Indian astronomers maximum parallax is 4 *ghaṭikās*).

is the more correct parallax. Apply it to the calculated time of the *amāvāsyā*, positively or negatively according as the Sun is in the western or eastern hemisphere. Repeat the process (till the parallax and, hence, the time of *amāvāsyā*, are fixed). This is the method in a solar eclipse.

[Corrected Celestial latitude]

8. The R sine of the zenith distance of the meridian ecliptic point multiplied by 21 and by the difference of the true motions of the Sun and the Moon, and divided by the difference of their mean motions

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Then to make it more correct another proportion is used : R : p : : R sin a : ?.

$$\text{So, } p' = \frac{p \times R \sin a}{R}$$

p' should be added to or subtracted from the calculated time of *amāvāsyā*, according as it is after or before midday. This process should be repeated till the parallax and consequently the time are fixed. This time is the time for mid-eclipse.

The *Pauliśasiddhānta* and *Romakasiddhānta*, as depicted in the *Pañcasiddhāntikā*, give a somewhat similar rule to find the parallax (*PS*, vii. 1, and viii. 9). There, at first, the hour-angle is obtained by getting the difference between the midday and the time of conjunction. This is changed into degrees by multiplying by 6. Then from the proportion

R : maximum parallax in *ghaṭikās* : : R sin hour-angle : ?,
the parallax is calculated.

8. This explains how to calculate the parallax in latitude in *aṅgulas* by a short process.

From the proportion

R : maximum parallax in latitude : : R sin z : ?,

$$\text{Parallax in latitude} = \frac{\text{maximum parallax} \times R \sin z}{R}$$

and the radius, gives the parallax in latitude. Its direction is the same as that of the zenith distance. The remaining process is the same as before.

[Probability of an eclipse at any time]

9-10. (If one wants to know whether there will be an eclipse after 6 months), one must first find the mean longitudes of the Sun, Moon, its apogee and node on that day. Then one must add to the first three longitudes $5^s, 24° 27' 6''$, $5^s 22° 12' 53''$, $0^s 19° 42' 53''$ and subtract $0^s 9° 22' 41''$, respectively (from the longitude of the node).

(The process must be reversed to determine the eclipses of the previous 6 months).

11. (If one wants to find out whether there was an eclipse before half a lunar month or there will be one after half a lunar month), one must first find the mean longitudes of the Sun, Moon, its apogee and node on that day. Then one must add or subtract $0^s 14° 47' 2''$, $6^s 17° 38' 42''$ and $100' (=1° 40') 13''$, respectively, to or from the first

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$$=\frac{21 \times R \sin z}{R} \text{ } a\dot{n}gul\bar{a}s$$

49′ is the maximum parallax in latitude and, as shown in the third verse, $2\frac{1}{3}'$ are equal to 1 *aṅgula*. So, 49′ will be equal to 21 *aṅgulās*. Mallikārjuna says the same in the commentary on this verse. This is made more correct from the proportion :

Difference in the mean motions : parallax in latitude : : difference in true motions : ?.

9-11. Verses 9 and 10 give the mean motions of the Sun, Moon, its apogee and node, during six months and verse 11 gives the motions during half a lunar month.

Brahmagupta gives only the six-monthly motions in *BSS*, xvi. 30-32. Śrīpati gives both in *SŚe*, vii. 3-4. They are slightly different from those given by Lalla.

three mean longitudes, according as the eclipse (to be determined) is after or before half a lunar month; (The process should be reversed in the case of the Moon's node whose motion is) 47′ 45″. This is the rule to determine an eclipse.

Thus ends Chapter Seven

on

THE POSSIBILITY OF AN ECLIPSE

CHAPTER VIII

RISING AND SETTING OF PLANETS

[Heliacal rising and setting of planets]

1. The planet, which has a motion slower than that of the Sun, always rises (heliacally) in the east and sets in the west. The planet which has a faster motion rises in the west and sets in the east. So, Mars, Jupiter and Saturn, (which have a motion slower than that of the Sun), always rise heliacally in the east and set in the west. Mercury and Venus, when retrograde, do the same. But when they have direct motion, (which is faster than that of the Sun), they rise heliacally in the west and set in the east. And so does the Moon.

(The daily rising and setting) is due to the wind called *Pravaha*. Then the planet with a lesser longitude than that of the

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MATHEMATICAL NOTES

1. In the commentary on verse 1, Mallikārjuna explains that there are two kinds of rising and setting of the planets. One is the daily rising and setting, which are always in the east and west, respectively. And this is due to the *Pravaha* wind.

The other is heliacal rising and setting. In this case, the planet which has a slower motion than that of the Sun, always rises heliacally in the east and sets in the west. This is the case with Mars, Jupiter and Saturn and also with Mercury and Venus when they are retrograde.

On the other hand, the planets which are faster than the Sun always rise in the west and set in the east. This is the case with the Moon and also with Venus and Mercury when they have direct motion.

Sun rises before sunrise and sets before sunset ; and that with a greater longitude rises after sunrise and sets after sunset.

[Planet corrected for ecliptic deviation]

2. Multiply the radius diminished by the R sine of the true longitude of a planet, as calculated, by 1397 and by the latitude of the planet. Divide the product by 1,18,19,844. The quotient in minutes etc. should be added to the true longitude of the planet if its latitude and declination are in opposite directions, and should be subtracted if they are in the same direction. The result is the longitude of the planet corrected by the visibility correction due to the ecliptic deviation or *Āyanadṛkkarmakalā*.

The English translation is in accordance with this explanation. Thus *ūna* means less or slower and *adhika*, greater or faster ; and, *prāk*, before or east, and *paścād*, after or west.

See *BSS*, vi. 2 ; Bhaṭṭotpala's commentary in the beginning of chapter vi of *KK* (edn., vol. II, pp. 135-36), *SŚe*, ix. 2-3, and *SŚi*, I. viii. 4 cd-5ab.

2-3 cd. These give the method to calculate the *āyanadṛkkarmakalā* or visibility correction due to ecliptic deviation.

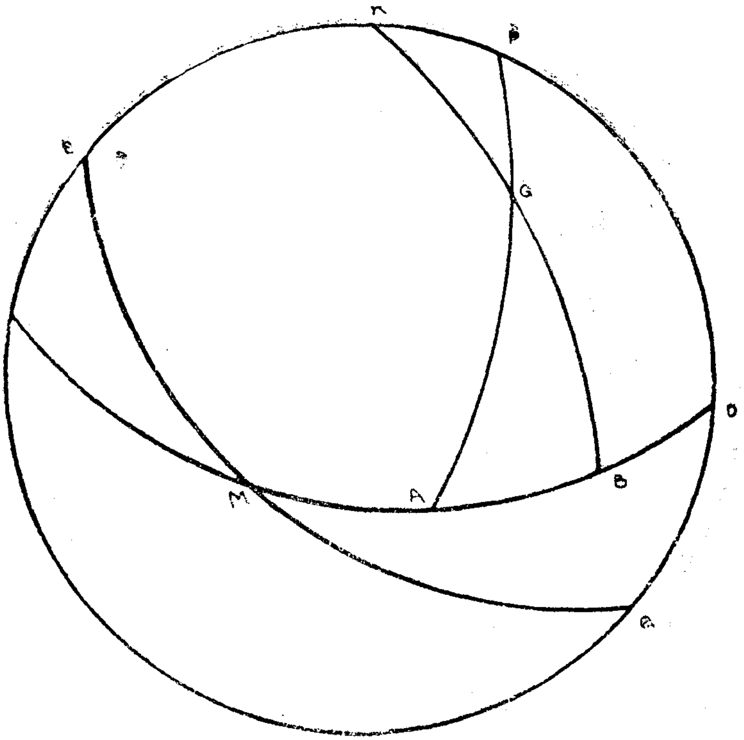

FIG. 22

Let EMQ be the equator and P its north pole, and CMD the ecliptic and K its north pole. Let G be the planet and KGB a secondary to the ecliptic through K and G, and PGA a secondary to the equator through P and G. Then, B is the position of the planet on the ecliptic and A its position after visibility correction due to ecliptic deviation. Then AB is the correction or *āyanadṛkkarmakalā*. MA is called the polar longitude of the planet. GB is the latitude of the planet but GA has approximately been taken as the latitude.

The triangle GAB is considered plane,

$$\text{So, } AB = \frac{R \sin G \times GA}{R \sin B}$$

[Planet corrected for visibility]

3. Or, multiply the R versed sine of the true longitude of a planet increased by 3 signs by the latitude and divide by 8461. Add the quotient in minutes to or subtract it from the true longitude

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$$=\frac{\text{R sin } \textit{āyanavalana} \times \text{latitude}}{\text{R}}$$

$$=\frac{1397 \times \text{R versed sin (longitude}+90^\circ) \times \text{latitude}}{1,18,19,844} \text{(v. 25)}$$

$$=\frac{\text{R versed sin (longitude}+90^\circ) \times \text{latitude}}{8461}$$

as given in the next verse 3 (ab). In the form in verse 2, Bhāskara II in his commentary says that R—R sin (longitude+node) is to be taken as equal to R versed sine (longitude+90°). Bhāskara's words are : *Pātagrahabhujajyonā trijyā satrirāśigrahotkramajyātulyā bhavati.*

When the correction is applied to the planet, positively or negatively as the case may be, the polar longitude of the planet is obtained. The directions of the latitude and declination are in accordance with verse v. 25.

See *MB*, vi. 2cd-3 ; *LB*, vi. 3-4 ; and *SŚe*, ix. 4. Brahmagupta gives (*BSS*, vi. 3) :

$$\frac{\text{R sin } \textit{āyanavalana} \times \text{latitude}}{\text{R}}$$

$$\text{Or } \frac{\text{R sine (longitude}+90^\circ) \times \text{R sin } 24^\circ \times \text{latitude}}{\text{R}^2}$$

The same is in *SS*, vii. 10.

Bhāskara II gives a better method in *SŚi*, I. vii. 3-5. He criticises Lalla for using versed sine and also for considering the latitude on the secondary to the equator. (*SŚi*, II. ix). But in his commentary on the verses he supports Lalla's formula.

3cd. The second part of verse 3 gives the method to find the second correction due to the latitude of the place or *ākṣadṛkkarmakalā.*

of the planet following the rule given above. Multiply the latitude (of the planet) by the R sine of the local latitude and divide by the R sine of the colatitude.

Or, multiply it (the latitude) by the equinoctial midday shadow expressed in *aṅgulas* and divide by 12. (The result in each case is the visibility correction due to the local latitude) or *Ākṣadṛkkarmaka* ā It is called *Saurikā* or 'due to the Sun'.

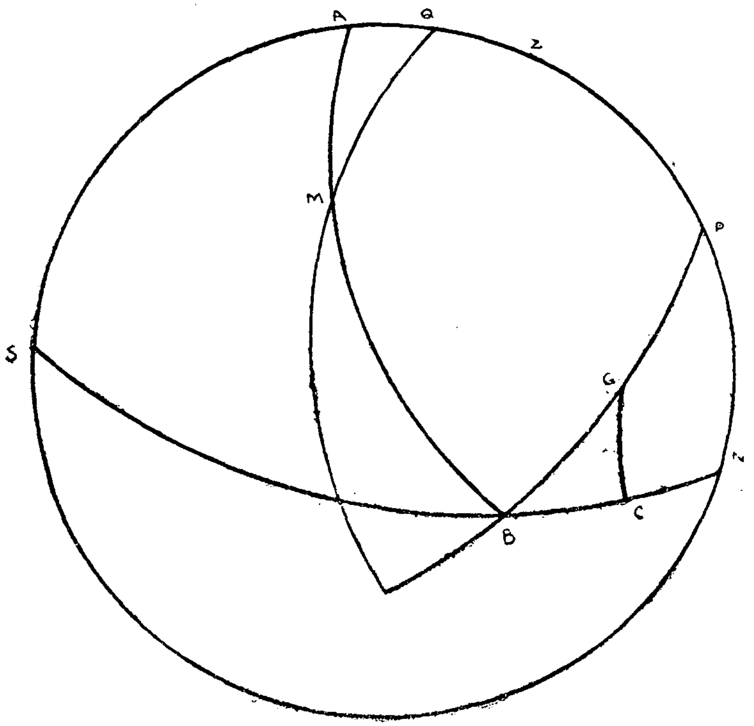

FIG. 23

Let ZSN be the meridian, SN the horizon, QME the equator and AMB the ecliptic. Let Z be the zenith, G the planet's disc and B the planet after first correction. Let GC be the part of the diurnal circle between PGB, the secondary to the equator and the horizon. Then GC is taken as the correction due to local latitude or *ākṣadṛkkar-māsu*. The angle B denotes the latitude of the place and C the

4. If the second visibility correction is calculated from a northern latitude (of a planet), it should always be subtracted from the corrected longitude of the planet on the eastern horizon. If, however, it is calculated from a southern latitude, it should be added to the same. The reverse is the process for the western horizon. The planet thus corrected is called *Dṛggraha.*

5. Corrected by the two visibility corrections, the Moon, Mars, Jupiter and Saturn are heliacally visible when separated from the Sun by a *Kālāṃśa* (or degrees of time) of 12°, 17°, 11° and 15°, respectively. Mercury and Venus (are heliacally visible) when in direct motion (when separated from the Sun) by a *Kālāṃśa* of 13°

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colatitude. GB is taken as the approximate latitude of the planet. Then, in the triangle GCB considered plane,

$$GC = \frac{R \sin B \times GB}{R \sin C}$$

$$= \frac{R \sin \text{local latitude} \times \text{latitude}}{R \sin \text{colatitude}} \quad (1)$$

$$= \frac{\text{equinoctial shadow} \times \text{latitude}}{12} \quad (2)$$

(triangles U and Z being similar)

As GC is on the diurnal circle, the correction GC is in terms of *asus*, which should be changed into minutes by means of the time of rising of the sign of the zodiac in which B is. But Lalla takes the *asus* nearly equal to the minutes. Śrīpati gives this correction to the *āyanadṛkkarmāsu* (*SŚe*, ix. 6).

When this correction is applied to the once corrected planet (verse 4), that point of the ecliptic which rises or sets with the planet is obtained.

Both the corrections are very approximate.

See *MB*, vi. 1-2 (ab) ; *LB*, vi. 1-2; *BSS*, vi. 4 ; *SŚe*, ix. 7 ; and *SS*, vii. 8-9. Bhāskara II gives a better method in *SŚi*, I. vii. 6-8,

5. See Table VII, at the close of this volume.

and 90°, respectively, and, when retrograde, by a *Kālāṃśa* of 12° and 8°, respectively.

(If the heliacal rising and setting of a planet) on the western horizon is considered, the true longitude of the Sun and that of the *dṛggraha* should each be increased by 6 signs.

6. (Find the Sun's true longitude at sunrise or sunset and the *dṛggraha* whether on the eastern or the western horizon, as the case may be). From the lesser of the two, find the time to be passed in *asus*, and, from the greater, the time passed. To the sum of the two times must be added the times of rising of the intervening signs of the zodiac. The result divided by 60 gives the required *kālāṃśa* or degrees of time of the planet.

Remember that when a planet is separated from the Sun by a *kālāṃśa* less than its *kālāṃśa* for visibility, as mentioned above, the planet remains invisible.

[Rising and setting corrected by degrees of time]

7. Find the difference expressed in minutes between the *kālāṃśa* of a planet at any time and the *kālāṃśa* for its visibility. Divide it by the difference of the true motions of the Sun and the planet, if they are moving in the same direction, and by the sum, if they move in opposite directions. The quotient in days gives the time elapsed or to come (as explained in the next verse). This is to be understood logically.

6-8. Lalla now gives the method to ascertain the heliacal visibility or disappearance of a planet.

If the rising or setting of a planet is considered on the eastern horizon, find the true longitudes of the Sun and the planet at sunrise of the given day. The former is the longitude of the orient ecliptic point or *tātkālikalagna*. Then, from the true longitude of the planet, following the methods given above, calculate the *dṛggraha* or the longitude of the ecliptic that rises (or sets) with the planet. Now, find the

8. When the setting of a planet is considered, if the given *kālāṃśa* for visibility is greater than its calculated *kālāṃśa*, know then that the planet has set heliacally before the number of days as found above ; if the former is less, the planet will set after these days. When the rising is considered, in the former case, the planet will rise after the days calculated, and, in the latter case, the planet has risen before the days calculated.

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distance in oblique ascension between these two, *tātkālikalagna* and *dṛggraha*, following the method in iv. 11-12. This distance in in *asus*. Dividing by 60, according to Lalla, the result is in degrees or *kālāṃśa*. (Lalla is obviously using the *asus* as minutes).

If the rising or setting of a planet is considered on the western horizon, find the true longitudes of the Sun and the planet at sunset. Then add 6 signs to each. From these calculate the *kālāṃśa* following the above method.

Find the difference between this calculated *kālāṃśa* and the *kālāṃśa* for visibility as given in the fifth verse. Express it in minutes. Though this is really the distance between the Sun and the planet in the oblique ascension, Lalla takes it approximately to be equivalent to the distance in longitude. Then, dividing it by the rate of approach of the Sun and the planet, which is the sum of their daily motions if they are moving in opposite directions, or is the difference otherwise, one gets the days for visibility or disappearance, as the case may be.

The eighth verse is quite clear. It explains how to decide whether the rising or setting is over or is yet to take place. Mallikārjuna, in his commentary, converts the distance in oblique ascension in terms of longitude in minutes.

The same method is given by Brahmagupta in *BSS*, vi. 5-7. But he calculates the days repeatedly till they are fixed. So his result is more correct. Śrīpati and Bhāskara II also give the same method, but they change the distance in oblique ascension first in terms of the longitude and then divide it by the sum or difference in motions. (See *SŚe*, ix 10-11, *SŚi*, I. viii, 7-10). See also *SS*, ix. 10-11,

[Corrected time for the rising and setting of the Moon]

9. If the true longitude of the Moon, (corrected by the two visibility corrections), is the same as the true longitude of the Sun at sunset, increased by 6 signs, the Moon rises at the same time as the Sun sets ; if greater, it rises after, and if less, it rises before sunset.

10. If the true longitude of the Moon (corrected by the two visibility corrections) and increased by 6 signs is the same as the true longitude of the Sun while rising, the Moon sets at that time ; if greater, it sets after and if less, before sunrise. The times (for these phenomena) should be calculated as before.

11. Multiply the daily motion of the Moon by the number of hours elapsed or to elapse, as (calculated above), either in the day or in the night, and divide the product by 60. The quotient should be added to or subtracted from the true longitude of the Moon, either

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9-11. Here Lalla discusses the rising and setting of the Moon on *pūrṇimā* or the Full Moon day.

First the case of rising. Find the true longitudes of the Sun and the Moon at sunset. The Moon's longitude should be corrected by the two visibility corrections. The corrections should be so applied as it would be for the eastern horizon. Let this result be x. To the longitude of the Sun, add 6 signs to find the corresponding point of the ecliptic on the eastern horizon. Let this result be y. Then it is evident that if $x=y$, the Moon will rise when the Sun sets ; if $x<y$, the Moon has risen before sunset ; if $x>y$, the Moon will rise after sunset. This time, which consists of some hours before or after sunset, can be found approximately by the method given before.

Now, the case of setting. Find the true longitudes of the Sun and the Moon at sunrise or at the end of the Full Moon. Then correct the Moon's longitude by the two visibility corrections. The corrections should be so applied as it would be for the western horizon. Add 6

at the end of the day or at the end of the night, (as the case may be). Then again, the longitude of the Moon corrected by the two visibility corrections should be calculated and hence the time. The process must be repeated till the time is fixed.

12. If the true longitude of the Moon, corrected by the two visibility corrections, is less than the *udayalagna* or the longitude of the rising point of the ecliptic and greater than the *astalagna* or the longitude of the setting point of the ecliptic, the Moon is visible,

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signs to it to find the corresponding point of the ecliptic on the eastern horizon. Let the corrected longitude of the Moon be u and that of the Sun be v. Then if $u=y$, the Moon sets with sunrise ; if $u<v$, the Moon will set before sunrise ; if $u>v$, the Moon will set after sunrise. This time, which consists of some hours, before or after sunrise can be found approximately by the method given before.

In the eleventh verse, Lalla gives the method to find the time more accurately. Multiply this time, before or after sunrise or after sunset, by the true daily motions of the Sun and the Moon. Divide each product by 60. The result from the Sun's motion should be applied to the Sun's true longitude positively or negatively as the case may be ; and, the result from the Moon's motion should be applied to the longitude of the Moon. The results are the true longitudes of the Sun and the Moon at that time.

From this corrected longitude of the Moon, the visibility corrections should be calculated and applied. Then the time should be calculated from the corrected longitudes of the Sun and the Moon. The process should be repeated till the time is fixed.

Śrīpati gives the same method in *SŚe*, ix. 14-15.

12. The following is the method to find, at any given time, the hours elapsed since the rising of the Moon or its *unnatakāla*.

Find the true longitude of the Sun at that time and hence the longitude of the orient ecliptic point or the *lagna* (iv. 11-12). Find

The time passed since the Moon was on the eastern horizon and to pass when it will be on the western horizon, can be found from the longitudes of these points and of the Moon ; and so also in the case of a planet.

Thus ends Chapter Eight

on

RISING AND SETTING OF PLANETS

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also the true longitude of the Moon at that time and correct it by the two visibility corrections. From these two longitudes calculate the time (iv. 13). This is the required time.

CHAPTER IX

CUSPS OF THE MOON

[R sine declination of the Moon]

1. The R sine of the longitude of the Moon multiplied by 13 and divided by 32 gives the R sine of the declination of the Moon's (position on the ecliptic) or its *madhyamakrāntijyā*.

MATHEMATICAL NOTES

1-2. These verses give the rule to calculate the true declination of the Moon.

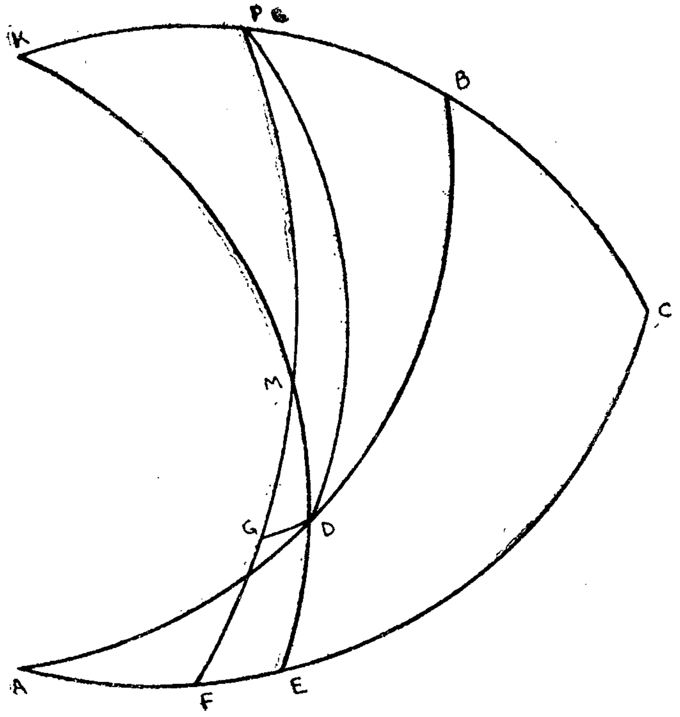

FIG. 24

[Measure of the Moon's day]

2. The sum or difference of the Moon's *madhyamakrānti* and its latitude, according as they are in the same or opposite directions, gives the Moon's *sphuṭakrānti* or the declination of the Moon's centre. Find its R sine. The result is the correct R sine declination. Hence find (the half variation of the Moon's day) following the methods given before.

Let AB be a quadrant of the ecliptic and AC that of the equator. Let K and P be their north poles, respectively. Let KPBC be the secondary to the ecliptic and equator. Let M be the Moon. Let KMD be a secondary to the ecliptic, so that D is the position of the Moon on the ecliptic. Let PMF and PDE be secondaries to the equator. Let DG be parallel to AC.

Then DE is the declination of the Moon's position on the ecliptic or *madhyamakrānti* and MF is its true declination or *sphuṭa-krānti*.

From spherical triangles ADE and ABC,

R sin DE or *madhyamakrāntijyā*

$$= \frac{\text{R sin BC} \times \text{R sin AD}}{\text{R sin AB}}$$

$$= \frac{\text{R sin } 24^\circ \times \text{R sin longitude of M}}{\text{R}}$$

$$= \frac{1397 \times \text{R sin (Moon's longitude)}}{3438}$$

$$= \frac{13 \times \text{R sin (Moon's longitude)}}{32}, \text{ nearly,} \qquad (1)$$

So, DE or GF is the arc corresponding to (1) as R sine.

Now, Lalla takes MG nearly equal to MD, which is the latitude. So, MF or true declination = MD + DE = latitude + mean declination or the declination of the Moon's centre. When the Moon has a south latitude, it would be the difference.

From this true declination the Moon's ascensional difference can

[Half variation of the Moon's day]

3. Or, calculate the Moon's half variation of the day (from its *madhyamakrānti* or the declination of its position on the ecliptic) using the tabular differerces to be given later on. Then add to it or subtract from it the Moon's second visibility correction, according as the Moon and its latitude are in the same or different spheres. The result is the correct half variation of the Moon's day.

[R sine altitude of the Moon]

4. As in the case of the Sun, calculate the R sine altitude of

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be calculated (ii. 17-18). This would be the ascensional difference of the Moon's disc or *sphuṭacaradala.*

See *MB*, vi. 8 ; *BSS*, vii. 5; *SŚe*, x. 7. Bhāskara II in *SŚi*, II. ix. 11-15 has pointed out the assumption for real latitude. He calculates the real latitude in *SŚi*, I. vii. 2 and hence real declination in vii 13 ab.

3. This gives another method to find the ascensional difference of the Moon more correctly. It is based on the principle that the difference between the ascensional difference calculated from the mean declination and that from the true declination is the visibility correction due to the local latitude or *ākṣadṛkkarmāsu.* (For the proof, see notes on xi. 12-13).

While commenting on this verse Mallikārjuna says that the following is an easy method for calculating the *ākṣadṛkkarmāsu.*

Multiply the equinoctial shadow in *aṅgulas* by the latitude of the divide by 10. The result may then be applied to the ascensional difference positively if the Moon and its latitude are in the same sphere, but otherwise negatively. From this the lengths of the day and night of the Moon can be calculated (ii. 20-21).

Śrīpati gives the same rule as Lalla in *SŚe.*, x. 8 and Bhāskara II in *SŚi*, I vii. 6 ab.

4-5. To find the length of the shadow of the gnomon caused

the Moon for the time passed or to pass in the day of the Moon. Then find the are corresponding to this as the R sine. Subtract from it the longitudinal parallax in minutes of the Moon as calculated before. Find the R sine of the remainder. The result is the correct R sine altitude of the Moon.

[Moon's shadow]

5. When the time equal to half the day of the Moon has passed since its rising, then it is midday. Find the shadow of the gnomon caused by the Mocn for that time as explained before (in the case of the Sun). In the same manner, the shadows caused by the stars and planets must be found.

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by the Moon at midday or at any other time during the Moon's day.

First at midday. Find the Moon's true declination at midday as given above. Hence following iv. 16, calculate its zenith distance and the altitude. Following iv. 17 find the length of the midday shadow and its hypotenuse.

Now to find the shadow at any other time of the Moon's day. Find the Moon's *sphuṭakrānti* or declination of its centre at that time and hence the ascensional difference (ii. 17-18). Then find the length of the day (ii. 20-21). Then as in viii. 12 find the time of the Moon's day that has elapsed or its *unnatakāla*. *Unnatakāla* subtracted from half the day gives the *unnatakāla*. The altitude and hence the zenith distance should be calculated according to iv. 24-26.

The longitudinal parallax for the Moon should then be calculat ed following the method in the case of the Sun given in vi. 1-7. The altitude should be corrected by the parallax. Hence according to iv. 30, calculate the length of the hypotenuse of the shadow at that time and then that of the shadow.

The same method is to be followed for stars and planets.

While commenting on verse 4, Mallikārjuna says that for parallax 1/15 of the true motion would do, though it will be only

[Computation of the Moon's cusps]

6. To know the cusps of the Moon, one must first find the true longitudes of the Sun and Moon on the second day of the light half of the luner month, at sunset.

7. Multiply the R sine of the Sun's declination and that of the Moon separately by the radius. Divide each product by the R sine of the local colatitude. The results are respectively the R sines of their amplitudes. (Hence find the amplitudes). Then find the base of the altitude of the Moon as explained in the case of the the Sun).

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approximate. 1/15 of the true motion is accepted as the maximum parallax.

In his commentary Mallikārjuna gives two practical observations According to the first, the *Nalakayantra* is used to observe the star or the planet as given in iv. 47-48. The second observation relates to the Moon. If the then longitude of the Moon is not known, that longitude of the midnight Moon should be found and then the two visibility corrections applied. The position of the fourteenth *nakṣatra* from the one in which the Sun is at the time of sunset should be observed. This *nakṣatra* rises with the sunset. So the time when this *nakṣatra* reaches the zenith is generally considered midnight. When it is seen to the east of the zenith, that time should be considered as the Moon's *natakāla*. The Moon's motion should be multiplied by this time and the product divided by 60. The result should be subtracted from the midnight Moon. If the star appears to the west of the zenith, similar result should be added. Thus corrected, the Moon's longitude would be for the time when the shadow is required. This longitude should be used for further calculations.

For the rules in the above verses see also *SŚe*, x. 28-35.

6-12. These give the method to find the measurement of certain lines required to draw, on a plane surface, a diagram representing the Moon's illuminated part,

8. In the case of the Moon, the sum or difference of the R sine of the amplitude and the base of the altitude, according as they are in the same or opposite directions, is called *bāhu* (*bhuja*). In the case of the Sun, the R sine of the amplitude itself is the the *bāhu* (*bhuja*).

First the calculation for the second day in the light half of the lunar month. The true longitudes of the Sun and Moon on that day at sunset should be calculated. Then find the R sines of the amplitudes of the Sun and the Moon from the similar triangles U and V, and hence the amplitudes. Then, according to the verse 4 above, and using the triangles U and X, calculate the base of the Moon and hence the Moon's *bhuja*. In the case of the Sun, since at the time of rising and setting, its altitude is nil, the base is also nil. Hence the R sin of its amplitude is its *bhuja*. Thus are found the *bhujas* of the Sun and Moon, that is, their distances from the east-west line on the second day at sunset.

The ninth verse gives the rule for finding the *bhujas* at times other than the sunrise or sunset. As before, the true longitude of the Moon at that time is calculated. From the longitude of the Moon and the given time, the longitude of the then orient ecliptic point is calculated. From this longitude the R sine of the amplitude is calculated. If the Moon is in the western hemisphere, the longitude of the orient ecliptic point should be increased by 6 signs and then the R sine of the amplitude should be calculated.

The *Śaṅku* or R sin altitude in both the cases would be nil and hence the base. So the R sine of amplitude is the *bhuja* in each case and it is to be assumed as the *bhuja* of the Sun.

From the Moon's longitude corrected by the two visibility corrections as before, its R sin altitude, R sin amplitude and base are calculated and hence the *bhuja*.

The next two verses explain that the sum or difference of the Sun's and Moon's *bhuja's* according as they are in the opposite or in the same direction, is the north-south difference of the two bodies.

9. (If at any other time besides sunrise and sunset, the cusps of the Moon are to be known), calculate the *lagna* for that time if the Moon is in the eastern hemisphere. But if the Moon is in the

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According to the two verses given in the commentary the length of the perpendicular or *koṭi* is calculated as follows.

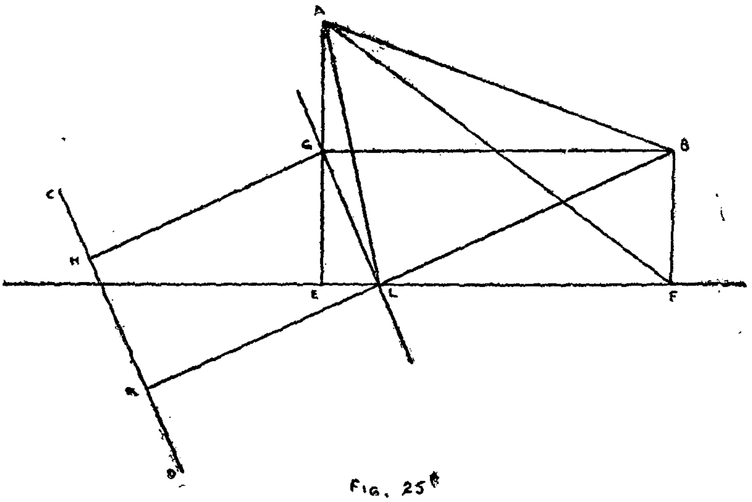

Fig. 25

Let A be the Moon and B the Sun. Let CD be the east-west line. Let AE be the Moon's R sin altitude and BF that of the Sun. Let BG be perpendicular to AE. Let GH and BK be perpendiculars to CD. Let GL be parallel to CD meeting BK in L. Then BL is the difference between the Sun's and the Moon's *bhujas*.

Now if in the diagram on p. 51, K′Q is drawn perpendicular to ZN, the north-south line, and K′Z is joined, then, as explained before, K′Q or C′C is the Sun's *bhuja*, K′Z is its R sin zenith distance or *dṛgjyā*, and QZ is its *koṭi*. Thus the Sun's *koṭi* is the square root of the difference of the squares of the R sin zenith distance and the *bhuja*. Similarly is obtained the Moon's *koṭi*. The sum or difference of these two *koṭis*, according as they are of different or same denomination, is the east-west difference of the Sun and Moon.

Now, since in this diagram, GL is perpendicular to BL, and BL represents the north-south difference of the Sun and Moon, GL

western hemisphere, increase the *lagna* by 6 signs. (Each result should be taken as the longitude of the Sun.) Then calculate the R sines of the amplitudes as explained in the case of the Sun.

10-11. The Sun's R sine amplitude is its *bāhu* (*bhuja*). If the *bāhu* (*bhuja*) of the Moon is in a direction different from the Sun, find their sum. If their directions are the same, then take their difference. The result in each case is called the 'corrected *bāhu*' (*bhuja*) or *sphuṭabāhu*.

In the first case, its direction is the same as that of the Moon's *bāhu* (*bhuja*). In the second case, its direction is the same as that of the Moon's *bāhu* (*bhuja*) if the latter is greater than the Sun's is *bāhu* (*bhuja*); if the Sun's is greater, the direction is opposite to that of the Moon's *bāhu* (*bhuja*).

(Subtract the square of the *bāhu* (*bhuja*) of the Sun and that of the Moon, respectively, from the squares of the R sines of their zenith distance. Find the square root of each difference. Find the

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represents the difference of their *koṭis*,. and according to the first verse, it is called *prathama* or 'first'.

AG is the difference of the R sine altitudes of the Sun and the Moon. At night, when the Sun is below the horizon, the sum of the R sine altitudes is considered. This is called *anya* or 'the other'. Let AL be joined. Then, in the right-angled triangle, AGL,

$$AL = \sqrt{AG^2 + GL^2}$$

AL is the *koṭi* or perpendicular.

Mallikārjuna, while giving these two verses in the commentary, says that this is the method according to *Brahmasiddhānta*. These two verses are not available in the *BSS*. The method, however, is given by Brahmagupta in *BSS*, vii. 6-9. Śrīpati follows him in *SŚe*, x. 10-13.

sum or difference of these results according as the Sun and the Moon are in the same or different hemispheres. The result is called *prathama* or 'first'.

Again, if it is day time, find the difference of the R sine altitudes of the Sun and Moon. But if it is night, then find their sum. The result is called *anya* or 'the other'. The square root of the sum of the squares of the *prathama* and the *anya* is called *koṭi* or perpendicular (and is parallel to the east-west line).

12. Now, they say that the Moon's R sine altitude is the perpendicular. The square root of the sum of the squares of the base (*bāhu*) and perpendicular (*koṭi*) is the hypotenuse.

When the lengths of the hypotenuse, perpendicular and base are each divided by 200, the results are respectively the lengths in *aṅgulas*.

13. The difference in the true longitudes of the Sun and Moon in degrees, multiplied by the radius of the Moon and divided

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According to Mallikārjuna's commentary these two verses do not form a part of the *ŚD*. They are, however, found in some mss. and Bhāskara II has commented on them.

In the next verse, number 12 in the text, Lalla says that according to some the Moon's R sine altitude or *śaṅku* itself is the *koṭi*, the Sun's *śaṅku* being nil when it is on the horizon. The square root of the sum of the squares of *bhuja* or base and *koṭi* or perpendicular, is the *karṇa* or hypotenuse. This follows from the right-angled triangle ALB.

Bhāskara II takes the difference of the R sine altitudes of the Sun and Moon as the *koṭi* and criticizes Brahmagupta's method (*SŚi*, I. ix. 3-4 and commentary).

Bhāskara I gives the methods to calculate *bāhu* and *koṭi* in *MB*, vi. 9-12 and in *LB*, vi. 8-13 ab.

13. This gives the usual formula to find the illuminated portion of the diameter of the Moon.

by 90, gives the illuminated portion (in the light half). When the above difference is diminished by 180° and the remainder is multiplied by the radius of the Moon and divided by 90, the result is the dark portion in the dark half.

14. Or, multiply the R versed sine of the difference (in degrees) in the longitudes of the Sun and Moon by the Moon's radius and

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When the Moon's distance from the Sun is 180°, its whole diameter is illuminated. So, for any given distance, say d, the illuminated portion visible from the earth is :

$$= \frac{\text{diameter in } \textit{angulas}}{180} \times d$$

$$= \frac{\text{radius} \times d}{90} \ \textit{angulas}$$

In the dark half of the lunar month when the dark portion is to be calculated, let the difference between the Sun and Moon which is now greater than 180°, first be diminished by 180° and then the above procedure followed. The illuminated portion calculated above is from the eighth day in the dark half to the eighth day in the light half of the lunar month. The dark portion is from the eighth day in the light half till the eighth day in the dark half.

Brahmagupta gives the same method in *BSS*, vii. 11 ab : and Śrīpati in *SŚe*, x. 16.

14. This gives a better formula to find the breadth of the illuminated portion of the Moon which is generally used for measurement in the day time.

divide by the radius. The result is the breadth of the illuminated portion of the Moon. This may be converted into *angulas* as before.

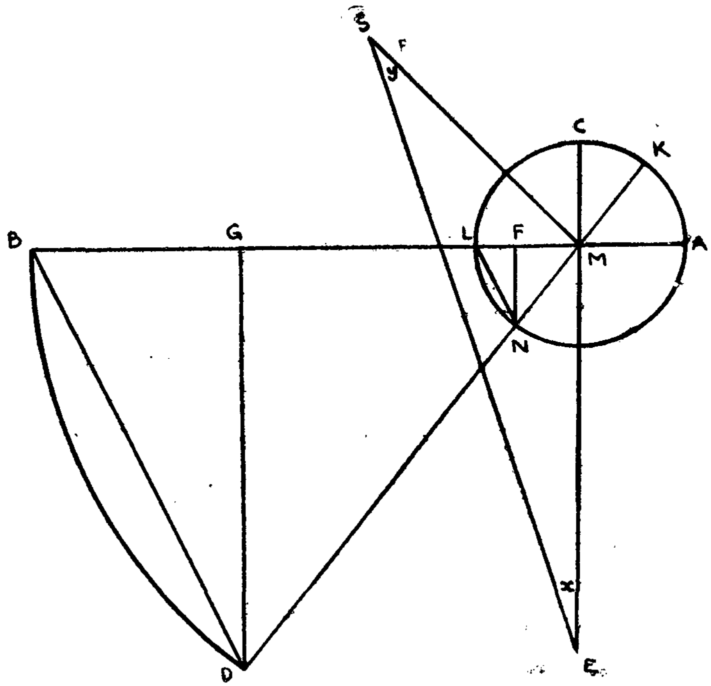

Fig. 26

Let S be the Sun, M the Moon's centre and E the earth. Let EM be joined and produced to meet the Moon's disc at C. Let AB be perpendicular to EC, meeting the Moon's disc in L. Let SM be joined. Let DMK be perpendicular to SM cutting the disc in K and N. Let MD be R. With M as centre and MD as radius cut AB in B. Let NF and DG be perpendiculars to AB. Let DB and NL be joined. Let the angle at E be x, the angular distance of the Moon from the Sun, and the angle at S be y.

Now, the angle GMD is equal to the angle SMe$=x+y$ and measures the portion of the Moon seen illuminated from the earth. Lalla neglects y and takes angle GMD to be equal to x approximately.

From the similar triangles MNE and MDG,

15. Mark a point on a level surface and assume it to be the Sun. From this point draw a straight line equal to the correct base (*bhuja*) and in its own direction. At the end of this line draw another straight line at right angles to it and equal to the perpendicular. If the Moon is in the eastern hemisphere, the perpendicular should be drawn to the west. If the Moon is in the western hemisphere, the perpendicular should be drawn to the east. Then draw the hypotenuse from this point to the Sun.

$$\frac{NF}{MN}=\frac{DG}{MD}$$

$$\text{or } NF=\frac{DG}{MD}\times MN \qquad (1)$$

Again, from the similar triangles NFL and DGB,

$$FL=\frac{GB}{DG}\times NF$$

$$=\frac{GB}{DG}\times\frac{DG}{MD}\times MN \qquad \text{from} \quad (1)$$

$$=\frac{GB}{MD}\times MN$$

So, the breadth of the illuminated portion

$$=\frac{\text{R versed sine GMD}\times\text{Moon's radius}}{R}$$

This is given by Bhāskara I in *MB*, vi. 5 cd-7 and in *LB*, vi. 6-7. Brahmagupta and Śrīpati also give the same in *BSS*, vii. 11 cd-13 and in *SŚe*, x. 17, respectively.

While commenting on this verse, Mallikārjuna says that the method given in verse 13 gives only an approximate result and is suitable for calculating the phase in the night, whereas the present method is more accurate and should be used for calculating the phase at night. He further continues that in the evening the illuminated portion in both the ways should be calculated and half of their sum taken. That should give the correct result.

15-18. The correct *bhuja* or base, the perpendicular or *koṭi* the hypotenuse or *karṇa* and the illuminate portion have been obtained.

16. The Moon's centre is at the point of intersection of the perpendicular and the hypotenuse. The direction of the hypotenuse is the east-west line (in the Moon's disc). Draw the north-south line at right angles to it (and passing through the centre of the Moon's disc) by means of fish-figures. All this should be drawn with chalk and very clearly.

17. If the breadth of the illuminated portion as calculated above is for the bright half of the lunar month, it should be marked along the hypotenuse from the west point. From the same point should be marked the breadth of the dark portion, if it is for the dark half of the lunar month. Then draw two fish-figures using this point and the north and south points.

Now a method is given to draw a diagram on plane surface to show the cusps of the Moon.

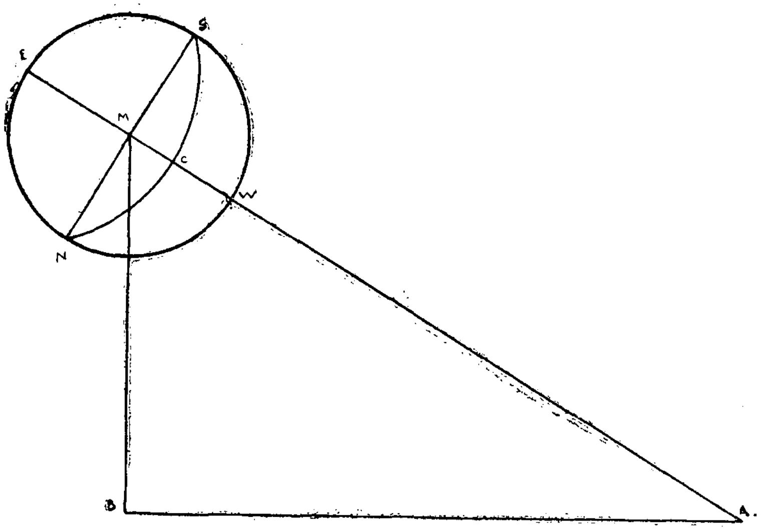

Fig. 27

Let A be the Sun. AB is the *bhuja* drawn north, if it is north, or south, if it is south. AB is then a north-south line on the plane surface. BM is drawn perpendicular to AB at B and equal to the *koṭi* or

18. With the point of intersection of the two lines joining the mouths and tails of the two fish-figures as centre, draw a circle to pass through the point marked above. This is the circle which shows the illuminated portion in the Moon's disc.

19. The lower cusp of the Moon is on that side on which lies the base (*bhuja*) measured in *aṅgulas*, and the other is the higher cusp.

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perpendicular. It is drawn to the west, if the Moon is in the eastern hemisphere, and to the east, if the Moon is in the western hemisphere. AM is joined and is the *karṇa* or hypotenuse. Then, M is the centre of the Moon's disc. Let AM cut the disc at W and at E when produced. Then EW is the east-west line in the Moon's disc. Draw NS at right angles to WE through M. Then NS is the north-south line.

Now, WE is the line along which the breadth of the illuminated or dark portion is to be marked. This breadth should be marked from W, the west point, along WE, if it is the breadth of the illuminated portion for the bright half of the lunar month. But if the illuminated portion is for the dark half of the lunar month, then it should be marked from E, the east point along EW. The dark portion should be marked in the reverse manner.

Let WC be the portion. Then a circle is drawn passing through N, C and S. Thus SWNC is the illuminated portion in the Moon's disc.

Bhāskara I's methods are given in *MB*, vi. 13-17, 23-25 and *LB*, vi. 12 cd-17. The same method as that of Lalla is given in *SS*, x. 10-13. *BSS*, xvii. 2-6; and in *SŚe*, x. 24-25. For Bhāskara II's improved method see *SŚi*, I. ix. 7-9.

19 ab. N is the lower cusp of the Moon and is on the side of AB, the *bhuja*.

19 cd. Śrīpati gives a similar comparison in *SŚe*, x. 26.

When the illuminated portion is equal to half of the Moon's disc which is bisected by the diameter, the Moon has the beauty of the forehead of a woman belonging to the *Lāṭa-deśa*.

20. The Moon's higher cusp rises first and sets last. The cusp of the Moon, when its crescent is very narrow, has the rich beauty of the middle portion (centre) of *ketakī* flowers.

Thus ends Chapter Nine

on

CUSPS OF THE MOON

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20. Mallikārjuna, while commenting on verse 20, says that on the thirteenth and fourteenth days of the dark half and on the second and third days of the light half of the lunar month, the cusps of the Moon look as beautiful and white as *ketakī* flowers.

Śrīpati gives the same comparison in *SŚe*, x. 27.

CHAPTER X

CONJUNCTION OF PLANETS

[Planetary conjunction]

1. Find the *manda* hypotenuse of a planet following the method for the *śīghra* hypotenuse. Then multiply it by the *śīghra* hypotenuse and divide the product by the radius. The wise astronomers say that the quotient is the distance between the centre of the earth and the planet.

[Computation of the time of planetary conjunctions]

2. If the diameter of the Moon measured in *yojanas* is divided by 5, 10, 15, 20 and 25, the results are, respectively, the diameters of

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MATHEMATICAL NOTES

1. A method to find the distance of the earth and the planet is given here. The *manda* hypotenuse is calculated as in iii. 17 and the *śīghra* hypotenuse as in iii. 3-7. The formula follows from the proportion :

R : *manda* hypotenuse : : *śīghra* hypotenuse : ?

The *manda* hypotenuse is the one calculated in the third operation for true planets and the *śīghra* hypotenuse is the one calculated in the fourth operation. This is explained in the commentary by Mallikārjuna.

Āryabhaṭa gives the same method in *ĀBh., Kālakriyāpāda*, 25, and Bhāskara I in *MB*, vi. 48, and in *LB*, vii. 8.

2. This gives the diameters of the planets in *yojanas*. According to Lalla, the diameters of Venus, Jupiter, Mercury, Saturn and

Venus, Jupiter, Mercury, Saturn and Mars (upon the Moon's orbit, when they are at their mean distances from the earth).

3. When the diameter of a planet (expressed in *yojanas*) is multiplied by the square of the radius and divided by the product of the distance between the centre of the earth and the planet, and the Moon's distance from the earth, (both in *yojanas*), the result is converted into minutes of the true angular diameter of the planet.

4. Or, (when the diameter of a planet) in *yojanas* is divided by 10, the quotient is the mean angular diameter in minutes. This again, when multiplied by the radius and divided by the distance between the centre of the earth and the planet, gives the true angular diameter in minutes.

Mars are, respectively, 64, 32, $21\frac{1}{3}$, 16 and $12\frac{4}{5}$ *yojanas*, when the Moon's diameter is 320 *yojanas*. In v. 6 if the reading is *nakhaguṇam*, then the diameter is 320 *yojanas* and if *tithiguṇam* then it is 315 *yojanas*. Mallikārjuna there takes it as 315 and here as 320 *yojanas*.

Āryabhaṭa gives the same method in *ĀBh., Daśagītikā*, 7, but since according to him the diameter of the Moon is 315 *yojanas*, the diameters of Venus etc. would be 63, $31\frac{1}{2}$, 21, $15\frac{8}{4}$ and $12\frac{3}{5}$ *yojanas*, respectively. See Table VIII, below.

3. A method to convert the diameters in *yojanas* into minutes, is given here. If D is the diameter in *yojanas*, then the diameter in minutes

$$= \frac{D \times R^2}{\underset{\text{(in } yojanas\text{)}}{\text{Moon's distance}} \times \underset{\text{(in } yojanas\text{)}}{\text{planet's distance}}}$$

4. Another method to convert the *yojanas* into minutes is given here. In v. 2, Lalla assumes that in the orbit of the Moon 10 *yojanas* are equivalent to 1′. Thus, the mean diameter of a planet expressed in *yojanas*, when divided by 10, gives the same in minutes.

5. The latitudes of Mars, (Mercury, Jupiter, Venus and Saturn, when at their mean distances from the earth), are, respectively, 9, 12, 6, 12 and 12 minutes, each multiplied by 10.

4, 2, 8, 6 and 10 each multiplied by 10, are, respectively, the degrees (in the longitudes) of the nodes (of the above planets).

6. The nodes of Mars, Jupiter and Saturn become more accurate when corrected by adding to their longitudes (given above), or subtracting from them, the respective *śīghraphalas* of the planets, according as these are additive or subtractive. But the nodes of Mercury and Venus become more accurate when corrected by adding to the longitudes, or subtracting from them, their respective *mandaphalas*, according as they are additive or subtractive.

[Time of conjunction and equality of longitudes]

7-9. If two planets are moving in the same direction, divide the difference of their true longitudes in minutes by the difference of their true daily motions. The quotient gives the days elapsed since

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The *SS* assume 15 *yojanas* as equivalent to 1′ (iv. 3, vii. 14).

The second formula follows from the proportion :

Distance of planet from earth's centre : mean diameter in minutes : : R : true diameter in minutes. See also *MB*, vi. 58 cd.

5. See Tables IX and X, below.

6. For the proof see the notes on the verses 9-10 of this chapter.

7-10. The first two and a half verses deal with the *kadambaprotīyayuti* of two planets or when their geocentric longitudes are equal. The translation makes the rule clear.

See *MB*, vi. 49-51 ; *LB*, vii. 4-5 ; *BSS*, ix. 5-7 ; *SŚe*, xi. 11-14 ; and *SŚi*, I. x, 3-4ab.

the conjunction, if the faster of the two planets is ahead. If the slower is ahead, the conjunction will take place after so many days.

If, again, one planet is direct and the other retrograde, divide the difference of their true longitudes, expressed in minutes, by the sum of their true daily motions. If the retrograde planet has the greater longitude, the conjunction will take place after the days in the quotient ; if lesser, the conjunction has taken place.

Multiply the difference between the true longitudes of the two planets severally by the true daily motion of each planet and divide each product by the sum or difference of their true daily motions, as the case may be. Add the quotients to the respective longitudes of the planets if the conjunction is to take place, and subtract if the conjunction has taken place. If the planet is retrograde, add the quotient if the conjunction has taken place and subtract if it is to take place. Then the two planets will have true equal longitudes expressed in minutes.

Now, in the case of Mars, Jupiter and Saturn, subtract the correct longitudes of their respective nodes from their true longitudes when thus made equal. But in the case of Mercury and Venus, subtract the correct longitudes of their nodes from the longitudes of their respective *śīghroccas* at the time when the planets themselves have true equal longitudes.

10. Find the R sine of the remainder in each case. Multiply it by the greatest mean (geocentric) latitude of the planet and divide by its distance from the centre of the earth. The result is the true

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Then is given the method to find the correct geocentric latitude of a planet. First to find the argument of latitude or *vikṣepakendra*. Then a superior planet, say, Mars.

Argument of latitude of Mars

=its true heliocentric place—node's longitude

=*mandaspaṣṭa* Mars—node's longitude (obtained after third operation)

(geocentric) latitude of the planet. This again should be corrected by the latitudinal parallax of the planet as in the case of the Moon, following the method given (in the chapter on) solar eclipse.

=*mandaspaṣṭa* Mars ± *śighrāphala*

— node's longitude ∓ *śīghraphala*

=true longitude of Mars — (node's longitude ± *śīghraphala*)

(The terms within the brackets prove the method in the sixth verse above).

=true Mars—(corrected node).

Now, take an inferior planet, say, mercury. In this case before finding the argument it is necessary to state that according to the ancient Indian astronomers, the stated revolutions of nodes of Mercury and Venus should be increased by the motions of their *śīghra* anomalies, and the sum, in each case, gives the true revolutions of the nodes.

Thus, the argument of the latitude of Mercury

=*mandaspaṣṭa* Mercury—(node's longitude—*śīghra* anomaly)

(In this case the *śīghra* anomaly is subtracted because the node's longitude as given by Lalla is measared in the same direction as that of the planet).

=*mandaspaṣṭa* Mercury—node + *śīghrocca*

— mean Mercury

= —(mean Mercury—*mandaspaṣṭa* Mercury)

± *śīghrocca*—node

= —(±*mandaphala*) + *śīghrocca*—node

=*śīghrocca*—[node + (± *mandaphala*)]

(The terms within the brackets prove the formula in the sixth verse).

Thus is calculated the argument of the latitude of a planet.

The next verse gives the method to calculate the true geocentric

11. The difference of the latitudes (of the two planets of equal longitudes), when of the same denomination, or their sum when of different denominations, is said to be the distance between the centres of the two planets. When this distance is less than the sum of their radii, the conjunction is called *Bhedayuti*.

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latitude. The method is the same for every planet. From the proportion :

R : R sin mean maximum latitude : : R sin
argument : ?,

or, R : mean maximum latitude (nearly) : : R sin
argument : ?,

the heliocentric latitude of the planet, say x, is found.

Then again from the proportion :

Distance of planet from earth : x : : R : ?,

the true geocentric latitude of the planet is calculated.

Lalla corrects the latitude by the latitudinal parallax, the method for calculating which is given in vi. 11. While commenting on the correction to be given to the latitude Mallikārjuna says that this correction is for the case of *Bhedayuti* only. Astronomers differ from one another in calculating the argument of the latitude of a planet.

See *MB*, vi. 52-53 : *LB*, vii. 6-9 ab ; *BSS*, ix. 8-10 ; *SŚe*, xi. 15-16 ; *SŚi*, II. vi. 21-26 ; and *SS*, ii. 56-67.

11. This defines the distance between two planets and *bhedayuti*. The distance is measured by the sum or difference of the latitudes (converted into *aṅgulas*). This formula for the distance is given in *MB*, vi. 54; *LB*, vii. 9 cd; *BSS*, ix. 11; *SŚe*, xi. 18; and *SŚi*, I x. 6.

When the distance between the centres of two planets having equal longitudes is less than the sum of the radii of the planets, the conjunction is called *bhedayuti*. In this case, the upper planet is eclipsed by the lower planet.

[Obscuration of the asterisms by planets]

12. When the latitudes of two planets are of different denominations, each planet is said to be in the same direction of the ecliptic as its latitude. But when they are of the same denomination, the planet having the shorter latitude is said to be in a direction opposite to that of its latitude with reference to the centre of the other planet.

[Half duration of planetary conjunctions]

13. In a *bhedayuti* (as defined in verse 11) the time elapsed since the rising of the obscured planet till the moment of equality of

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While commenting on this verse, Mallikārjuna says that the conjunction of planets with equal longitudes is of various kinds. (1) It is called *bheda* when the distance between the centres is less than the sum of the radii. (2) It is called *ullekha* when the distance is equal to the sum of the radii. (3) It is called *aṃśumarda* when the distance is greater than the sum of the radii. (4) It is called *apasavya* if the distance is greater than the sum of the radii by a number less than 60. (5) When the distance between two planets is subtracted from the radii and the remainder is greater than 60, the conjunction is called *samāgama*.

SS also gives several kinds of conjunction in *SS*, vii. 18-19. Bhāskara II deals with *bhedayuti* in *SŚi*, I. x. 7-9.

12. While commenting on this verse, Mallikārjuna says that during the conjunction of planets, the planet to the north of the other planet is called *jayī* or victorious. But Venus even when to the south is called victorious. In the conjunctions called *ullekha*, *aṃśumardá* and *apasavya*, the planet to the north is the obscuring body and is called victorious. Then Mallikārjuna gives his own verse regarding the orbits of the planets from below upwards and comments that in the case of conjunction the lower planet obscures the upper planet.

13. Just as in a solar eclipse, the *tithi* is measured from the rising of the Sun till the end of the *amāvāsya* and is used for calculating

the longitudes is taken to be equivalent to the duration of the *tithi* (as in a solar eclipse). From this time the parallax etc. should be calculated.

[Direction of planetary conjunctions]

14. When the north-south distance between the centres of two planets of equal longitudes is great, (their conjunction as obtained previously does not agree with observation). This conjunction is not the same in the eastern and western hemispheres. So, in order to make the calculated result agree with observation, (one must proceed as follows).

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the parallax etc., in a *bhedayuti* when the distance between two planets of equal longitudes is less than the sum of their radii, the time elapsed since the rising of the obscured planet till the conjunction with the lower planet is used for calculating the parallax etc. Then, as in a solar eclipse, the correct time of conjunction etc. should be determined. Śrīpati gives this in *SŚe*, xi. 33-34.

14. This verse says that the *kadambaprotīya* conjunction, i.e., when the longitudes are equal, does not agree with observation, especially when the distance between the two planets is great.

Mallikārjuna says in his commentary on this verse that not only in the case of planets, but in the case of certain stars also this is so. The north-south distance between Citrā and Svāti is 39°. To the north of the Vindhyas, Svāti rises first, then Citrā. There is conjunction only in the morning of their day. In the afternoon, Citrā sets before Svāti. To the south of the Vindhyas, Citrā rises before Svāti. They have no conjunction there. It is therefore necessary to find a method so that calculation agrees with observation.

Brahmagupta has pointed out that *kadambaprotīya* conjunction is not correct and so *samaprotīyayuti* (*BSS*, ix. 12). (Śrīpati also has said this in *SŚe*, xi. 19-20).

[Visibility correction in planetary conjunctions]

15. For the eastern horizon, the true equal longitudes of the two planets should be corrected by their respective visibility corrections (calculated as above) to find their respective *udayalagnas* or rising ecliptic points. But, for the western horizon, their true equal longitudes, each increased by 6 signs, should be corrected in the same way to find their respective *astalagnas* or setting ecliptic points.

16. The sum of the time to pass calculated from the *udayalagna*, the time passed calculated from the *astalagna* and the times of rising of the intervening signs of the zodiac between the *udayalagna* and the *astalagna* is the measure of the day of the planet. The time calculated in the same manner from the *udayalagna* of the planet and the then orient ecliptic points is the time elapsed since the rising of the planet. Thus say (the wise).

17-20. Multiply the elapsed portion, expressed in *ghaṭikās*, of the day of the planet which has the shorter day, by the length of the day of the planet which has the longer day, and divide the product

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15-19. Having dealt with the *kadambaprotīyayuti* of two planets (that is, when they are on the same vertical to the ecliptic or have the same longitude), Lalla now gives a method to find their *samaprotīya-yuti* or conjunction when they are on the same secondary to prime vertical, because, as he says in the last part of the fourteenth verse, the results calculated in this manner would be in accordance with the observation.

In verse 15 is calculated the *udayalagna* of each planet or that point of the ecliptic which rises with the planet, and also the *astalagna* of each planet or that point of the ecliptic which sets with the planet. These are to be found when the planets have true equal longitudes. The methods are given in viii. 2-4.

by the length of the shorter day. If the quotient is less than the elapsed portion of the day of the planet which has the longer day, the conjunction is to take place ; if greater, the conjunction has taken place.

The difference between these two is called *prathama* or 'the first'. Now, assume a certain time for the conjunction. (Find the longitudes of the planets at that time, and, hence, as before, the lengths of their days and the portions of the days elapsed). From these find the second difference, as above. This is *anya* or 'the other'.

In verse 16, the length of the day of each planet is found. This length is obviously the time between the planet's rising and setting and, consequently, it is calculated from its *udayalagna* and *astalagna* following the method given in iv. 13. In this verse is also given the method to find, at any time the portion of the day of a planet that has passed. This is found from the *udayalagna* of the planet and the orient ecliptic point at the given time. The method is as given above.

In the next three verses is given the method to calculate the more correct time of conjunction.

An approximate time has been obtained when the two planets have equal longitudes (see *verses* 7-9). The length of the day of each planet and also the portion of the day elapsed at this time are also found. Let the longer day be a, the shorter b, the elapsed portion of longer day c, and the elapsed portion of the shorter day d.

Then, from the proportion :

$b : d : a : ?$, d is converted in terms of a. Let the result be x.

If $x=c$, there is conjunction.

If $x > c$, the conjunction has passed.

If $x \angle c$, the conjunction will take place.

The difference between x and c is called *prathama* or 'first'.

If both the *prathama* and the *anya* show that either the conjunction has taken place or will take place, find their difference; otherwise, find their sum. Divide by this sum or difference the product of the *prathama* and the assumed time. The quotient in hours etc. gives the correct time of conjunction, before or after the time, when the two planets have equal longitudes before, if the *prathama* shows that the conjunction has taken place, and after, if it shows that the conjunction is yet to take place.

When the elapsed portion of the day of one planet is multiplied by the length of the day of the other planet and divided by the length

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Then assume some time before or after the time when the longitudes are equal. Find the longitudes of the planets as that time, and from that the lengths of their days and the portions of the day elapses. Then convert the elapsed portion of the shorter day in terms of the longer day. Then find the difference between this result and the elapsed portion of the longer day. This is called *anya* or 'the second difference'.

Then from the porportion :

prathama ± *anya* : assumed time : : *prathma* : ?' find the time when the first difference or *prathama* is made up. This is the time of conjunction before or after the time when the planets are of equal longitude—before, if the *prathama* shows that the conjunction has taken place but otherwise after. Lalla has definitely borrowed this formula from Brahmagupta.

While commenting on these verses, Mallikārjuna says that the method should be repeated till the time is fixed.

This is the formula given by Brahmagupta in *BSS*, ix. 22-24 and by Śrīpati in *SŚe*, xi. 29-31.

20. This defines conjunction. Let a be the length of the day of one planet and b of the other. Let c be the elapsed portion of the former and d of the latter.

of its own day, and if the quotient is equal to the elapsed portion of the day of the other planet, the conjunction takes place. Otherwise follow the method as before.

Thus ends chapter Ten

on

CONJUNCTION OF PLANETS

Then, from the proportion :

$a : c :: b : ?$, c is converted in terms of b. Let the result be x. If $x = d$, the conjunction takes place.

Brahmagupta gives this formula in *BSS*, ix. 25 and Śrīpati in *SŚe*, xi. 32,

CHAPTER XI

CONJUNCTION OF STARS AND PLANETS

[Positions of the stars in minutes]

1-3. It has been said by the ancient sages that the positions of the *nakṣatras*, Asvinī etc., in their respective portions are, respectively, 48′, 40′, 56′, 54′, 52′, 20′, 72′, 70′, 44′, 48′, 36′, 44′, 78′, 66′, 62′, 72′, 52′, 8′, 6′, 4′, 4′, 19′, 18′, 40′, 42′, 12′, and 74′, each multiplied by 10. They also say that the polar longitude of each *nakṣatra* is equal in minutes to the sum of its portion and 800 times the number of the *nakṣatras* preceding it (that is, its own number less 1).

[Time of conjunction of stars and planets]

4ab. If the true longitude of a planet is greater than the polar longitude of a *nakṣatra*, their conjunction has taken place ; if less, it will take place. (This is so, when the planet has direct motion). When retrograde, the contrary is the case. The method (of finding the time of conjunction etc.) is the same as that in the case of planets.

[Latitudes of the stars]

4c-d. The following are, respectively, the polar latitudes of the *nakṣatras*, Aśvinī etc., measured from the ends of their respective declinations on the ecliptic corresponding to their polar longitudes.

5-7. (They are) 10°, 12°, 5°, 5°, 10°, 11°, 6°, 0°, 7°, 0°, 12°,

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MATHEMATICAL NOTES

1-10. Each *nakṣatra* is said to occupy 1/27 of the circle, that is, 800′. See Tables XI and XII, below.

13°, 8°, 2°, 37°, 1°30′, 3°, 4°, 8°30′, 5°20′, 5°, 30°, 36°, 20′, 24°, 26°. and 0°.

1° is equivalent to 24 *aṅgulas*.

[Latitudes of Agastya, Mṛgavyādha and Abhijit]

7c-d. The polar longitude of Agastya is 2^s 27°. (When the true longitude of a planet is the same as this), there is conjunction of the planet and Agastya.

8. The polar longitude of Mṛgavyādha (Lubdhaka) is 2^s 26° and that of Abhijit is 8^s 27°. The polar latitudes of Agastya, Mṛgavyādha and Abhijit are respectively 80° S, 40° S and 64° N.

[North-South latitudes of the stars]

9-10. The latitudes of Ārdrā, Āśleṣā, Jyeṣṭhā, Uttara-Āṣāḍha, Pūrva-Āṣāḍha, Rohiṇī, Mūla, Hasta, Anurādhā, Citrā, Śatabhiṣā, Mṛgaśirā and Viśākhā are to the south.

The latitudes of Puṣya, Revatī and Maghā are each 0°, that is, they are seen at the ends of their respective declinations. The remaining stars have northern latitudes.

The last quarter of Uttara-Āṣāḍha added to the first 4 *ghaṭikās* of Śravaṇa is the polar longitude of Abhijit, which is seen here.

[Occultation of the stars by the planets]

11. The Moon, when in the middle of the asterism of Rohiṇī, pierces the Cart (of Rohiṇī), if its southern latitude is 2° 40′, and occults the principal star, Rohiṇī, when its southern latitude is 4° 30′.

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11. The occultations of stars are also given in *MB*, iii. 71cd-75 ab ; *LB*, viii. 11-16 ; *BSS*, x, 11-12 ; *KK*, II. x. 15-16 ; *SŚe*, xii. 8-9 ; *SS*, viii. 13.

Again, the Moon occults the middle star of the asterism, Maghā, when it has a north latitude of 4° 30′. When it has no latitude, it occults Revatī, Puṣya and Śatabhiṣā.

[Visibility correction for latitude from half asc. difference]

12. The R sine of the sum or difference of the declination of a *nakṣatra*, calculated from its polar longitude, (known as *madhyama* or mean declination), and the latitude, according as they are in the same or opposite directions, is the R sine of the correct declination of the *nakṣatras*.

From each of these R sine declinations, (*viz.*, *madhyama* or mean and corrected or *sphuṭa*), calculate the ascensional difference of the *nakṣatra* following the same method as that in the case of the Sun.

13-15. Find the sum or difference of the *madhyama* and *sphuṭa*

Lalla's method is more or less the same as that of Brahmagupta. Bhāskara I's are in greater detail. He also says that these are based on actual observations made by the instrument *Yaṣṭi*.

ascensional differences according as they are of opposite or same denomination.

From the result subtract the time passed (*bhuktakāla*) calculated from the polar longitude of the *nakṣatra*. From the remainder again subtract the times of rising of as many signs of the zodiac as possible, (beginning with the one next to that in which the *nakṣatra* is and) going in contrary direction.

12-15. These give the method to calculate the rising ecliptic point or *udayalagna* and the setting ecliptic point or *astalagna* of a *nakṣatra*. The method given here is more accurate than that given in viii. 3-4, especially in the case of a *nakṣatra*, which has a considerable latitude.

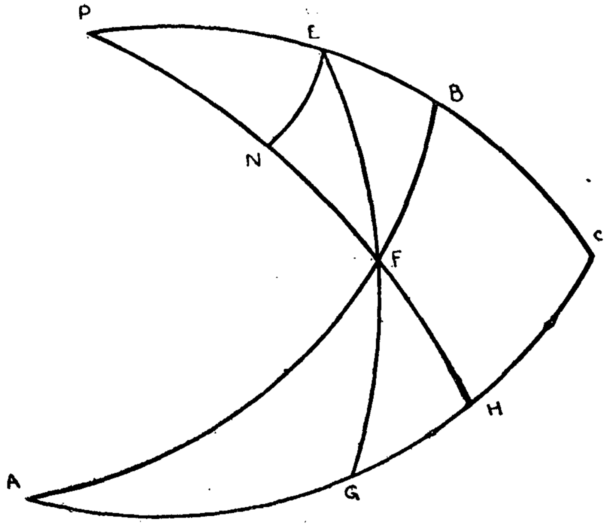

FIG. 28

Multiply the remainder by 30 and divide by the time of rising of the sign that is not subtracted. The result is in degrees etc. Subtract it from the polar longitude of the *nakṣatra*. Then subtract from the remainder the degrees etc. in the sign of the zodiac in which the *nakṣatra* is (*i. e., bhuktabhāga*). From the remainder again subtract the number of signs the times of rising of which have already been subtracted. The result is the *udayalagna* of the *nakṣatra* corrected by the visibility correction when its latitude is to the north.

If the latitude is to the south, calculate the *lagna* from the

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Let AB be the ecliptic, AC the equator, P the north pole, N the *nakṣatra* and PNFH the secondary through P and N, meeting the ecliptic and equator in F and H, respectively. Let NE be the diurnal circle of N meeting EFG, the horizon, in E. Then AF is the *polar* longitude or *dhruvaka* of N, FH its mean declination, NF its latitude and NE the visibility correction due to the local latitude or *ākṣadṛkkarmāsu*.

Then, NH = true declination or *sphuṭakrānti*

= NF + FH

= latitude + mean declination, calculated from AF the *polar* longitude.

If N and F were on opposite sides of AC, the true declination would be the difference between the latitude and the mean declination.

Brahmagupta calls them *tatkrānti* and *svakrānti* (*BSS*, x. 15).

Now, GH is the ascensional difference due to the mean declination and GK is the ascensional difference due to the true declination NH or EK.

So, NE = HK = GK—GH = *ākṣadṛkkarmāsu*.

If N and F were on opposite sides of HC, it would be the sum of the two.

polar longitude of the *nakṣatra* (by the direct method). The results in this case are to be added to the polar longitude. (The final result is the *udayalagna*).

To find the *astalagna*, the polar longitude *nakṣatra* must be increased by 6 signs, (and the *lagna* calculated by the direct method).

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applying corrections to F using the *ākṣadṛkkarmāsu* or visibility correction due to the local latitude.

Now, to find the rising ecliptic point and the setting ecliptic point

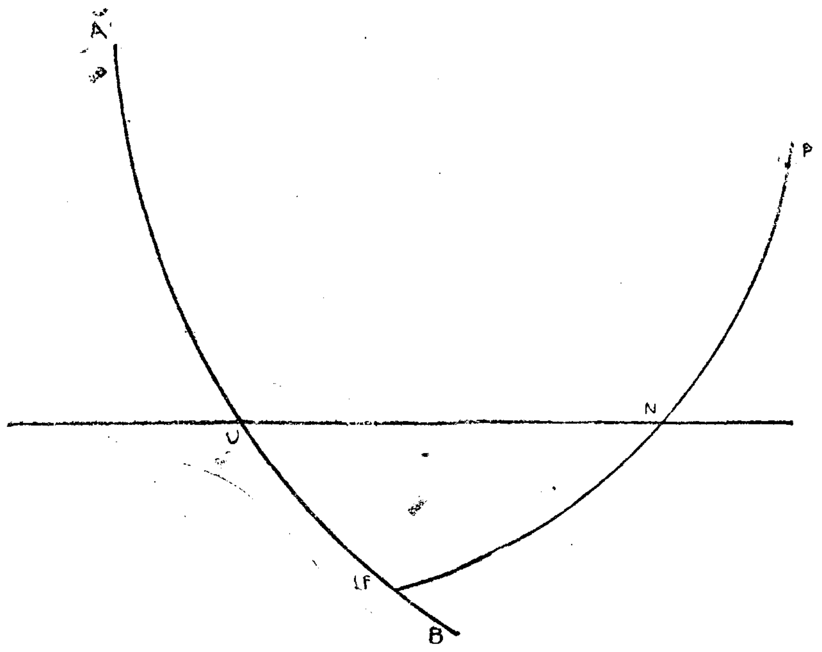

Fig. 29

Let AB be the ecliptic, A the first point of Aries, P the north pole, N the *nakṣatra* on the eastern horizon or *pūrvakṣitija*, which cuts the ecliptic in U, and let PNF be the secondary through P and N meeting the ecliptic in F. Then AF is the *polar* longitude of the *nakṣatra* and U is the rising ecliptic point.

If the latitude is to the south, a method contrary to this should be followed.

[Time of rising and setting of the stars]

16. Because of the motion of the whirl wind, Pravaha, (round the universe), the *nakṣatras* rise and set with their respective *udaya-lagna* and *astalagna*.

The *kalāṃśa* or degrees indicative of time in *ghaṭikās* of a *nakṣatra* is $2\frac{1}{3}$; that of Mṛgavyādha is $2\frac{1}{6}$ and that of Agastya is 2.

17. When the *lagna* calculated from the *udayalagna* or a

The diagram shows that when a *nakṣatra* has a northern latitude, and is on the eastern horizon, its *dhruvaka* is below the horizon by a time equal to the visibility correction or *ākṣadṛkkarmāsu* and its rising ecliptic point has a lesser longitude than its *dhruvaka*. So, if the *nakṣatra-dhruvaka* is assumed as the Sun, and the *ākṣadṛkkarmāsu* as the time, and the *lagna* is calculated from these following the *indirect process*, the result will be the rising ecliptic point of the *nakṣatra*.

The *astalagna* is that point of the ecliptic which is on the western horizon or *paścimakṣitija*. So, it should be found by assuming the *dhruvaka* increased by 6 signs as the Sun, the *ākṣadṛkkarmāsu* as the time and calculating the *lagna* from these by the direct method as given in iv. 11-12 above.

It is evident that when the *nakṣatra* has a southern latitude, methods contrary to those for the northern latitude must be followed, that is, the direct process for the rising ecliptic point and the indirect process for the setting ecliptic point.

The same method is given in *BSS*, x. 18-20 ; *SŚe*, xii. 13-15 ; and in *SŚi*, I. vii, 6-8.

17. The *lagnas* calculated by the method given here are called

nakṣatra (for a time equal to the *kālāṃśa* in *ghaṭikās*) following the additive method is the same as the true longitude of the Sun, the *nakṣatra* rises heliacally with the Sun.

When the *lagna* calculated from the *astalagna* of a *nakṣatra* (for a time equal to the *kālāṃśa* in *ghaṭikās*) following the subtractive method and diminished by 6 signs, is the same as the true longitude of the Sun, the *nakṣatra* sets heliacally with the Sun.

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udayārka and *astamayārka*, or the true longitudes of the Sun when the *nakṣatra* rises and sets with it. Suppose the Sun is on the eastern horizon. Then a *nakṣatra* will rise heliacally with the Sun, if its *udayalagna* is above the horizon at a distance of 2⅓ *kālāṃśaghaṭikās* or 14° *kālāṃśa* from the Sun.

Now, to find the true longitude of the Sun in this position. The Sun is at that point of the ecliptic, which following the *udayalagna* is at a distance of 2⅓ *kālāṃśaghaṭikās* from it. Therefore, the longitude of the Sun is the *lagna* calculated from the *udayalagna* using 2⅓ *kalāṃśa-ghaṭikās* as the time and following the method in iv. 11-12. This is called *udayārka* of the *nakṣatra*, because when this is the true longitude of the Sun, the *nakṣatra* rises heliacally with it.

Again, suppose the Sun is on the western horizon. Then a *nakṣatra* will set heliacally with the Sun if its *astalagna* has not yet reached the eastern horizon, but is still below it by a distance of 2⅓ *kālāṃśaghaṭikās*. So, at that time the point of the ecliptic already on the eastern horizon is the point less than the *astalagna* by 2⅓ *kālāṃśa-ghaṭikās*. Evidently, it is the *lagna* calculated from the *astalagna* using 2⅓ *kālāṃśaghaṭikās* as the time and following the indirect method. Thus the true longitude of the Sun on the western horizon is this *lagna* minus 6 signs. This is called *astamayārka* of the *nakṣatra*, because when this is the true longitude of the Sun, the *nakṣatra* sets heliacally with it.

See also *BSS*, x. 36-37 ab ; *SŚe*, xii. 17 ; and *SŚi*, I. xi. 12-14.

[Time of heliacal visibility of the stars]

18. As long as the true longitude of the Sun is between *udayārka* or that longitude of the Sun with which the *nakṣatra* rises and *astārka* or that longitude of the Sun with which the *nakṣatra* sets, so long the *nakṣatra* is heliacally visible.

The sum or difference of its correct declination and the local latitude, according as they are in the same or opposite directions, must then be less than 90°.

[Time of heliacal invisibility of the stars]

19. As long as the true longitude of the Sun is between the *nakṣatra's astārka* and *udayārka*, so long is the latter heliacally invisible.

The difference between the *udayārka* and *astārka* of a *nakṣatra*, expressed in minutes, when divided by the true daily motion of the Sun, gives the days (for which the *nakṣatra* is visible or invisible).

[Heliacal visibility of stars at any place]

20. The *nakṣatra*, whose *astārka* is greater than its *udayārka*, never sets heliacally. When the declination of a *nakṣatra*, corrected by its latitude and the local latitude, is greater than 90°, the *nakṣatra* is not visible at the place.

18-20. A *nakṣatra* rises heliacally with the Sun, when the Sun's true longitude is the same as its *udayārka*. Again, it sets heliacally with the Sun when the Sun's true longitude is the same as its *astamayārka*. Consequently, the *nakṣatra* remains heliacally visible as long as the true longitude of the Sun is between its *udayārka* and *astamayārka* and it remains heliacally invisible as long as the true longitude of the Sun is between its *astamayārka* and *udayārka*.

The days of invisibility are obtained by subtracting its *astamayārka* from its *udayārka* and dividing the remainder by the Sun's true daily motion. The days of invisibility are obtained by subtracting its

[Different view on the rising of Agastya]

21. Some say that Agastya sets when the true longitude of the Sun is 2 signs *minus* the local latitude. And it rises when the true longitude of the Sun is 6 signs *minus* this (remainder).

[Stars passed by Marīci and other sages]

22. Subtract 14 from the years passed since the beginning of

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udayārka from its *astamayārka* and dividing the remainder by the Sun's true daily motion.

The sum or difference of the mean declination and the latitude of a *nakṣatra* gives its true declination. Again, the sum or difference of this declination and the latitude of the place gives its zenith distance. If the zenith distance is less than 90°, the *nakṣatra* will be visible at that place. If it is greater than 90°, it will be below the horizon and consequently, will not be visible at that place.

While commenting on verse 20, Mallikārjuna says that Svāti, Abhijit, Śravaṇā, Dhaniṣṭhā, Pūrvabhādrapada and Uttarabhādrapada these six stars, because of their large northern latitudes, do not set in certain regions.

When the visibility correction of one of these is greater than 14° and the *udayārka* is less than 14° and the *astārka* is greater than 14°, the star is not visible in the evening even when the *udayārka* is the same as the true longitude of the Sun ; but it is visible in the morning. When the visibility correction is less than 14° and if the *astārka* is greater than the *udayārka*, the *nakṣatra* is not always visible. It is not visible when its declination is to the south and its zenith distance is greater than 90°.

In those regions where the equinoctial shadow is 8 *aṅgulas*, there the southern zenith distance of Agastya is greater than 90° and in those regions Agastya is not visible.

21. In the commentary Mallikārjuna says that by 'some' is meant 'pupils of Āryabhaṭa'.

22. Millikārjuṇa gives the following example ;

Kaliyuga and divide the remainder by 100. The wise say that the quotient gives the number of stars from Rohiṇī passed by the sages beginning with Marīci, the sages who decorate the sky.

Thus ends chapter Eleven

on

CONJUNCTION OF STARS AND PLANETS

To calculate the *nakṣatra* in the Śaka 1035 in which the seven sages, *viz.*, Marīci, Bhṛgu, Vasiṣṭha, Aṅgirā, Atri, Pulastya, and Pulaha, are.

Number of years passed from the beginning of *Kaliyuga* in the *śaka* $=3179+1035=4214$

$$\frac{(4214-14)}{100}=42$$

Thus the present *nakṣatra* will be the sixteenth from Rohiṇī, which is Mūla.

CHAPTER XII

VYATĪPĀTA AND VAIDHṚTA

[Possibility of Vyatīpāta and Vaidhṛtā]

1. When the sum of the true longitudes of the Sun and Moon is 12 signs and they are in different hemispheres of the ecliptic but on the same side of the solstitial points, there is *Vaidhṛtayoga*. But when

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MATHEMATICAL NOTES

1. This verse defines the *Vyatipātayoga* and the *Vaidhṛtayoga*. The mean declination or *madhyamakrānti* of the Moon is that declination which is calculated from its longitude, and hence it is the declination of the position of the Moon on the ecliptic. Its true declination or *sphuṭakrānti* is the sum or difference of its *madhyamakrānti* and latitude or *vikṣepa*. The Sun has no latitude and so its *madhyamakrānti* is its *sphuṭakrānti* or, briefly, *krānti*.

According to the Indian astronomers, that whole period is called *vyatīpātakāla* during which the sum of the true longitudes of the Sun and the Moon is somewhere near 6 signs and the true declination of any point on the disc of the Moon is equal both in magnitude and in direction to the declination of any point on the disc of the Sun.

Vaidhṛtakāla is that period during which the sum of the true longitudes of the Sun and the Moon is somewhere near 12 signs and the true declination of any point on the disc of the Moon is equal in magnitude but opposite in direction to the declination of any point on the disc of the Sun.

Both *vyatīpāta* and *Vaidhṛta* are denoted by *pāta*,

the sum is 6 signs and they are in the same hemisphere of the ecliptic but on the opposite sides of the solstitial points, there is *Vyatīpātayoga*.

[Time of mid-pāta]

2. Find the difference in minutes (between the sum of the true longitudes of the Sun and Moon at any given time) and 6 signs or 12 signs. Multiply it by 60 and divide by the sum of the true daily motions of the two. The result in *ghaṭikās* etc. gives the time elapsed since the *vyatīpāta* or *vaidhṛta* took place or to elapse, (according as the sum is greater than 6 or 12 signs or less). Then find the true longitudes of the Sun, Moon and its node for that time.

Pātadhruvakakāla is the time when the sum of the true longitudes of the Sun and Moon is exactly 6 or 12 signs. At this time the mean declination of the Moon is equal to the declination of the Sun. *Pātamadhyakāla* is the time when the true declination of the Moon's centre is equal to the declination of the Sun's centre. *Pātasparśakāla* is the time when true declination of the lowest point on the Moon's disc is equal to the declination of the highest point on the Sun's disc. *Pātamokṣakāla* is the time when the true declination of the highest point on the Moon's disc is equal to the declination of the lowest point on the Sun's disc.

Thus, at *pātasparśakāla* and *pātamokṣakāla* the difference of the true declination of the Moon's centre and the declination of the Sun's centre is equal to the sum of their radii.

See *BSS*, xiv. 33-34 ; *SŚe*, viii. 1-2 ; *SŚi*, I. xii. 8 ; and *SS*, xi. 1-2.

2. This gives the method to find the time when the true longitudes of the Sun and Moon may be 6 or 12 signs. At any given time find the true longitudes of the Sun and Moon and then their sum.

3a-b. At that time, if the true declination of the Moon's centre is equal to the declination of the Sun's centre, both expressed in minutes, the time is called middle of *pāta*.

[Equality of declination]

3c-d. When the Moon's true declination is obtained by subtracting the mean declination from its latitude, even if the true declination is numerically greater than that of the Sun, it must be considered as less.

4. If the Moon's true declination at the beginning of an even

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If it is near 6 signs, find the difference between the sum and 6 signs. Then, from the proportion :

Sum of daily motions : 60 : : difference : ?

The number of *ghaṭikās* will be obtained when the sum of the longitudes was 6 signs, if at present it is greater than 6 signs ; or when the sum of the longitudes will be 6 signs, if at present it is less than 6 signs. If the sum of the longitudes is near 12 signs, find the difference between the two and hence the time when the sum will be 12 signs or was 12 signs.

See *BSS*, xiv. 35 ; *SŚe*, vii. 5 ; and *SŚi*, I. xii. 9-10 ab.

3. Now find the true longitudes of the Sun, the Moon and its node when the sum is 6 or 12 signs, as the case may be. Hence calculate the declination of the Sun and the true declination of the Moon. If the two declinations are the same, the time is *pātamadhyakāla*. If this is not the case, the method to find the time, when this will be so, is given in the verses 6-9. When the Moon's true declination is obtained by subtracting its mean declination from its latitude, it is negative. So, even if it is numerically greater than the Sun's declination, it must be considered less.

See *BSS*, xiv. 37 ; *SŚe*, vii. 5-6 ; and *SS*, xi. 8 cd.

4-5. Perhaps the following is the argument. When the Moon

quadrant, (that is, at the end of an odd quadrant), is less than that of the Sun in an odd quadrant, there is no possibility of *pāta*. If in this case the Moon's true declination is greater, there is a possibility of the declinations being equal and hence of *pāta*.

5. When the true declination of the Moon in an odd quadrant is greater or in an even quadrant less than that of the Sun, the *pāta* has taken place. In the contrary cases, it will take place.

[Time of Vyatīpāta and Vaidhṛta]

6. (Find the Sun's declination and the Moon's true declination at the time when the sum of their true longitudes is 6 or 12 signs). In the case of *vaidhṛta*, find the sum of the declinations, if in the same

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is at the end of Gemini and Sagittarius, its declination is maximum. If in either of these positions, its true declination is less than that of the Sun there is no possibility of their becoming equal in the near future, and hence there is no possibility of *pāta*.

When the Moon is in the first or third quadrant, its declination increases till it reaches the end of the quadrant. If in either case, its true declination is less than the Sun's declination there is a possibility of *pāta* ; if greater, *pāta* may have taken place.

In the second or fourth quadrant, the Moon's declination decreases till it reaches the end of the quadrant. If in either case its true declination is less than the Sun's declination *pāta* may have taken place ; if greater, there is a possibility of *pāta*.

See *BSS*, xiv. 36-38; *SŚe*, viii. 3 and 7; *SŚi*, I. xii, 10 cd-11 ab ; and *SS*, xi. 7-8 ab. See also Bhāskara II's criticism of Lalla's verse 4 (*SŚi*, I. xii, commentary on 12-14).

6-9. These give the method to find the *pātamadhyakāla*. By the method given above, ascertain the time when the sum of the true longitudes of the Sun and Moon is 6 or 12 signs. This time is known

direction, but difference, if in opposite directions. In the case of *vyatīpāta*, find the difference, if in the same direction, and the sum, if in opposite directions.

7. In each case, the result is called *prathama* (or 'the first'). In the same manner find *apara* (or 'the other'), by assuming some time (before or after that when the sum of the longitudes is 6 or 12 signs ; by calculating the longitudes of the Sun, Moon and its node for that time and hence the declinations); and by taking their sum or difference (as above).

If both the groups of declinations indicate that the *pāta* has either taken place or will take place, find the difference of the first (*prathama*) and the other (*apara*). But find their sum, if one group indicates that *pāta* has taken place and the other that the *pāta* will take place.

8. By this sum or difference divide the product of the first and the assumed time. The result in *ghaṭikās* gives the time of the middle of *pāta*, before or after the time when the sum of the longitudes of the Sun and Moon is 6 or 12 signs, *i. e.*, before, if the first group of declinations shows that the *pāta* has taken place, and after, if it shows that the *pāta* will take place.

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as *pātadhruvakakāla*. Then find the true longitudes of the Sun, Moon and its node at that time. From these longitudes calculate the declination and latitude of the Moon and hence its true declination. If the latter is not equal to the Sun's declination find their difference. Call it the 'first difference' or *prathama*.

Now, assume some time, say x *ghaṭikās*, before or after the *pātadhruvakakāla*, when *pātamadhyakāla* might take place. Then find the true longitudes of the Sun, Moon and its node at this time. Hence find the Sun's declination and the Moon's true declination. Then find their difference. Call it 'the second difference' or *apara*.

Now, the difference between the *prathama* and the *apara* is the

9. Repeat the above process by repeatedly finding the true longitudes (of the Sun, Moon and its node for the corrected time and hence their declinations, till the time is fixed). Multiply the sum of the diameters of the Sun and Moon by the fixed time and divide the product by the *prathama* or 'first'. The result in *ghaṭīkās* is the duration of the *pāta*. Hence find the times for the beginning and end of the *pāta*, as in the case of an eclipse.

[Time of Pāta : Things to be done and avoided]

10. That whole time during which the Moon is in the same

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increase or decrease in the difference of the Sun's declination and the Moon's true declination. Let this difference be y. Then, from the proportion :

y : x : : prathama : ?,

that time is obtained when the difference in the Sun's and Moon's declinations is made up. That is, the *pātamadhyakāla* takes place either before or after the *pātadhruvakākāla* by this time.

This time is approximate. Hence the method must be repeated till the time is fixed. And that will be the correct time for the *pātamadhyakāla*. Let this time be z. Now, at the *pātasparśakāla* and the *pātamokṣakāla*, the difference between the Sun's declination and the Moon's true declination is equal to the sum of their radii. So, from the proportion :

prathama : z : : sum of radii : ?,

the half duration of the *pāta* is found. So, the total duration is

$$= \frac{z \times \text{sum of diameters}}{prathama}$$

Then by adding and subtracting the half duration to and from the *pātamadhyakāla*, the times for the beginning and end are found.

The same method is given in *BSS*, xiv. 39-43 ; *SŚe*, viii. 8-11; and in *SŚi*, I. xii. 11 cd-17.

10. This indicates the inauspicious times when a *pātā* takes place. While commenting on this Mallikārjuna says that according to

diurnal circle as the Sun is considered to be the duration of the *pāta*. Giving gifts, saying prayers, and performing sacrifices bear fruit during this time ; otherwise it is harmful. Three days are not favourable for travelling etc.

[Time of entrance and exit of the Sun and Moon in Pāta]

11. Multiply the (angular) diameter of a planet by 60 and divide by its true daily motion. The result in *ghaṭikās* gives the time

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some, three days are inauspicious, *viz.*, the day of the *pāta*, the day before and the day after. But according to *SS* the time of *pāta* alone is evil. This is supported by the modern *SS*, according to which (xi. 16), only the whole duration of the *pāta* is absolutely inauspicious. During this time only good things such as worship, sacrifice, giving gifts, etc. should be done (xi. 18).

See also *BSS*, xiv. 44 ; *SŚe*, viii. 14 ; and *SŚi*, I. xii. 21.

Vyatīpāta and *vaidhṛta* are very important from the point of view of *Pañcāṅga* or calendar. Most of the *Siddhāntas*, *SS*, *SŚe*, *SŚi*, etc., have a separate chapter called *Pātādhikāra* on this topic. *BSS* has included it in the *Uttara* chapter on true motions of planets. The method to find *pātamadhyakāla* as given in verses 6-9 by Lalla is borrowed from *BSS*. The same has been followed by Śrīpati and Bhāskara II.

11. Here, the first half gives the method to find the time during which some point or other on the disc of a planet is at the junction of two signs of the zodiac or the duration of the planet's *saṅkrānti*.

This follows from the proportion :

Planet's true daily motion : 60 : : its diameter : ?.

The second part gives the method to find the duration when the Sun leaves the last point of a sign and the Moon the last point of a *nakṣatra*. In the case of the Sun the time is :

$$\frac{\text{diameter of the Sun} \times 60}{\text{Sun's true daily motion}}$$

during which some point or other of the disc of the planet is at the junction of two signs of the zodiac ; (or this is the duration of the planet's *saṅkrānti*). The time taken by the Sun to cross the last point of a sign and that by the Moon the last point of a *nakṣatra* are also obtained in this manner by using their respective daily motions.

12. Multiply the sum of the radii of the Sun and Moon by 60

In the case of the Moon the time is

$$\frac{\text{diameter of the Moon} \times 60}{\text{Moon's true daily motion}}$$

12. This gives the method to find the times taken by the Moon to leave the end of a *yoga*, *tithi* and *karaṇa*.

First the *yoga*. This follows from the proportion :

Sum of Sun's and Moon's true motions : 60 : :
sum of the radii of the Sun and Moon : ?.

Now, the *tithi* and *karaṇa*. This follows from the proportion :

Difference of the Sun's and Moon's true motions : 60 : :
sum of radii of the Sun and Moon : ?.

In the first case the sum is taken because there is a *yoga* when the sum of the Sun's and Moon's longitudes is 800′. In the second case the difference is taken because there is a *tithi* when the difference of the Sun's and Moon's longitudes is 12° and a *karaṇa* is ½ *tithi* during which the difference of the Sun and Moon is 6°.

It will be observed that instead of taking Moon's diameter, Lalla takes the sum of the radii of the Sun and Moon.

In this connection see *BSS*, xiv. 29-31 ; *SŚe*, iii. 81 (last verse) ; and *SŚi*, I. ii. 76-77.

Surprisingly, Mallikārjuna's commentary on these verses does not touch these points. He explains and illustrates how to find the beginning and end of the *pāta*.

and divide by the sum of their true daily motions. The result gives the time in *ghaṭikās* at the end of which the Moon leaves the last

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There are some errors in the portion of the manuscript giving the illustration. It has not been considered worthwhile correcting these errors.

A similar example given by Varuṇa in his commentary on *KK*, *Uttara*, 16-20, is being extracted below from the edition of *KK* with Bhaṭṭopala's commentary, Pt. I, pp. 141-45.

Data : The true longitudes of Sun, Moon and its node, corrected by *Deśāntara* and *Bhujāntara* at midnight are respectively $5^s\ 15°\ 51'\ 11''$, $6^s\ 15°\ 7'\ 53''$, and $10^s\ 9°\ 46'\ 40''$ and their true motions are $59'\ 28''$, $805'\ 18''$, and $3'\ 11''$.

To find *Pātamadhyakāla* :

Sum of Sun's and Moon's longitudes $=12^s\ 0°\ 59'\ 4''$.

Difference between this and 12 signs $=59'\ 4''$.

Sum of Sun's and Moon's motions $=864'\ 46''$

Pātadhruvakakāla $=\dfrac{59'\ 4''\times 60}{864'\ 46''}$

$=4$ *ghaṭikās*, 6 *vināḍis* before midnight

Sun's longitude at this time $=5^s\ 15°\ 47'\ 7''$.

Moon's longitude $=6^s\ 14°\ 12'\ 53''$

Node's longitude $=10^s\ 9°\ 46'\ 53''$

Sun's *krānti* $=343'\ 3''$ north,

Moon's *sphuṭakrānti* $=343'\ 3''+243'\ 39''$ south

$=586'\ 42''$ south

It is greater than Sun's *krānti*. Since the Moon is in the third quadrant, *vaidhṛtamadhyakāla* has taken place.

Suppose it has taken place 30 *ghaṭikās* before *pātadhruvakakāla*. Now, the difference between the above *krāntis* is $243'\ 39''$ (*prathamarāśi*).

point of a *yoga*. Similarly, multiply the sum of the radii of the Sun and Moon by 60 and divide by the difference of their true daily

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At the time assumed, Sun's longitude $=5^s\ 15^\circ\ 17'\ 23''$

Moon's longitude $=6^s\ 7^\circ\ 30'\ 14''$

Node's longitude $=10^s\ 9^\circ\ 48'\ 28''$

Sun's *krānti* $=355'$ north

Moon's *sphuṭakrānti* $=181'\ 6''+228'\ 9''$ south

$=409'\ 15''$ south,

greater than Sun's *krānti*. So the *vaidhṛtamadhyakāla* has taken place.

Difference between the above *krāntis*

$=54'\ 15''$ (*dvitīyarāśi*).

Both *prathamarāśi* and *dvitīyarāśi* show that *pāta* has taken place.

Difference of *rāśis* $=189'\ 24''$ (*cheda* or divisor)

Vaidhṛtamadhyakāla after first approximation

$$=\frac{243'\ 39''\times 30}{189'\ 24''}$$

$=38$ *ghaṭikās*, 35 *vināḍīs* before *pātadhruvakakāla*.

The above process should be repeated.

At the above time,

Sun's longitude $=5^s\ 15^\circ\ 8'\ 53''$

Moon's longitude $=6^s\ 5^\circ\ 35'\ 2''$

Node's longitude $=10^s\ 9^\circ\ 48'\ 55''$

Sun's *krānti* $=358'\ 26''$ north

Moon's *sphuṭakrānti* $=134'\ 45''+223'\ 5''$ south.

$=357'\ 50''$ south

It is less than Sun's *krānti*. So, *vaidhṛtamadhyakāla* will take place.

Dvitīyarāśi $=358'\ 26''-357'\ 50''$

$=36''$

Prathamarāśi indicates that *vaidhṛtamadhyakāla* has taken place and *dvitīyarāśi* the opposite. Taking their sum 243′ 39″+36″=244′ 15″ is the *cheda* or divisor.

Vaidhṛtamadhyakāla after the second approximation is 38 *ghaṭikās*, 29 *vināḍīs* before *pātadhruvakakāla*.

The above process should be repeated. At the above time,

Sun's longitude $=5^s\ 15°\ 8'\ 59''$

Moon's longitude $=6^s\ 5°\ 36'\ 22''$

Node's longitude $=10^s\ 9°\ 48'\ 55''$

Sun's *krānti* $=358'\ 23''$ north

Moon's *sphuṭakrānti* $=135'\ 17''+223'\ 10''$ south

$=358'\ 27''$ south,

and is nearly equal to the Sun's *krānti*.

∴ *Vaidhṛtamadhyakāla* is 38 *ghaṭikās*, 29 *vināḍīs* before *pātadhruvakakāla*.

Now, to find half duration of *pāta*.

Sum of Sun's and Moon's radii

$$=\frac{32'\ 42''+32'\ 36}{2}$$

$=32'\ 39''$.

∴ Half duration of *vaidhṛta*

$$=\frac{32'\ 39''\times 38\ gh.\ 29\ vin.}{243'\ 39'}$$

$=5$ *ghaṭikās*, 9 *vināḍīs*

At 5 *ghaṭikās*, 9 *vināḍīs* before *vaidhṛtamadhyakāla*,

Sun's longitude $=5^s\ 15°\ 3'\ 53''$

Moon's longitude $=6^s\ 4°\ 27'\ 14''$

Node's longitude $=10^s\ 9°\ 49'\ 11''$

Sun's *krānti* $=360'\ 26''$ north

Moon's *sphuṭakrānti* $=107'\ 29''+219'\ 59''$ south

$=327'\ 28''$ south

∴ difference $=32' \; 58''$

∴ duration of first half after first approximation

$$= \frac{32' \; 39'' \times 5 \; gh. \; 9 \; vin.}{32' \; 58''}$$

$=5$ *ghaṭikās*, 6 *vināḍīs*.

At that time,

Sun's longitude $=5^s \; 15^\circ \; 3' \; 56''$

Moon's longitude $=6^s \; 4^\circ \; 27' \; 55''$

Node's longitude $=10^s \; 9^\circ \; 49' \; 11''$

Sun's *krānti* $=360' \; 25''$ north

and Moon's *sphuṭakrānti* $=107' \; 45''+220' \; 1''$ south.

$=327' \; 46''$ south.

∴ difference $=32' \; 39''$

∴ duration of the first half after second approximation

$$= \frac{32' \; 39'' \times 5 \; gh. \; 6 \; vin.}{32' \; 39''}$$

$=5$ *ghaṭikās*, 6 *vināḍīs*, which is the same as before and is the correct duration.

Now to find the duration of the second half. At 5 *ghaṭikās*, 9 *vināḍīs* after *vaidhṛtamadhyakāla*,

Sun's longitude $=5^s \; 15^\circ \; 14' \; 5''$

Moon's longitude $=6^s \; 6^\circ \; 45' \; 30''$

Node's longitude $=10^s \; 9^\circ \; 48' \; 39''$

Sun's *krānti* $=356' \; 20''$ north

and Moon's *sphuṭakrānti* $=163' \; 6''+226' \; 14''$ south.

$=389' \; 20''$ south

∴ difference $=33'$

∴ duration of the second half after first approximation

$$= \frac{32' \; 39'' \times 5 \; gh. \; 9 \; vin.}{33}$$

$=5$ *ghaṭikās*, 5 *vināḍīs*.

motions. The result in *ghaṭikās* gives the time after which the disc of the Moon leaves the end of a *tithi* or of a *karaṇa*.

Thus ends chapter Twelve

on

VYATĪPĀTA AND VAIDHṚTA

At that time,

Sun's longitude $=5^s\ 15^\circ\ 14'\ 1''$

Moon's longitude $=6^s\ 6^\circ\ 44'\ 35''$

Node's longitude $=10^s\ 9^\circ\ 48'\ 39''$

Sun's *krānti* $=356'\ 22''$ north

and Moon's *sphuṭakrānti* $=162'\ 44''+226'\ 12''$ south

$=388'\ 56''$ south.

∴ difference $=32'\ 34''$

∴ duration of the second half after second approximation

$$= \frac{32'\ 39'' \times 5\ gh.\ 5\ vin}{32'\ 34''}$$

=5 *ghaṭikās*, 5 *vināḍīs*, which is the same as before and is the correct duration.

As the lengths of the day and half nights are 31 *ghaṭikās*, 16 *vināḍīs*, and 14 *ghaṭikās*, 22 *vināḍīs*, respectively, *vaidhṛtadhruvakāla* is

31 *gh.* 16 *vin.*+(14 *gh.* 22 *vin.*—4 *gh.* 6 *vin*)

=41 *gh.* 32 *vin.* after sunrise.

The *vaidhṛtamadhyakāla* being 38 *gh.* 29 *vin.* before, is 3 *gh.* 3 *vin.* after sunrise.

Vaidhṛta begins 5 *gh.* 6 *vin.*—3 *gh.* 3 *vin.*=2 *gh.* 3 *vin.* before sunrise.

Vaidhṛta ends 5 *gh.* 5 *vin.*+3 *gh.* 3*vin.*=8 *gh.* 8 *vin.* after sunrise.

CHAPTER XIII

RATIONALE OF CORRECTIONS

[The Laghujyās]

1. I have given the easier way of calculating the mean places of planets by the method called *Pratyabdaśuddhi* or corrections given to the longitudes of planets at the end of the each year. Now, I shall explain how to calculate the true places of planets (more easily) by using the *Laghujyā* method. In this case, the R sine of arcs of 10° are used. It must be remembered that when the value of an arc corresponding to a given R sine is to be found, the multiplier must be 10°.

[The Khaṇḍajyās and Piṇḍajyās]

2-3. The R sines of the nine consecutive arcs, (each of 10°, in a quadrant), are, respectively, 26, 25, 24, 21, 19, 15, 11, 7 and 2.

Now, I give the R sines of arcs of 10°, 20°, 30°, etc. upto 90°. They are, respectively, 26, 51, 75, 96, 115, 130, 141, 148 and 150.

[True longitudes and Rates of motion of the Sun and Moon]

4. (In the *laghujyā* method), multiply the R sine of the mean

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MAHEMATICAL NOTES

1-3. The table of R sines given in ii. 1-4 is borrowed by Lalla from *Āryabhāṭīya*. There the radius is 3438 units and the R sines of arcs are calculated at intervals of 225′ or 3° 45′. Here Lalla takes 150 units as the radius and arcs are of 10° each. This method is known as *laghujyā* method.

Bhāskara II also gives R sines of arcs of 10° but his radius is 120 units (*SŚi*, I. ii. 13).

4. This gives the method to calculate the *mandaphala* and the

anomaly of the Sun or Moon by its *mandaguṇaka*. Then multiply each product by 2 and divide by 7. The results in minutes etc. are their respective *mandaphalas*. Hence find their true longitudes as before.

Divide the Sun's *bhogyakhaṇḍa* by 11. The result is the Sun's *mandagatiphala* or correction to be given to its mean motion.

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mandagatiphala or the first correction to the planet's longitude and to its motion, respectively, by the *laghujyā* method.

From ii. 14, the circumference of the Sun's epicycle is $13\frac{1}{2}^\circ$ when the circumference of the concentric is 360°, and the radius is 3438.

So; R sin Sun's maximum *mandaphala*

$$=\frac{13\frac{1}{2}\times 3438}{360}=\frac{27\times 3438}{2\times 360} \qquad (1)$$

So, the maximum *mandaphala* is the arc corresponding to (1), as R sine and it is taken as

$$\frac{27\times 3438}{2\times 360} \text{ minutes, nearly.}$$

Thus, from the proportion :

$$150 : \frac{27\times 3438}{2\times 360} :: \text{given R sin } manda \text{ anomaly}$$

$$: \ ? \ ,$$

the required *mandaphala*

$$=\frac{27\times 3438\times \text{R sin } manda \text{ anomaly}}{150\times 2\times 360}$$

$$=\frac{3\times 2}{\dfrac{50\times 2\times 120}{1719}}\times \text{R sin } manda \text{ anomaly}$$

$$=\frac{3\times 2}{7}\times \text{R sin } manda \text{ anomaly, nearly,}$$

3 is the *mandaguṇaka* or multiplier (ii. 14) for the Sun.

In the case of the Moon, since the circumference of the epicycle

Multiply the Moon's *bhogyakhaṇḍa* by 3 and subtract from the product $\frac{1}{7}$ of the *bhogyakhaṇḍa*. The result is the correction to be given to the Moon's motion.

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is $31\frac{1}{2}^{\circ}$, the radius 3438, and the circumference of the concentric 360°, the R sin maximum *mandaphala*

$$=\frac{31\frac{1}{2}\times 3438}{360}=\frac{63\times 3438}{2\times 360} \qquad (2)$$

So, the maximum *mandaphala* is the arc corresponding to (2) as R sine. Lalla takes it equal to

$$=\frac{63\times 3438}{2\times 360} \text{ minutes, nearly.}$$

Then from the proportion :

$$150 : \frac{63\times 3438}{2\times 360} :: \text{given R sin mean anomaly} : ?;$$ the required *mandaphala*

$$=\frac{63\times 3438}{150\times 2\times 360}\times \text{R sin mean anomaly}$$

$$=\frac{7\times 1}{\frac{150\times 2\times 40}{1719}} \text{ R sin mean anomaly}$$

$$=\frac{7\times 2}{7}\times \text{R sin mean anomaly, nearly.}$$

7 is the *mandaguṇaka* or multiplier for the Moon (ii. 14).

Now, to the correction to the Sun's motion. The Sun's *mandaphala* on any day is $\frac{6}{7}$ R sin mean anomaly (on that day). So, the difference between the *mandaphalas* on two consecutive days or the *mandagatiphala* or the correction to the motion

$$=\frac{6}{7}\times \text{difference of R sines of the mean anomalies}$$

$$=\frac{6}{7}\times\frac{bhogyakhaṇḍa}{10^{\circ}=600'}\times \text{motion of the anomaly (ii. 15)}$$

$$=\frac{6}{7}\times\frac{59'\times 8''}{600'}\times bhogyakhaṇḍa$$

$$=\frac{59'\ 8''\times 11}{700'\times 11}\times bhogyakhaṇḍa$$

These corrections should be added to or subtracted from the respective mean motions of the Sun and Moon as instructed before.

5. Multiply the R sines of the mean anomalies or the *manda-kendras* of Mars, Mercury, Jupiter, Venus and Saturn by their respective corrected *mandaguṇakas*. Then multiply each product by 2 and divide by 7. The results in minutes are the respective *mandaphalas* of these

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$=\frac{1}{11}\times$ *bhogyakhaṇḍa*, nearly.

Now, to the Moon's correction to the Motion. It will be

$$=\frac{7\times 2}{7}\times \text{motion of mean anomaly}\times\frac{bhogyakhaṇḍa}{600}$$

$$=\frac{2\times 783'\ 24''}{600}bhogyakhaṇḍa$$

$$=\frac{2\times 20\times 7\times 783'\ 54''}{600\times 20\times 7}bhogyakhaṇḍa$$

$$=\frac{20}{7}\ bhogyakhaṇḍa, \text{ nearly.}$$

$$=3bhogyakhaṇḍa-\frac{1}{7}\ bhogyakhaṇḍa, \text{ nearly.}$$

See Bhāskara II's methods in *SŚi*, I. ii. 30-31.

5. This explains the method to find the *mandaphala* of planet like Mars. According to iii. 3, the *mandaphala* of a planet

$$=\frac{\text{correct } mandaguṇaka\times \text{R sin mean anomaly}}{80}\text{ minutes.}$$

So, the maximum *mandaphala*

$$=\frac{\text{correct } mandaguṇaka\times \text{R}}{80}\text{ minutes}$$

$$=\frac{\text{correct } mandaguṇaka\times 3438}{80}\text{ minutes.}$$

So, the *mandaphala* for any given mean anomaly, when the radius is 150 will be

$$=\frac{\text{correct } mandaguṇaka\times 3438}{80\times 150}\times\text{R sin mean anomaly}$$

planets. These should be added to or subtracted from their respective mean longitudes as explained before.

6. As before, multiply the R sine and R cosine of the *śīghra-kendra* of a planet by its correct *śīghraguṇaka* and divide the product by 80. Hence find the R sine *śīghraphala*, etc. as said before. And hence the corresponding arc. Thus the true longitudes of the planets are calculated.

[Mandaphala by an alternative method]

7. Or, multiply the radius by the correct *mandaguṇaka* of a

$$= \frac{\text{correct } mandaguṇaka \times 2}{\frac{80 \times 150}{1719}} \times \text{R sin mean anomaly}$$

$$= \frac{\text{correct } mandaguṇaka \times 2}{7} \times \text{R sin mean anomaly}$$

While commenting on verse 5, Malikārjuna says that another way of getting the corrected *mandaguṇaka* is to add the circumference of the epicycle in the first or third quadrant and that in the second or fourth quadrant. Thus they will be 32° (14°+18°), 12° (7°+5°), 15°(7°+8°), 6°(4°+2°) and 22°(9°+13°). These measurements are given by Āryabhaṭa, as has already been pointed out in the notes on iii. 1-2.

Then the mean anomaly of each planet should then be respectively multiplied by these and divided by 7. The result will be the respective *mandaphala*.

For the correction to motion, multiply each *bhogyakhaṇḍa* by the motion of the planet and divide by 10. Then the results should each be multiplied by 32, etc. and divided by 7. The final result in each case is the correction to be given to the motion (see iii. 11).

6. See, iii. 3-8.

7. Another method to calculate *mandaphala*. From iii. 3, R sin *mandaphala* of a planet

planet and divide by 80. The result is the R sine of its maximum *mandaphala*. Multiply it again by the R sine of the mean anomaly of the planet and divide by the radius (*viz.*, 150). The result converted into arc is the *mandaphala* of the planet.

[Śīghraphala by an alternative method]

8. If the *śīghrakendra* is within six signs beginning with Capricorn, find the sum of its R cosine and the R sine of the maximum

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$$=\frac{\text{correct } mandaguṇaka \times \text{R sin } manda \text{ anomaly}}{80}$$

So, R sin maximum *mandaphala*

$$=\frac{\text{correct } mandaguṇaka \times 3438}{80}$$

So, R sin *mandaphala* for a given mean anomaly and radius 150

$$=\frac{\text{correct } mandaguṇaka \times 3438}{80 \times 150} \times \text{R sin mean anomaly}$$

and *mandaphala* is the arc corresponding to this as R sine.

8. This gives the same method as that given in iii. 3-8 for

śīghraphala. If it is within six signs beginning with Cancer, find their difference. Add the square of the result to the square of the R sine of

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finding the *śīghraphala*, with the difference that the *śīghra* hypotenuse is found in another way.

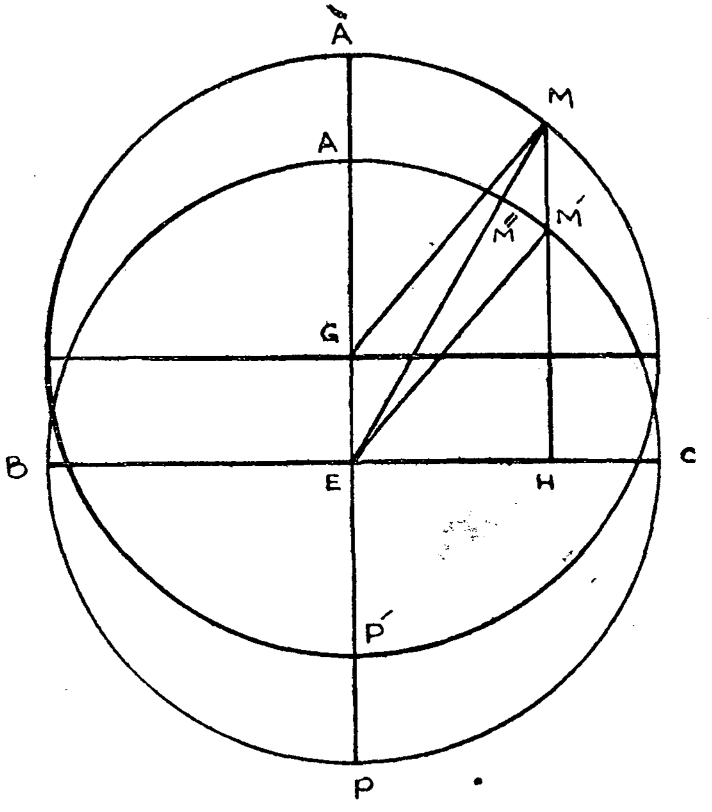

FIG. 30

Let ABPC be the circle with E, the centre of the earth, as centre or concentric or *kakṣāvṛtta*. Let AEP be the vertical diameter. Let EG be the R sin maximum *śīghraphala* of a planet. Let A′MP′ be a circle with G as centre and equal to the former circle. This is the eccentric circle or *śīghrapratīvṛtta*. Let AP cut this circle in A′ and P′.

the *śīghrakendra*. The square root of the sum is the hypotenuse. Divide the product of the R sine of the *śīghrakendra* and the R sine of

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Then A and P and A′ and P′ are *śīghrocca* and *śīghranīca*, respectively, in the concentric and eccentric.

Let the thrice-corrected planet and the real planet start from A and A′, respectively, and move along the concentric and eccentric with the same velocity and in the same direction. Let the new places be M′ and M, respectively.

Let EM be joined, cutting the concentric in M″, which is the true place of the planet. Join GM, EM′ and MM′. Produce MM′ to meet BC in H. It is evident that GE is equal and parallel to MM′ and, so, GM is equal and parallel to EM′. So, the angle A′GM = the angle AEM′ = *śīghrakendra*.

So, EH is R sin *śīghrakendra* and M′H is R cosine *śīghrakendra*.

EM, the *śīghra* hypotenuse $=\sqrt{EH+H^2M^2}$

$=\sqrt{EH^2+(MM'+M'H)^2}$

$=R^2 \sin^2$ (*śīghrakendra*) + (R sin maximum *śīghraphala* + R cos *śīghrakendra*)^2.

If the *śīghrakendra* is greater than 3 signs but less than 9 signs, the difference must be taken. The remaining method is the same as that given in iii. 3-8.

While commenting on this verse, Mallikārjuna says that three methods have been explained for calculating the true longitude of a planet: (1) by calculating R sin of maximum *phala* by using correct circumference ; (2) by calculating R sin of maximum *phala* by using half of the sum of the two types of circumference, odd and even ; (3) by using the measurements given in *KK*. The last method is followed by most people, especially by those to the north of the Vindhyas.

The following are the maximum *manda* and *śīghra* equations of

its maximum *śīghraphala* by the hypotenuse. The arc corresponding to the result is the *śīghraphala*.

[Ascensional difference at any time]

9. The equinoctial midday shadow (at the observer's station) multiplied severally by 60, 49 and 20 gives, respectively, the ascensional

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Sun etc., as said to be given in the *KK* according to Mallikārjuna and those found in the present *KK*.[1]

	Given by Mallikārjuna		Given in KK	
	Manda	*Śīghra*	*Manda*	*Śīghra*
Sun	131′	—	134′	—
Moon	302′	—	296′	—
Mars	670′	2430′	670′	2430′
Mercury	268′	1371′	268′	1290′
Jupiter	306′	690′	306′	690′
Venus	131′	2775′	134′	2775′
Saturn	459′	380′	574′	380′

Further on, Mallikārjuna says that even according to *KK* the rising and setting of Venus and the conjunction of the planets Mars, etc. do not tally with the observations. He adds that the calculation of the true longitude of a planet is very difficult and so several methods for such calculations are given in the *Āryabhaṭasiddhānta*. He then gives three verses composed by him for calculating the true motion from the mean motion. (Text, p. 184). The rule is the same as that already given before.

9. This gives the method to find the ascensional differences

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1. The figures given by Mallikārjuna agree with those given in *Uttara-khaṇḍakhādyaka* ; the figures ascribed to *KK* by Bina Chatterjee are those mentioned in *Pūrva-khaṇḍakhādyaka*. So there is no discrepancy as supposed by Bina Chatterjee. —*KSS*

differences in *asus* of the first three signs of the zodiac beginning with Aries, (at the local latitude). By means of these and from the longitude of the Sun, the ascensional difference at any time may be found.

[Moon's latitude]

10. The R sine of 24° is $60\frac{50}{60}$. Its square is 3700 (nearly).

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when the Sun is at the end of 30° or Aries, of 60° or Taurus, and of 90° or of Gemini, when the radius is 150. By ii. 17,

R sin declination of 30°

$$=\frac{\text{R sin } 30^\circ \times \text{R sin } 24^\circ}{\text{R}}$$

$$=\frac{75\times 60\frac{50}{60}}{150}=\frac{365}{6\times 2}=\frac{365}{12}.$$

Suppose the midday equinoctial shadow is 1 *aṅgula*. Then, earth-sine

$$=\frac{\frac{365}{12}\times 1}{12}\left(cf., \text{ similar triangles U and Z}\right)=\frac{365}{144}$$

So, radius day-circle

$$=\sqrt{150^2-\frac{365^2}{144}}=147, \text{ nearly.}$$

Thus, R sin ascensional difference

$$=\tfrac{365}{144}\times\frac{1}{147}\times 150$$

$$=\frac{54750}{21168}=2\tfrac{35}{60}$$

So, the ascensional difference is the arc corresponding to $2\frac{35}{60}$ as the R sine or it is 60′, nearly.

Now, along the equator 1′=1 *asu*, and so the ascensional difference when the Sun is at the end of 30° is 60 *asus*, nearly, assuming that the midday equinoctial shadow is 1 *aṅgula*. The other differences can be found as before.

10. R sin 24° is 1397, when the radius is 3438. Now, since the radius is 150, R sin 24° is $60\frac{50}{60}$ or $61-\frac{1}{6}$.

The R sine of the true longitude of the Moon minus that of its node, multiplied by 9 and divided by 5, gives its latitude in minutes.

[Parallax in longitude]

11. The radius and the *valanajyā,* when divided by 5, are converted into *aṅgulas.* The R sine of the *dṛggati,* multiplied by 2 and divided by 75, gives in *ghaṭikās,* the parallax in longitude.

[Parallax in longitude and latitude : Alternate method]

12. Or, the R sine of the hour angle multiplied by the R sine of the altitude of the meridian ecliptic point and divided by 5625, gives the parallax in longitude.

The difference of the true motions of the Sun and Moon, multiplied by the R sine of the *dṛkṣepa* and divided by 2250, gives the parallax in latitude.

The formula for the latitude of the Moon as given here follows from that given in verse 11 by substituting 150 for 3438. (See notes on v. 11).

11. In v. 27, Lalla considered 110 units equivalent to 1 *aṅgula,* when 3438 units was the radius. Now, since the radius is 150 units, the measurement would be

$$=\frac{110\times 150}{3438}=5, \text{ nearly.}$$

So, 5 units are assumed to be equal to 1 *aṅgula.*

The formula for the longitudinal parallax follows from the one given in vi. 9, by substituting 150 for 3438.

12. The first part gives another formula to find the longitudinal parallax, and the second part the latitudinal parallax, by using 150 as the radius.

The formula for the latitudinal parallax follows from that given in vi. 11b by substituting 150 for 3438.

[Parallax in longitude and latitude : Still another method]

13. Multiply the radius of the earth by the R sine of the *dṛkkṣepa* and divide the product severally by the correct distance of the Sun and the Moon from the earth. Find the corresponding arc in each case. The difference of the two results is the parallax in latitude (in minutes).

Again multiply the *dṛggati* by the radius of the earth and divide the product severally by the correct distance of the Sun and the Moon from the earth. Find the corresponding arc in each case. The difference of the two results is parallax in longitude. Then follow the procedure as given before.

[Visibility correction]

14. Multiply the R versed sine of the true longitude of a planet increased by 90° by its latitude and by $60\frac{50}{60}$ and divide the product by 22500. The result in minutes, etc. is the *dṛkphala* or visibility correction of the planet due to the obliquity of the ecliptic.

15. Or, multiply the difference of the radius and the R sine of

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14. This gives the formula for the visibility correction of a planet due to the obliquity of the ecliptic when the radius is 150.

This form of the formula is obtained by substituting $60\frac{50}{60}$ for R sin 24° and 150 for 3438 in the formula given in viii. 3.

15. This gives an alternate form for the formula for the visibility correction. According to viii. 2, as given by Lalla, visibility correction

$$= \frac{\text{R sin } 24° \times (\text{R} - \text{R sin longitude of planet})}{\text{R}^2} \times \text{latitude (of planet)}$$

$$= \frac{60\frac{50}{60} \times (\text{R} - \text{R sin planet's longitude}) \times \text{latitude}}{150 \times 150}$$

$$= \frac{(\text{R} - \text{R sin planet's longitude}) \times \text{latitude}}{370}, \text{ nearly.}$$

the true longitude of a planet by its latitude and divide by 370. The result is the visibility correction (in minutes).

[Elongation of the Moon's horns]

16. When the cusps of the Moon are to be calculated, first find the base, perpendicular and the hypotenuse by the *laghujyā* method and then divide each of them by 9, to convert them into *aṅgulas*. One *aṅgula* is equivalent to the width of 8 grains of barley without husk.

[Biodata of the Author]

17. Thus, having bowed at the lotus-like feet of the Goddess (Sarasvatī), daughter of Brahmā, and auspicious source of all the good qualities, I complete the calculations of the motions etc. of the planets, through Her blessings alone.

18-19. Tāladhvaja was born in a noble Brāhmaṇa family and innumerable learned men bowed at his lotus-like feet. Sāmba was his son. He was as dear to the people as is the Moon to the white lotus (blossoming at night). He was known as Bhaṭṭa Trivikrama in the world.

His son Lalla, having made obeisance at the feet of Śiva who has the Moon on His head and is the husband of the daughter of Himālaya, the king of mountains, has written this *Tantra* which will give the same results as those obtained (from the formulas) in the *Āryabhaṭasiddhānta*.

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16. In ix. 10, Lalla takes 200 as equivalent to 1 *aṅgula* while calculating the cusps of the Moon, when the radius is 3438. So, here, from the proportion 3438 : 200 : : 150 : ?, the equivalence is 9, nearly.

20. I, Lalla, after completing *Śiṣyadhīvṛddhida,* propitiate Pārvatī Who is Herself the sacred knowledge and is worshipped by Brahmā, Viṣṇu and Maheśvara and is also known as Mahāmāyā.

Thus ends chapter Thirteen

on

RATIONALE OF CORRECTIONS

CHAPTER XIV

GRAPHICAL REPRESENTATION OF THE MOTION OF THE PLANETS

[Invocation]

1. Having made obeisance to Him by Whose grace this entire universe, with everything in it, moves fast, and also to Her who dwells in the lotus-like mouths of the learned men, I write (this section) on the sphere of universe.

[Importance of the armillary sphere]

2. No astronomical treatise (is complete) without a section on the sphere of universe, just as the night is not (lovely) without the rays of the Moon, no woman without breasts (is pretty to look at), and no feast (is enjoyable) without sweets.

3. As a disputant without of knowledge of grammar, as a sacrificer without the knowledge of the *Śrutis* (Vedas) and *Smṛtis*, (*Purāṇas*, etc.), as a physician without experience, so is an astronomer totally unsuccessful, without the knowledge of the sphere of universe.

4. The astronomers say that the sphere of universe is solely (an object) of mathematical calculations ; and those who wish to study the planets must be experts in the subject of the sphere. Those who know neither mathematics nor the essence of the sphere of universe are ignorant. How can they ever detect the motion of the planets ?

MATHEMATICAL NOTES

1. Herein Lalla makes obeisance to Brahmā and Sarasvatī.

2-6. These verses emphasise the importance of the study of the sphere. Brahmagupta, Śrīpatī and Bhāskara II have also explained

5. He who has a comprehensive knowledge of this sphere containing the Sun, etc. sees in front of his eyes, as it were, the whole universe, beautiful with a variety of exquisite situations. Moreover, he becomes the recipient of religious fruit, wealth, salvation and fame.

6. The mean motions of the planets, etc., that have been discussed above, are clearly perceptible on the armillary sphere, especially to him who has mastered the science of their (geometrical) representation.

[Method of graphical representation]

7. (With any point as centre) and the given radius, describe a circle which is the circular orbit of the planet or *kakṣāvṛtta*. Divide its circumference into 12 signs (following the 12 signs of the zodiac). Assume that the earth is at the centre of this circle.

8. From the centre of the earth draw a vertical line upwards equal in length to the radius of the epicycle of the planet or *antyaphalajyā* or R sine of the maximum correction. With the extremity of this line as centre and the given radius (equal to that of the *kakṣāvṛtta*), describe a circle. It is the eccentric circle or *kendravṛtta*.

9. Or, (draw a vertical diameter in the circular orbit). With the upper end of that diameter as centre and radius equal to the radius of the epicycle, describe a circle. This is known as the epicycle or *nīcoccāvṛtta* of the planet. If the mean place of the planet is at the apogee or *ucca* of the circular orbit, its true place is at the apogee or *ucca* of the epicycle.

10. When a planet is at a distance of 6 signs from its apogee, it is said to be at the perigee or *nīca*. When a planet is at the apogee,

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that such knowledge is essential (*BSS*, xxi. 1; xxii. 1-4 ; *ŚSe*, xv. 1-6; and *SŚi*, II. i. 2-9).

7-18. These describe epicyclic and eccentric theories relating to the motions of planets, which have been explained in the Appendices below.

it is farthest from the earth ; when at the perigee, it is nearest to the earth. This is so because of the (length of the) hypotenuse in each case.

11. Thus, the planets appear small or large accordingly. In the same manner, they appear small or large, according as they are near the Sun or removed from it.

12. A planet moves in the direction of the signs of the zodiac from its *manda* apogee or *mandocca* (apex of slow motion) and in the opposite direction from its conjunction or *śīghrocca* (apex of quick motion).

The mean planet moves along the circular orbit or concentric and the true planet along the eccentric.

13. The rate of motion of the true planet in the eccentric is the same as that of the mean planet in the concentric. When the planet is at the points of intersection of these two circles, its mean motion is the same as its true motion.

14. When the anomaly is either 6 signs or 12 signs, the observer sees the mean planet and the true planet in the same straight line. Therefore, at these places the mean planet has no correction.

15. Since, (for other values of the anomaly) an observer sees the true planet either ahead or behind its mean place, the difference of

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According to Lalla, the formula for the first correction to the motion of a planet is

$$\frac{\text{correct manda epicycle}}{80} \times \frac{\textit{bhogyakhaṇḍa}}{225} \times \text{motion of mean anomaly}$$

and the formula for the second (*śīghra*) correction to the motion of a planet is

$$\frac{\text{correct } \textit{śīghra} \text{ epicycle}}{80} \times \frac{\textit{bhogyakhaṇḍa}}{225} \times$$

$$\frac{\text{R}}{\text{hypotenuse}} \times \text{motion of } \textit{śīghra} \text{ anomaly.}$$

these positions, that is, the correction, should either be added to or subtracted from the mean place (to obtain the true planet).

16. When the anomaly is within 6 signs beginning with Capricorn, its R cosine is above the R sine *antyaphala*. So their sum is the correct R cosine. Again, when the anomaly is within 6 signs beginning with Cancer, its R cosine is below the R sine *antyaphala*. So, their difference is the correct R cosine.

17. When the anomaly is either in the first or fourth quadrant the *koṭiphala* is above the radius. So, their sum is the correct *koṭi*. When the anomaly is either in the second or third quadrant, the *koṭiphala* is below the radius. So, their difference is the correct *koṭi*.

18. The distance between the centre of the earth and the true position of the planet is known as the hypotenuse or *karṇa*.

The correction or *phala* is the (arc of the concentric intercepted between the point) where the hypotenuse cuts it and the position of the mean planet.

The R sine of the anomaly, *bāhujyā*, is always the perpendicular distance between the place of the planet and the diameter passing through the apogee and perigee.

19. At the end of a revolution of a planet there is no difference

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Thus, the values of both the corrections are 0, when the *bhogya-khaṇḍa* is 0. This is not so when the planet is at the points of intersection of the concentric and eccentric and, hence, at these points, the mean motion is not equal to the true motion. Thus, Lalla's statement (13b) does not follow from his own formulae. Moreover, the mean motion of a planet is equal to its true motion at the points where the straight line passing through the centre of the concentric and at right angles to the line of apse, cuts the eccentric circle. This follows from Bhāskara II's formula. He criticizes Lalla's statement in his commentary on *SŚi*, II. v. 39. See *BSS*, xxi. 24-30 ; *SŚe*, xvi. 1-8, 18-21 ; and *SŚi*, II. v. 7-37.

19. Śrīpati says the same in *SŚe*. xvi. 22.

between its mean and true motions. (The same revolution is achieved by means of mean or true motion).

The various phenomena relating to a planet, such as its rising (in a particular quarter), regression, etc. are only apparent. They do not cause changes in the number of revolutions.

Thus ends chapter Fourteen

on

GRAPHICAL REPRESENTATION OF MEAN MOTION

CHAPTER XV

CONSTRUCTION OF THE ARMILLARY SPHERE

[Sphere of the sky]

1. The great circle passing through the east and west points (of the horizon) is said to be the prime vertical or *samamaṇḍala.* The great circle passing through the north and south points (of the horizon) is the meridian or *yāmyottaramaṇḍala.* In the same manner, the great circles (passing through the north-east and south-west points and through the north-west and south-east points) are called verticals in intermediate directions or *koṇamaṇḍalas.*

2. The great circle that goes transversely through the middle of these is the horizon or *kṣitija.* The planets rise and set here.

3. The six o' clock circle or *unmaṇḍala* passes through those two points on the prime vertical where the horizon and the celestial equator meet it ; it also passes through the north pole at a distance equal to the latitude of the place, from that point on the horizon, where the meridian meets it ; it also passes through the south pole at a distance equal to the latitude of the place, below that point on the horizon, where the meridian meets it.

The six o'clock circle causes the increase and decrease in the lengths of day and night.

MATHEMATICAL NOTES

This chapter describes the great circles necessary for determining the longitude, motion, etc. of planets. All the circles have already been described in the notes on Sn. I : Mathematics, above.

In this connection see *BSS,* xxi. 49-58, 67-69 ; *SŚe,* xvi. 29-47 ; and *SŚi,* II. vi. 1-31.

4. These six circles in the celestial sphere are each divided into 360°.

The celestial equator or *viṣuvadvṛtta* or *nāḍīvṛtta* is divided into 60 *nāḍīs* or *ghaṭikās*. It is the *bhagola* or the sphere of the fixed stars, which is below the *bhagola* or celestial sphere.

[Sphere of asterisms]

5. The celestial equator in the *bhagola* is to the south of the prime vertical (at a distance equal to the) latitude of the place. (It intersects the meridian at a distance equal to the) latitude of the place, to the south of the zenith and (again at a distance equal to the) latitude of the place to the north of the nadir.

6. There is to be another circle known as meridian which completely encircles the (other circles) like the horizon. The *dhruva-yaṣṭi* or the polar axis is the line joining the north and south poles and passing through the earth's centre. (The earth is placed loosely in the middle of it).

7. The ecliptic or *apamaṇḍala* intersects the celestial equator at the first points of Aries and Libra. It is to the north of the equator by 24° where is the first point of Cancer. It is also to the south of the equator by 24° where is the first point of Capricorn.

8. The Sun moves along the ecliptic and so do the Moon, the planets and the nodes of the Moon and the superior planets. The shadow of the earth also moves on the ecliptic at a distance of 6 signs from the Sun. The nodes of Mercury and Venus move along their respective *śīghravṛttas*.

9. The first half of the orbit or *vikṣepamaṇḍala* (of a planet) measured from the node is to the north of the ecliptic and (the greatest distance between them) is the greatest latitude of the planet to the north.

The second half of the orbit measured from the same node increased by 6 signs is to the south of the ecliptic and (the greatest

distance between them) is the greatest latitude of the planet to the south.

10. The three day-circles of the first three signs beginning with Aries are drawn parallel to the celestial equator, at distances equal to their respective declinations. (The same circles considered in a) contrary direction are the day-circles of the next three signs. In the same way the day-circles of the other six signs beginning with Libra can be drawn.

11. The day-circle for any given declination should be drawn (parallel to the celestial equator) at a distance equal to that declination. The spheres of the planets are drawn by means of great circles in the *bhagola.*

[Sphere of the planets]

12. In the plane of the equator make a circle equal to the planet's orbit. (This is the equator in the sphere of the planet). Also draw the meridian circle going round (this circle) like the horizon.

13. Now, fix a great circle at the same distance from the circle already drawn equal to the planet's orbit as is the ecliptic from the celestial equator in the celestial sphere. This will be the ecliptic in the sphere of the planet. The planet moves along this (circle).

14. (Then, fix) epicycles for mean anomaly and for *śīghra* anomaly and also eccentrics for mean anomaly and for *śīghra* anomaly.

The sphere of earth is beneath that of the planets.

15. The circle attached to the six o'clock circle at a distance equal to the planet's declination, in the eastern and the western hemisphere, and to the planet's meridian at a distance equal to the planet's meridian zenith distance, is the diurnal circle of the moving planet.

[Armillary sphere in general]

16. The meridian is also called *madhyajyāvṛtta.* The *madhyalagna* or meridian ecliptic point is on this circle. The *dṛṅmaṇḍala* is above it. The planet is seen on this circle.

17. The *dṛggatijīvāvalaya* is just ahead of the meridian ecliptic point.

The circle on which the extremity of the shadow (of the gnomon caused by a planet) moves, is opposite to that on which the planet moves.

18. The amplitude or *agrā* is (measured) on the celestial horizon. Its R sine is the R sine of the arc of the horizon intercepted between the east point and the day-circle of the rising planet or the west point and the day-circle of the setting planet. That is, its extreme points are the east (or west) point and the point where the planet rises (or sets).

19. The earth-sine or *kujyā* is the R sine of the arc of the day-circle intercepted between the horizon and the six o'clock circle. The R sine of the same arc, when converted to the great circle or *trijyāvṛtta*, is the R sine of the ascensional difference.

20. The orient ecliptic point or *lagna* is the point on the ecliptic where the eastern horizon meets it. The *astalagna* or the setting ecliptic point is the point on the ecliptic where the western horizon meets it.

21. The R sine of the degrees of the arc (or altitude) by which the Sun is raised either above the eastern or the western horizon, is called *śaṅku*. The Sun is at its extremity. Or, (it can be said that the) *śaṅku* is the perpendicular support of the Sun's disc.

22. The *samaśaṅku* is the R sine altitude of the Sun when it is on the prime vertical ; and the *koṇaśaṅku* is the R sine altitude when it is on the *koṇamaṇḍala* or the vertical circle in an intermediate direction. When the Sun is equidistant from the north and south points of the horizon, the R sine of the degrees by which it is raised above the horizon is called '*śaṅku* at midday' or *dyudalaśaṅku*.

23. When the Sun is on the meridian, the R sine of the degrees measured from the zenith is called *natajyā* or R sine zenith distance. The *dṛgiyā* is the distance on the earth between the foot of the R sine altitude and the centre of the earth, that is, the point where the directions meet.

24. When the Sun is on the equinox, its zenith distance at midday is equal to the latitude of the place, and its altitude is the colatitude of the place. Thus, the R sines of the zenith distance and of the altitude are, respectively, the R sines of the latitude and the colatitude of the place.

25. The line that joins the zenith to the point of intersection of the lines of direction is the radius or *trijyā* and so is also the line that joins the top of the R cosine latitude to the same point. The same *trijyā* is the hypotenuse in a rectangular figure.

26-27. In the north, measure out an arc below the horizon equal to the elevation of the pole. (Let it be the lower point). In the south, measure out an arc of the same length above the horizon. (Let it be the upper point). One half of each of the vertical lines joining one of the poles to one of the points measures the R sinel atitude of the place ; and one half of the horizontal lines measures R sine colatitude. One half of the oblique line joining one upper point to one lower point is the radius or *trijyā*.

28. When seen from the equator, the radius of the orbit of a planet is the radius of its diurnal circle. The same is the R sine of the zenith distance of a planet when seen from the pole of the earth. The R sine of its declination is its R sine altitude.

29. The latitudinal parallax or *nati* is the north-south difference of the orbits of two planets from the middle of the sky. The parallax in longitude or *lambana* is their east-west difference.

30. The prime vertical, meridian, horizon, celestial equator and the six o'clock circle (all of the same size as the planet's orbit) are fixed for the stars and planets (at any given latitude).

31-32. There are 7 epicycles, 7 eccentrics, 7 *dṛṅmaṇḍalas* 7 *dṛkkṣepamaṇḍalas* and 7 *apamaṇḍalas* or *kakṣāmaṇḍalas*, one each for each planet beginning with the Sun. Then, there ara 5 *śīghra* epicycles and 5 *śīghra* eccentrics, one for each of the five planets. Then, there are 6 *vimaṇḍalas*, one for the Moon and 5 for the 5 planets. Thus, there are 51 great circles (which are not fixed).

Thus ends chapter Fifteen

on

CONSTRUCTION OF THE ARMILLARY SPHERE

CHAPTER XVI

RATIONALE OF RULES RELATING TO MEAN MOTIONS

[Rationale of the measure of the sidereal day etc.]

1. Since a civil day or *sāvanadina* is equal to a sidereal day or *nākṣatradina plus* the number of *asus* equal to the number of minutes in the Sun's daily motion, the number of sidereal days in a year is equivalent to the number of Sun's revolutions in the year, converted into days and increased by 1.

2. When a civil month is decreased by 28 *ghaṭikās* and 10

MATHEMATICAL NOTES

1. A civil day is the period between one sunrise and the next. So, in one year the number of civil days is the number of times the Sun rises. According to i. 9, this is equal to the difference between the number of sidereal revolutions and the number of revolutions of the Sun in one year, which is 1. Thus the required number of civil days is equal to the number of sidereal days *minus* 1.

Hence Lalla says that the length of a civil day is equal to that of a sidereal day *plus* the number of *asus* equal to the number of minutes in the Sun's daily motion. This would be so if the Sun were moving with uniform motion along the equator. But since the Sun is moving with non-uniform along the ecliptic, of which equal portions do not rise above the horizon in equal times, Lalla's formula gives the length of a mean civil day.

Bhāskara II notes this and gives the formula for finding the length of a true civil day (*SŚi*, II. iv. 5-6 and commentary). Śrīpati in *SŚe*, xv. 63 gives the same formula as Lalla.

palas, the conjunction of the Sun and Moon takes place. This duration is a lunar month or *cāndramāsa*.

3. A solar month or *sauramāsa* consists of 30 *dinas*, 26 *ghaṭikās* and 18 *vighaṭikās* (in civil units), and begins with Caitra.

4. As a solar month is always longer than a lunar month, there is an intercalary month or *adhimāsa* in 65 half lunar months or *pakṣas* of the Moon.

5. So, the number of solar months (during a period) increased by the number of intercalary months is equal to the number of lunar months (in the same period). And, so, the number of lunar days diminished by the number of civil days (during a period) is equal to the number of *avamadinas* or omitted *tithis* (in the same period).

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2. From i. 3-4 and 8, the length of a lunar month

$$=\frac{1,57,79,17,500}{5,77,53,336-43,20,000}$$

= 29 civil days, 31 *ghaṭikās*, 50 *palas*, nearly

= 1 civil month—28 *ghaṭikās*, 10 *palas*.

See *SŚe*, xv. 64 and *SŚi*, II. iv. 9.

3. 1 solar month $=\frac{1}{12}$ solar year

$$=\frac{1}{12}\times\frac{1,57,79,17,500}{43,20,000}\text{ civil days}$$

= 30 civil days, 26 *ghaṭikās*, 18 *palas*.

4-5. The difference between a solar month and a lunar month is 54 *ghaṭikās*, 27 *palas* or $54\frac{9}{20}$ *ghaṭikās*, and the length of a lunar month is 29 civil days, 31 *ghaṭikās*, 50 *palas* or $1771\frac{5}{6}$ *ghaṭikās*. one lunar month has two *pakṣas*. So, from the proportion :

$$54\tfrac{9}{20} : 2 :: 1771\tfrac{5}{6} : x,$$

one intercalary month occurs in 65 *pakṣas*. See *SŚe*, xv. 65-66 and *SŚi*, II. iv. 10-14.

[Rationale of Deśāntara etc.]

6. As the Sun rises first at a place which is to the east of the prime-meridian line, and then at a place which is to the west of it, the correction for terrestrial longitude is applied positively or negatively, as the case may be, to the mean place of a planet (as found at sunrise at Laṅkā).

7. Since the time of the mean Sunrise (is not the same as the time when the true Sun rises)—the latter time being either before or after—(the longitude of a planet should be corrected) by means of proportion using the Sun's *mandaphala* in minutes and the motion of a planet. The correction in minutes, (known as *bhujāntara* correction), is additive or subtractive, as the case may be.

8. (At the equator), the Sun rises and sets on the eastern and western six o'clock circle, respectively. The elevations and depressions therein are controlled by the north and south celestial poles.

9. In the northern hemisphere, the horizon is below the six o'clock circle and, in the southern hemisphere, it is above it. (So, when the Sun is in the northern hemisphere), at a place to the north of the equator, it rises earlier and sets later than it does on the equator.

10. But when the Sun is in the southern hemisphere, it rises later and sets earlier than it does on the equator. So, the result due to the ascensional difference is subtracted (from the mean longitude of a planet at sunrise on the equator in order to obtain its mean longitude at sunrise at the observer's station to the north of the equator, if the Sun is in the northern hemisphere ; for sunset), it is added. (When the Sun is in the southern hemisphere,) the contrary is the case.

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6. See the Mathematical Notes on i. 44-45.

7. This gives the *bhujāntara* correction, as explained in ii. 16. See Mathematical notes on this verse. See also *SŚe*, xvi. 23 and *SŚi*, II. v. 43.

8-12. The horizon as seen at the equator is called *unmaṇḍala*, So, at the equator, the Sun rises and sets on the *unmaṇḍala*. But at

11. Hence, when the Sun is in the northern hemisphere, the day is longer and the night is shorter ; the reverse is the case when it is in the southern hemisphere, (the place of observation being to the north of the equator). The cause can be explained clearly by means of a diagram.

[Rationale of ascensional difference etc.]

12. Along the latitude of Laṅkā which passes through the middle of the earth, (that is, along the equator), the horizon coincides with the six o'clock circle. So, there is no ascensional difference (at this place). The lengths of days and nights are always the same.

13-14. There, though the latitude is 0°, (the times of rising of the signs of the zodiac above the horizon are not the same). Since Aries and Taurus are more oblique because of their respective larger declinations, they take a shorter time to rise. The end of Gemini is more upright because of its comparatively shorter declination and because of being at the end of the quadrant. Therefore, it takes a longer time to rise. At places other than the equator, (the times of rising differ), because of difference in latitude.

15. (In the northern latitudes) the first and fourth quarters (of the ecliptic) pass the horizon in one fourth of the duration of the day and night (that is, one fourth of 60 *ghaṭikās*) *minus* the ascensional difference. The second and third quarters, however, (pass the horizon)

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other latitudes it rises and sets on their respective horizons. At the equator the days and nights are equal.

Verses 9-11 explain the reason for the application of the ascensional difference and the difference in lengths of days and nights (ii. 19-21). See *SŚe*, xvi. 25-28, and *SŚi*, II. vii. 1-9.

13-16. On account of the obliquity of the ecliptic even at the equator, its equal portions do not rise above the horizon at equal times.

in one fourth of the duration of the day and night *plus* the ascensional difference. This is so because of the latitude of the place and of the motion of the universe.

16. (Since in the northern latitudes) the six signs from Cancer are inclined towards the south and the six signs from Capricorn are inclined towards the north, the former take longer and the latter shorter times than those they take at the equator. This is so also because of the motion of the universe.

17. (The signs of the zodiac) that rise when they are inclined towards the north set when they are inclined towards the west. Again, those which rise when inclined towards the east or west set when they are inclined towards the north.

18. If the times of rising of the signs *at any latitude* are obtained by adding or subtracting their respective ascensional differences (to or from their times of rising at the equator), the times of their setting are obtained by subtracting or adding their respective differences.

19. If at any latitude, the ascensional difference of a sign is the same as its time of rising at the equator, expressed in *asus*, that sign is always visible at that place. At other places it rises and sets (as usual).

The protions more oblique take a shorter time to rise and set than those which more upright. At the other latitudes, the times very because of the difference in latitudes.

See *BSS*, xxi. 59-60 ; *SŚe*, xvi. 50-53, 58 and *SŚi*, II. vii. 16-23.

17-18. See *SŚe*, xvi. 54-55.

19. Śrīpati says the same in *SŚe*, xvi. 56. Bhāskara II refutes this statement in *SŚi*, II. vii. 31.

20. Scorpio, Sagittarius, Capricorn and Aquarius are not visible at a place where the latitude is 75°.

21. When planets in the southern hemisphere are not visible at a particular station, they do not set at that station when in the northern hemisphere and of the same declination.

22. When the Sun is at the end of Aries, it rises 108 *yojanas* to the north of the east point ; when at the end of Taurus, by 189 *yojanas* : and by 220 *yojanas* when at the end of Gemini.

[Rationale of visibility correction]

23. While an observer at the centre of the earth can see the Sun obscured by the Moon at the time of conjunction, an observer on the surface of the earth does not see it as such.

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20. In *SŚi*, II vii. 32, Bhāskara II says that according to Lalla, Sagittarius and Capricorn are never visible at a latitude of 66° 30′, and Scorpio and Aquarius are never visible at a latitude of 75°. This reading is slightly different from the present text. Bhāskara refutes these statements. Bhāskara's latitudes are given in *SŚi*, II. vii. 28-30.[1]

According to Śrīpati, Sagittarius and Capricorn are not visible where the latitude is 66½° ; and, these two as well as Scorpio and Aquarius are not visible at places where the latitude 75° (*SŚe*, xvi. 57).

21. See *SŚe*, xvi. 59. See also *SŚi*, II. vii. 25.

22. This would follow from the proportion :

360° : earth's circumference : : declination at the end of each sign : ?

See *SŚe*, xvi. 60. There the *yojanas* are 163, 286 and 333, since, the earth's circumference is taken differently from that of Lalla.

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* The latitudes given by Bhāskara are almost the same as those given earlier by Varāhamihira. It is surprising that Lalla and Śrīpati were unaware of the correct latitudes given by Varāhamihira.S

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24-25. The observer on the surface of the earth sees the disc of the Sun obscured by the Moon even before (the calculated time for conjunction), as he is elevated above the centre. (This is so, if the Sun is) in the eastern hemisphere.

But if the Sun is in the western hemisphere, he sees it after the calculated time, when the Sun has set, that is, has disappeared below the horizon.

So, the parallax in longitude due to the radius of the earth is subtracted (from the calculated time of conjunction) if the eclipse takes place in the eastern hemisphere ; but added if it takes place in the western hemisphere.

26. The line joining the observer at the centre of the earth and the zenith coincides with the line joining the observer on the surface of the earth to the zenith. Thus, there is no parallax at midday (when the Sun is at the zenith).

27. Whatever reasoning has been given (by the astronomers) for the parallax in longitude in minutes due to the eastern and western horizons, similar reasoning is to be understood for the parallax in latitude in minutes due to the northern and southern horizons.

[Rationale of eclipses]

28. Or, the parallax in latitude is (calculated) by means of the degrees in the zenith distance of the meridian ecliptic point. It is the north-south distance of the orbits of the Sun and the Moon at the time of an eclipse.

29. In a solar eclipse, since the Moon comes from the west like a cloud and obscures the Sun, contact takes place on the west and separation on the east.

30. (In a solar eclipse) the obscuring body, (that is, the Moon) being smaller in size, the Sun, when half eclipsed, has pointed horns.

23-49. Whatever has been stated here has already been made clear in Sn. I : Mathematics. In this connection see *SŚe*, xvii, 11-15,

The duration of the eclipse is short and the obscured portion appears different from different places.

31. (In a lunar eclipse) the Moon enters the circle of the earth's shadow cast on its own orbit as shown by calculations. Since the orbit (of the obscuring and the obscured bodies) is the same, there is neither parallax in longitude nor in latitude.

32. Since, in a lunar eclipse, the Moon moving eastward enters the shadow which is moving westward, contact takes place on the east and separation on the west.

33. As the disc of the earth's shadow (which is the obscuring body in a lunar eclipse), is big, the horns of the Moon, when half-eclipsed, are blunt. The duration of the eclipse is long and the obscured portion does not appear different from different places.

34. In a solar and lunar eclipse, the parts of the disc obscured are different ; there is a difference in the duration of each ; and the directions of the contact and separation vary in each case. The cause (for the eclipse) is the Moon (in a solar eclipse) and the earth's shadow (in a lunar eclipse). It is thus established that Rāhu is not the cause of the eclipse.

35. The Moon, when eclipsed, is thrown (to the north or south) of its obscuring body, (that is, the earth's shadow), according as its latitude is to the south or north. (So, in the projection of a lunar eclipse), the latitudes (must be marked) in a direction contrary to their own.

But in a solar eclipse, (the Moon is thrown to the north or south of the obscured body, the Sun, according as its latitude is to the north or south). (Hence, in the projection of a solar eclipse), the latitudes of the Moon should be marked in their own direction.

36. When the east point on the prime vertical moves to the

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xviii, and *SŚi*, II. viii, ix and x. The similarity between Lalla and Śrīpati is remarkable.

south (with reference to the east point on the ecliptic), the other directions (points) move to the right from the left ; when the point moves to the north, the other directions move to the left from the right. Therefore the *valana* (deflection) should be marked in a direction contrary to its own.

[Rationale for the brightness of the Moon]

37. Oh friend (*aṅga*) ! just as a pot placed in the Sun appears bright on the side facing the Sun and dark on the side away from it on account of its own shadow, the Moon in contact with the rays of the Sun appears bright on the side facing the Sun, white as a woman angry with her lover, and dark on the side away from it.

38. At the end of a lunar month (*amāvāsyā*) the (lower) half of the disc of the Moon visible to the people (of this earth) is completely dark, but at the end of the first fortnight (*pūrṇimā*), it is completely white.

Since the Moon approaches the Sun in the dark half of the lunar month, its dark portion gradually increases ; in the light half of the month, however, the Moon recedes from the Sun and so its illuminated portion gradually increases.

39. Just as the Sun's rays reflected by a mirror dispel the darkness in a room, the Sun's rays reflected by the Moon dispel the blinding darkness of the night caused on the earth.

40. All the spheres of planets together with the stars including the sphere of earth are like the Moon, bright on the side facing the Sun and dark on the side away from it.

41. That half of the disc of every planet or star having its orbit above that of the Sun, which is visible to the people (of the earth), is always seen illuminated by the rays of the Sun. Hence it is never dark.

As regards eclipses, see *BSS*, xxi. 36-48 (*Grahaṇavāsanā*) and

42. Venus and Mercury though (moving in orbits) beneath (that of) the Sun, do not appear dark like the Moon ; this is so because they are nearer the Sun and thus their whole discs are illuminated.

[Rationale of the visibility of the planetary discs]

43. The interval between the *lagna* and the rising or the setting Moon is the *bhuja* and that from its end directed towards the Moon is the perpendicular or *koti.*

44. Man (on the earth) sees the Sun having warm rays, difficult to look at and small in size there (in the zenith of the sky) where the whole disc is covered on all sides with innumerable rays.

45-46a. When the Sun is in the middle of the sky, it appears small (covered with rays) just like the pericarp of a lotus covered with filaments. Again, when it is in the middle of the sky, its rays pervade through the entire space. Thus man sees the Sun, the Lord of the day, while surrounded by its rays.

46b-47a. But when it (the Sun) is on the horizon, it is at a distance and its rays are obstructed by the earth. But it can be seen without discomfort, and looks big, red and less hot.

47b. Since (the Moon and the five planets) move in their respective orbits, (which deviate from that of the Sun), their declinations have to be corrected by their latitudes.

[Rationale for the correction to the visibility correction]

48. The wise men have laid down the rule that to a planet both on the eastern and western horizons, visibility correction due to deviation of the ecliptic or *āyanadṛkkarmāsu* and visibility correction due to the terristrial latitude or *ākṣadṛkkarmāsu* should be applied. The reason is somewhat similar to that for the application of ascensional difference.

64-65. Here, Brahmagupta's scientific attitude predominates. He, however, tries to reconcile it with the traditional view that Rāhu is the

49. When the latitudes (of two planets) are equal in magnitude and direction, the planets have the same day-circle. But when one planet has a latitude smaller than that of the other planet, it appears in a direction opposite to that of the latitude (with regard to the centre of the other planet) even if the latitudes are of the same denomination.

50. Thus have I given in brief the rationale (for the various rules enunciated in the section on Mathematics). The learned should make these more detailed after proper consideration. A scientific treatise instructs briefly. The learned enlarge the rules therein, just as one drop of oil spreads (extensively) on water.

Thus ends chapter Sixteen

on

RATIONALE OF RULES RELATING TO MEAN MOTIONS

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cause of eclipse. But he certainly does not succumb to this belief as has been presumed by some scholars. For visibility correction see *BSS*, xxi. 66.

CHAPTER XVII

SPHERE OF EARTH

[Nature of the Earth sphere]

1. This sphere of earth, made of ether, air, fire, water and clay and thus having all the properties of the five elements, surrounded by the orbits (of the Moon, etc), and extending up to the sphere of stars, remains in (the centre of) space.

2. Just as a ball of iron, when placed amidst pieces of magnet, remains suspended in space, in the same manner this globe of earth remains in space unsupported, while itself remaining the abode of all.

3. Yamakoṭi is to the east of Laṅkā (which is in the middle of the earth), and Romaka is to the west. Siddhapura is beneath Laṅkā (just opposite) ; the Meru (mountain) is to the north and the abode of the demons is to the south.

4. These (four cities) are on islands. Meru is on the land and the abode of the demons (in the south) is surrounded by water. All these six places are believed to be situated transversely at a distance of one fourth of the earth's circumference, (that is 90°), each from the next one.

5. Those who are at a distance of half the earth's circumference from each other are antipodes, just as the man (standing on the bank of a river) and his reflection in the water. The sky is above

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MATHEMATICAL NOTES

1-9. See *SŚe*, xv., 7, 21, 27, 28, 30 ; and *SŚi*, II. iii. 2-5, 17-20. The seventh verse refers to the attraction exerted by the earth. Bhāskara II also mentions it in *SŚi*, II. iii. 6.

all. This (globe of earth) is beneath it. The inhabitants are on the earth.

6. This globe of the earth is covered on all sides by trees, mountains, cities, goods, demons and men, just as a *kadamba* flower is beset with anthers and bees.

[Gravitational attraction of the Earth]

7. Just as a house-lizard, resting underneath the cross beams of the ceiling of a dwelling, runs forward without hesitation, so do people on the bottom side of the sphere of the earth.

8. In our daily life (we see) that the flame of fire goes towards the sky and a heavy weight falls towards the earth. In the same manner, everything that has a surface to reach, makes for it. The earth, however, has no such surface. Where can it fall ?

[Earth's power to support all]

9. Coolness is the natural characteristic of the Moon ; heat, of the Sun and of the fire, hardness, of stone, movement, of air, and fluidity, of water. Similarly, the characteristic of the spheres of the earth, stars, etc. is to remain suspended in space.

10. Why should the earth be supported by others, when it itself can bear the burden of all creatures. The wise say that it remains fixed, (though not supported), because of the power of Brahmā's boon.

[Measure of Earth's surface]

11. It is said that the area of the external surface of the earth, which is like a net enclosing a ball, is 2,85,63,38,557 square yojanas.

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11. Bhāskara II refers to this area of the surface of the earth, *viz.*, 2,85,63,38,557 square *yojanas*, and rightly says that this is wrong. Even one-hundredth of this is wrong. He points out that according to Lalla, as given in his *Pāṭigaṇita*, the area of the surface of a sphere is the

[The four cities round the Earth]

12. When the Sun rises at Laṅkā, the abode of the Rākṣasas, it sets simultaneously at Siddhapura. When it is midday at Yamakoṭi, it is midnight at Romaka.

Thus ends chapter Seventeen

on

SPHERE OF EARTH

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area of a circle multiplied by its circumference or $\pi r^2 \times 2\pi r$, which is surely incorrect. Bhāskara adds that the form should be $4\pi r^2$. See *SŚi*, II. iii. 53-57 and commentary.

12. This was held by all astronomers. See *ĀBh*, *Kālakriyāpāda*, 13 ; *SŚe*, xv. 31 ; and *SŚi*, II iii. 44.

CHAPTER XVIII

MOTION OF THE CELESTIAL SPHERE

[The Seven winds]

1. The seven winds (surrounding the earth) are *Āvaha*, *Pravaha*, *Udvaha*, *Saṁvaha*, *Suvaha*, *Parivaha* and *Parāvaha*. *Āvaha* is well known as *Ku-marut* (atmosphere) to others.

2. The circumference of the orbit of the *Āvaha* surrounding the earth is 3375 *yojanas*. The wise maintain that its diameter is 1074 *yojanas*.

[Movement of the Earth]

3. The celestial sphere at the earth's equator, is constantly carried towards the west by the *Pravaha* wind. To the gods (at the north pole) it appears to move (from left) to the right and to the demons (at the south pole from right) to the left.

[Situation of the Earth]

4. At the equator men always see the north and south celestial poles coinciding with the north and south points respectively of the horizon. The gods see the north celestial pole on their zenith and the demons see the south celestial pole on their zenith.

MATHEMATICAL NOTES

1-3. See *SŚe*, xv. 50-53 ; and *ŚSi*, II. iv. 1-3.

4-8. These verses describe the parallel sphere, the oblique sphere and the right sphere.

The abode of the gods is in the north pole and that of the demons in the south pole.

5. To the gods and the demons the equinoctial appears to coincide with their horizon. The gods never see the southern half of the ecliptic and the demons the northern half.

6-7. Whosoever proceeds from the abode of the gods or of the demons towards the equator, he would observe the celestial sphere gradually rising and the celestial pole more and more depressed from his zenith, the depression of the pole from the zenith being the same as the elevation of the celestial sphere.

The number of degrees in the colatitude of a place, multiplied by $9\frac{1}{6}$, gives the corresponding length in *yojanas.*

8. The elevation of the pole above the horizon is the same as the depression of the celestial sphere from the zenith. This latitude in degrees (elevation of the pole) multiplied as before by $9\frac{1}{6}$, gives the latitude of the place in *yojanas,* measured from the equator.

9. The zenith distance of the Sun multiplied by the (number of *yojanas*) in its orbit and divided by 360, gives the zenith distance (in *yojanas*).

(When the zenith distance is) $\frac{1}{4}$ of the orbit minus 525, the Sun is on the horizon.

7b gives the formula for changing the degrees in the colatitude of a place into *yojanas.* If follows from the proportion :

360 : 3300 : : given colatitude : ?

So, $1° = 9\frac{1}{6}$ *yojanas.*

See *BSS,* xxi. 3-5, 10 ; *SŚe,* xv. 54-57 ; and *SŚi,* II. iii. 48-51 and vii. 8-9.

9. This gives the formula for changing the degrees in the Sun's zenith distance into *yojanas.* The proportion is :

360° : Sun's orbit (*yojanas*) : : zenith distance (degrees) : zenith distance (*yojanas*).

10. When the Sun is at the first points of Aries and Libra, the gods and the demons see it simultaneously on their horizons. Then after $1\frac{3}{4}$ days[1] they gradually see it leaving the horizon.

[Motion of the Sun]

11. The Sun moves from the first point of Aries to the last point of Gemini in three months ; and again from the first point of Libra to the last point of Sagittarius in three months.

The greatest altitude of the Sun is 24° from the horizon during the day of the gods or demons.

12. The zenith is 90° from the horizon. This is equivalent to 15 *ghaṭikās* along the equator. So, when the Sun has its greatest declination of 24° and reaches its greatest altitude, it is seen by gods or demons for 2 *kṣaṇas* (or 4 *ghaṭikās*) of their day.

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10. When the Sun is at the beginning of Aries, the day of the dwellers at the north pole begins ; that is, on that day they see the Sun rising ; and the night of the dwellers at the south pole begins ; that is, on that day they see the Sun setting.

Similarly, when the Sun is at the beginning of Libra the night of the dwellers at the north pole begins ; that is, on that day they see the Sun setting ; and the day of the dwellers at the south pole begins ; that is they see the Sun rising.

According to Lalla, the Sun leaves the horizon of the observer at the pole after $1\frac{3}{4}$ days.

See *BSS*, xxi. 6-7 ; *SŚe*, xv. 59 ; and *SŚi*, II. vii.

11. Since in a parallel sphere, R sine of the declination of the Sun is its R sine altitude, R sine greatest declination is the R sine greatest altitude.

12. It follows from the proportion :

$$90° : 15 \textit{ ghaṭikās} :: 24° : 4 \textit{ ghaṭikās}.$$

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1. त्रिभागयुक्तेऽह्नि, in my opinion, means "in $1\frac{1}{3}$ days".—*KSS*

13. After that, on account of its motion along the ecliptic, the Sun, after three months, again reaches the horizon gradually. (If the gods saw it) rising (when it was on the horizon), (the demons will now see it) rising, and vice versa.

14. The gods see the Sun during the first half of the solar year ; their enemies, the demons, see it during the second half. The manes, who live (in the upper half) of the Moon, see the Sun for one half of a lunar month and the men during their own day.

[Motion of the Moon]

15. In this world the bright half of the lunar month is that during which the illuminated portion of the Moon increases and the dark half is that during which the dark portion increases. So, to the manes, the morning twilight is the eighth day of the Moon in the dark half of the lunar month and the evening twilight is the eighth day of the Moon in the light half of the lunar month.

16. Since on the day of the New Moon the Sun's disc is just above that of the Moon, to the manes the Sun appears to be on the zenith. This then is their midday as it is in the case of the men (when the Sun is on their zenith).

17. As the Moon recedes from its position below the Sun, to the manes, the Sun appears to decline. Then, on the middle of the eighth day when the Moon is at a distance of 90° from the Sun, it appears to be on their horizon or very near it.

18. On the Full Moon day the Sun is hidden by the Moon's disc which is at a distance of 6 signs from it. It is then midnight for

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13-18. These define the days of the gods, demons, manes and men. The manes or the dead ancestors are supposed to live on the upper part of the Moon. The duration of their day and night together is lunar 1 month. On the New Moon day, they see the Sun on their zenith. Then it is their midday. From that day till the eight day of the Moon, they see the Sun gradually declining, and, then, on the eighth day, it sets. Then their night starts.

the manes. Again, on the eighth day, when the Moon approaches the Sun, the latter is visible (to the manes). Then, gradually it rises above their horizon.

[The Creator's day and the creation and dissolution of the Earth]

19. Brahmā the creator, with four faces, stationed at a greater distance from the earth, sees the Sun at all times. So, His day is different from that of the gods. When He closes His eyes, it is night for Him. (And when He opens them, it is His day).

20. During the day of Brahmā, the earth increases 1 *yojana* on all its sides. Again, during the passage of His night, this artificial growth of the earth is destroyed.

21. With the destruction of Brahmā, all, including the earth, is destroyed; there is no continuity. At the end of one Manu, the three (old) worlds are destroyed and new (worlds) are created. The same are again destroyed at the end of the day of Brahmā.

22. The whole creation, including the world, etc., gradually becomes non-existent at that pure state (when the grand deluge occurs). It, however, gradually comes into existence again, born of Him, if He so wills.

23. With the onset of destruction is destroyed this air which pervades the sky ; as the body is a corpse, without life (breath), so is the ether without sound.

The Full Moon day is their midnight. On the eighth day after that they see the Sun again rising. Then their day begins.

See *BSS*, xxi. 8 ; *SŚe*, xv. 58, 61 ; and *SŚi*, II. vii. 10-14.

19-26. Here Lalla gives the story of the creation and destruction of the universe, as given in the Purāṇas. Brahmā is the Creator. His day and night each consists of 1000 *mahāyugas* or a *kalpa*. There are 14 *Manus* or Patriarchs in a *kalpa*. There are two kinds of destruction. The first takes place at the end of one day of Brahmā, that is, after 1000 *mahāyugas*. Then the three worlds *Bhūḥ*, *Bhuvaḥ*,

24. Men can exhaust their good and bad actions only by experiencing the consequences thereof. With the destruction of Brahmā, these actions also gradually come to an end.

25. In course of time everything is destroyed. Brahmā, however, creates again, if He so desires. So, the wise burn away completely their good and bad deeds with the fire of knowledge, (with a view to prevent their rebirth).

[Worldly advice]

26. The merit one obtains (by knowing *Brahman*) through meditation, *Brahman*, which is one, imperishable, beyond all qualities, and which is really in one's own body, that merit one can never achieve either through penance or by performing sacrifice or even through pilgrimage.

27. The Lord, Who is Himself the three qualities, *viz.*, (*Sattva*, *Rajas* and *Tamas*), creates everything, protects it and then destroys it.

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and *Svaḥ* are destroyed and everything enters into the body of Brahmā. Then, after the night, that is, after 1000 *Mahāyugas*, Brahmā again creates. The second deluge takes place at the end of the life of Brahmā, which consists of 100 of His years. In that grand deluge even Brahmā is extinct and everything is dissolved into nature. After this destruction, creation takes place again at His desire. At the end of each Manu there is also a deluge in which the three worlds are destroyed.

If wise men do not want to be reborn, they must acquire the knowledge of the Self and concentrate their thought on the supreme Lord.

See *BSS*, xxi. 1-10 ; *SŚe*, xv. 29, 62 ; and *SŚi*, II. iii. 62-65 and vii. 15.

27. The three *guṇas* or qualities are *Sattva* or goodness, *Raja* or activity and *Tamas* or darkness. The God creates with *Rajas*, protects the creation with *Sattva* and destroys it with *Tamas*.

Man may make every effort but he cannot create, whether by pleasure or by suffering.

28. The Vedas, (sages, such as Vyāsa) the son of Satyavatī and others, knew (the Lord) in various forms; they called Him Brahmā, radiant with a thousand rays, seated on the lotus ; they called Him Viṣṇu ; they called Him Śiva, the king of all beings, who has conquered Death.

29. Men should know that this life is not real and that it is the cause of all trouble. They should, therefore, learn from their teacher the truth about God Who alone can give eternal happiness and Who should be worshipped with respectful minds.

30. Reciting the *Praṇava* (*mantra, Om*) with resonance, as prescribed, and meditating on the Inner Self's brilliance, as bright as the rays of thousand Suns risen simultaneously, one should abandon this world together with one's riches.

31. He, who recites with devotion everyday the Sun's *mantra* (prayer) consisting of three syllables, and also (recites) the eternally existent though uncognised syllable which is the seed of *Śakti* or Power, and also recites the name of Durgā, fire-mouthed and united with Rudra, attains happiness and prosperity.

Thus ends chapter Eighteen

on

MOTION OF THE CELESTIAL SPHERE

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30. The mystic syllable *Praṇava* or *Om* is a symbol of the Supreme Being. The Upaniṣads advocate its worship. The *Yogasūtra* i. 27 of Patañjali says that a repeated muttering of this syllable helps one to concentrate one's mind.

31. The trisyllabic *mantra* is the *Gāyatrī* or *Sāvitrī*. The letter '*ā*' is the symbole of God Śiva and Śakti is the power of Śiva.

CHAPTER XIX

DESCRIPTION OF THE EARTH

[The Seven Oceans]

1. The Salt Sea is to the south of those who live on the equator. The land to the north of the (Salt Sea) is the *Jambū-dvīpa.* The Sea of milk is to the south of the Salt Sea. They say that the *Sākadvīpa* is between (the Salt Sea and the Sea of Milk).

2. The Sea of the Curd is to south of the Sea of Milk. The land between them is *Kauśa-dvīpa.* The Sea of Butter is to the south of the Sea of Curd. The land between them is called *Krauñca.* The Sea of Sugarcane Juice lies to the south of the Sea of Butter. The land between them is called *Śālmalī.*

3. Next to the Sea of Sugarcane Juice is the *Gomeda-dvīpa.* After that comes the Sea of Wine. To its south is the *Puṣkara-dvīpa.* To the south of the *Puṣkara-dvīpa* is the Sea of Sweet Water. In this sea is the *Vāḍava* fire, where dwell the demons. The sinners are punished here.

4. At the end of every half lunar month, when the Moon rises or sets, the waves of the sea go towards the shore in the north. At the middle of every half lunar month, however, when the Moon is in the middle of the sky, the waves go towards the *Vāḍava* fire.

5. The demons consider that the fire, water, land and the group of gods are respectively to their south, whereas all the gods consider

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MATHEMATICAL NOTES

1-30. These give the geographical description of the world as given in the Purāṇas. The fourth verse speaks of the tides. In the

that the land, water, fire, and the demone are, respectively, to their south.

[The inter-terranean regions]

6. Inside the earth there are seven *pātālas* or inter-terranean regions. They are the abodes of the *dānavas* and the serpents. Here there are divine plants, various precious stones, mercury, mines and other divine things.

7. There are charming and delightful *Vidyādhara* women sought by Cupid, with eyes as those of a deer, with dark and curly hair, with voice as sweet as that of a cuckoo, a little bent because of their heavy and plump breasts, with graceful bodies, beautiful hips and thick thighs.

[The mountains]

8. To the north of *Laṅkā* and the Salt Sea in the *Jambū-dvīpa* is the *Himālaya* mountain stretching up to the sea, like the string of a bow. The land between this mountain and the sea is called *Bhārata.*

9. To the north of the *Himālayas* lies the *Hemakūṭa* mountain. The land between these two is called *Kiṃmanuṣya.* To the north of the *Hemakūṭa* is the *Niṣadha* mountain. The land between these two is called *Hari.*

10. The *Śṛṅgavat* mountain is to the north of the *Siddhapattana* (opposite *Laṅkā*) just as the *Himālaya* is (to the north of *Laṅkā*). The land of the shape of the arc of a circle lying between the Salt Sea and the *Śṛṅgavat* mountain is called *Kuru.*

11. To the north of *Śṛṅgavat* is the *Śveta* mountain. They call the land between these two the *Hiraṇmaya.* To the north of the *Śveta* mountain is the *Nīla* mountain. They call the land between them *Ramya.*

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twentieth verse Lalla names only 7 deities. But according to the Purāṇas, each of the 8 directions is ruled by a deity, Indra rules the

12. It is maintained that the land between the *Nīla* and the *Niṣadha* mountains is divided into three parts. The *Gandha* mountain is to the north of *Romaka* and the *Mālyavat* is to the north of *Yamakoṭi*.

13. The land between the *Gandha* mountain and the sea is called *Ketumāla* and that between the *Mālyavat* and the sea is called *Bhadrāśva*.

14. The land between the *Mālyavat* and the *Sugandha* mountains is called *Ilāvṛta*. Here is the Meru, the mountain of gold and precious stones, on which is the abode of the gods.

15. The *Mandara* hill on the side of the *Mālyavat*, the *Gandhamādana* hill on the side of the *Niṣadha* the *Supārśva* hill on the side of the *Nīla* and the *Vipula* hill on the other side surround the mountain *Meru*.

16. On these hills are, respectively, the four lakes called *Aruṇa*, *Mānasa*, *Mahāhrada* and *Sitoda*. These lakes are beautiful with lotuses, birds and gold.

17. The water in these lakes appears to display a part of the beautiful bow of Indra (rainbow)—it is red with the saffron powder loosened from the high breasts and thighs of the women with big hips, who keep immersed in the water up to their nose ; it is yellow with the hue of the golden ornaments (on their bodies) ; and at some places in the eastern portion it is blue because of the excellent *indranīla* stones (sapphire) dropped there.

18. The wise say that on these hills (*Mandara*, etc.) grow *Kadamba*, *Jambū*, *Vaṭa* and *Pippala* trees. On these are, respectively, very beautiful forests of *Caitrartha*, *Sunandana*, *Dhṛti*, and *Bhrāja*.

19. The mountain *Sumeru* (or *Meru*) has three peaks on which

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east, Agni the south-east, Yama the south, Nairṛta the south-west, Varuṇa the west, Vāyu the north-west, Kubera the north and Īśa the north-east.

are three abodes of gods, made of golden tissues. The dwellings are expansive and very high, extending to several *yojanas.* (They are) the abodes of Śiva, Brahmā and Viṣṇu.

20. Beneath these (abodes) are the cities of Indra, Agni, Yama, Nairṛta, Varuṇa, Vāyu and Īśa. Indra rules over these cities of gods and the heaven.

[The place of Indra etc.]

21. His (Indra's) weapon is the thunderbolt, which, in battle, can rend asunder the surface of the temples of even the best elephants of the enemy, difficult to conquer. His swift horse is called Uccaiḥśravas and his elephant Airāvata. His minister is (Bṛhaspati), the Lord of learning.

22. The enemies of Indra may well possess intoxicated elephants, horses yoked to innumerable chariots, weapons to protect their bodies, strong fortresses and dwellings accessible with difficulty. (In spite of everything), if Indra desires, all can be annihilated in no time, just as snow can be melted away (by the heat) of the day.

23. The King Indra with thousand eyes and desirous of conquering (his enemies), has (only) one beloved (queen) Śacī whose face is (as beautiful as) the Moon, whose gait is (fascinating) as that of a swan, whose eyes are (as dark) as the dark lotus, whose ointment is sandal powder and whose breasts are heavy and white. She carries fragrance (with her) and adds beauty to the heaven as does the autumn season.

24. There are (nine) parts of (the country of) *Bhārata*, *viz.*, *Aindra*, *Kaseru*, *Tāmraparṇa*, *Gabhastimat*, *Kanyā*, *Saumya*, *Nāga*, *Vāruṇa* and *Gāndharva*.

25. There are seven principal mountains (in *Bhārata*), *viz.*,

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For similar descriptions see *SŚe*, xv. 26-27, 32-49 ; and *SŚi*, II iii. 17-43. Bhāskara II says that whatever is given here is from the Purāṇas.

Mahendra, Śukti, Sahya, Vindhya, Pāriyāta, Malaya and *Ṛkṣa. Kumārikā* (*Kanyā*) has all the four castes (as inhabitants); the remaining eight parts have only the outcastes.

[Regions above the Earth]

26. In the land between the mountains of the *Gandha, Mālyavat, Himālaya* and *Sṛṅgavat*, there live the *Apsarās, Vidyādharas, Munis, Kinnaras, Siddhas, Yakṣas* and *Gandharvas*.

27. Beyond the city of the *Gandharvas* in the atmosphere surrounding the earth, there are clouds, rainbow, lightning together with whirlwind, meteors and halo (surrounding the Sun and Moon).

28. The part of the earth to the south of the equator and the *Pātāla* or inter-terranean regions are known as *Bhūḥ*. This earth known to men is called *Bhuvaḥ*. *Sumeru*, the land of the gods, is called *Svaḥ*.

29. Above *Svaḥ* is *Mahaḥ*; then *Janaḥ* ; then *Tapaḥ* ; and then *Satyalokaḥ*. These regions are abodes of those with successively increasing merits.

30. All this that has been described above in order has as its orbit the space which is its boundray. It is inside the hollow of the universe. It is called *Lokāloka*.[1]

31. This sphere of universe is outside everything, being surrounded by three circles each of the same size. (Of these three circles), one stretches from east to west, the other from north to south and the third from side to side.

32. To the north of the inhabitants of the *Jambūdvīpa* is the

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32. Notes on Sm. II. Spherics, ch. xx. 14 may be referred to.

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1. Alternate translation : "All that has been described (above) in succession lies in the interior of the Brahmāṇḍa-kuṇḍa ('the cavity of the egg of Brahmā'). The periphery of that is the orbit of the sky; it is also called *Lokāloka*." —*KSS*

Sumeru mountain; to the south is the abode of the demons. Meru has no direction, since the Sun rises on all its sides.

[The Pole]

33. The feet of a man point downwards and his head upwards. These directions will, therefore, be up and down. The direction, in which the Sun rises is the east and that in which it sets is the west.

34. The celestial pole points to the north ; and the direction opposite to it is known as the south. All these directions meet at the point where the observer is.

[Planetary Motion]

35. Whatever is obtained by subtracting the earth's radius from the radius of the orbit of a planet, is the portion visible (to a man). And whatever is obtained by the semi-diameter of a planet's orbit increased by the semi-diameter of the earth, remains hidden from the earth.

36. The (mean) orbit of each planet multiplied by its mean motion and divided by its true motion, gives the corrected measure of the orbit.

Or, the (mean) orbit multiplied by the *manda* hypotenuse in minutes of the planet, and divided by the radius gives the corrected result.

37. A planet moves along its mean circular orbit at the rate of its mean motion. When it is above (its mean orbit), it moves at slower rate, and when below, at a higher rate.

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36. This follows from the proportion :

True orbit : mean orbit : : mean motion : true motion

OR true orbit : mean orbit :: *manda* hypotenuse : radius.

38. A planet naturally moves to the east, but, when retrograde, to the west. It is drawn towards north or south by its declination. Thus the motion of a planet is of six kinds.

39. The *ghaṭikās*, etc. obtained by dividing by the earth's true orbit (circumference) the product of the longitudinal distance expressed in *yojanas* between two places (*deśāntarayojanas*) and 60, give (the time due to the difference in longitudes between the two places). If a planet has already risen to set (in places through which the prime-meridian passes), it has still to rise or set in places (to the west of the prime-meridian, at a distance of the *deśāntarayojanas* after the time so determined).[1]

[City of Avanti and the prime meridian]

40. A planet reaches simultaneously the zenith at all the regions on the prime-meridian.

37-38. These give the six kinds of motion of a planet as observed by the ancient Indian astronomers. They are ; natural motion to the east ; motion to the west due to the motion of the universe ; motion to the north or south due to declination ; and also motion above or below the mean orbit, having earth as its centre, due to its eccentricity. See *SŚe*, xv. 71-72.

39. Notes on i. 43-45 may be referred to. See also *BSS*, i. 39.

40. Bhāskara II also says that Avantī is at a distance of 1/16 of the earth's circumference from the meridian of Laṅkā. His words

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1. Verse 39 should probably be translated as follows : Whatever *nāḍīs* are obtained from the division of the product of the Earth's semi-diameter and 60, by the true circumference of a planet's orbit, give the horizontal parallax of the planet (in terms of *nāḍīs*). If the planet has theoretically risen so many *nāḍīs* before, it has not risen actually; and if, theoretically, it is to set so many *nāḍīs* afterwards, it actually appears to set even now." —*KSS*.

(The city of) *Avantī* (or Ujjain) is at a distance of 1/16th part of the earth's circumference (from *Laṅkā*) to its north.

[Lunar Motion]

41. When the Moon has a north declination of 612′, it rises above the horizon of the gods (who live at the north pole). With the same declination, again, it sets, because of the smallness of its orbit.[1]

42. The Moon is seen by the gods during the lunar month for 2 *ghaṭikās,* 25 *vināḍīs,* when it has a north latitude and is above the Sun. The contrary is the case, when it is below.

[Sphere of Universe]

43. The whole sphere of universe round the earth is created by the Creator. According to interferences and non-interferences from them (the planets), there is good or evil (fortune) for all.

[Specialities for the Gods]

44. The rising of the heavenly bodies cannot always be seen (there); the solar eclipse is not always visible nor is (the star) Agastya. There are no light and dark halves of the lunar month and the Sun does not have its maximum altitude for 2 *kṣaṇas* (as seen at the north pole).

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are : *nirakṣadeśāt kṣitiṣoḍaśāṃṣe bhaved Avantī.* (*SŚi,* II. iii. 15 ab). According to *BSS,* xxi, 9 it is 1/15 of the earth's circumference.

43. This refers to the astrological belief that planets and stars have effect on the fate of man.

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1. There should probably be इभर्तु (=68) in place of इनर्तु (=612). For, Moon's horizontal parallax+Moon's semidiamiter =68′ approx.

[Sun, the Illuminator]

45. The Sun alone gives light to all who live in this hollow of the universe—to all the regions described above, whether in the earth, on the earth or above the earth ; to all, whether gods, demons, *Rākṣasas*, *Bhūtas*, *Piśācas*, *Kinnaras*, *Vidyādharas*, *Nāyakas* *Pitṛs*, *Siddhas*, sages or men; and to all the heavenly bodies.

Thus ends chapter Nineteen

on

DESCRIPTION OF THE EARTH

CHAPTER XX

FALSE NOTIONS

[Need for removing false notions]

1. Different situations have been ascribed (by different people) to the matters contained in the universe. When one hears the different statements made by (mutual) opponents, doubt, as it were, will assail one's mind.

2. For this reason, I shall first enumeratc the irrational views ; and then I shall put forward the truths together with proofs. That will make things clear and steady the mind.

[False notions]

3. Some say that the day of the demons (at the south pole) starts when the Sun goes (from the northern to) the southern solstice ; and, that of the gods (at the north pole) starts, when the Sun goes from there towards the northern solstice.

Again, others say that the solar eclipse and the lunar eclipse are caused by Rāhu.

1-7. In this chapter Lalla mentions some of the beliefs prevailing then with regard to astronomical phenomena and refutes them. Most of these beliefs occur in the Purāṇas.

The views are as follows :

1. The day of the observers at the north pole begins when the Sun starts *for* the summer solstitial point and that of the observers at the south pole begins, when it starts *for* the winter solstitial point.

2. The night comes when the mountain Meru covers the Sun.

3. Directions can be determined at Meru.

4. Some say that the Moon is above and the Sun is below. Again, some say that the night is caused by the darkness due to the mountain *Meru*. Again, others say that the disc of the Moon daily decreases because it is being drunk by the gods.

5. Some say that the mountain *Meru* has directions. Others think that there are two Suns and two Moons. Again, some say that the day of the manes commences with the dark half of the lunar month, and the night, with the bright half.

6. Some think that the earth is infinite ; others, that it is plane as a mirror. Again, others say that it extends to many *yojanas* and floats on water like a boat.

7. Some say that the earth is supported by a tortoise, a serpent a boar, an elephant or by mountain ranges. Again, others say that the earth goes up and down ; and some say that it goes round and round like the wheel of a chariot.

[Doubts on Solstice clarified]

8. If the Sun is always visible to the gods when near the end of Gemini, then, for what reason, pray, it is not visible when in Cancer ?

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4. The dark half of the lunar month is the day of the manes and the light half is their night.

5. Rāhu, a mythical demon or a snake is the cause of eclipses.

6. The Moon's orbit is above that of the Sun.

7. The illuminated portion of the Moon decreases because it is being sucked by the gods.

8. The earth is infinite.

9. The earth is plane like a mirror.

10. The earth is supported in various ways.

11. The earth moves.

12. There are two Suns and two Moons.

9. If the Sun is visible to the gods while ascending due to (the increase) in the declination, why should it not be visible to them while descending through the same declination.

[Doubts on night clarified]

10. How is it that the gods see the same Sun, even when the wheel of darkness (approaches the world), just as one sees a horse attached to a rope going round and round, sometimes near and sometimes at a distance.

11. If the night of the men in this world is caused by the *Meru* covering the Sun, tell me how the night of the gods, (who live on the *Meru*) and feed on nectar, is caused. (Therefore the statement) is irrational.

12-13. When the Sun goes to the southern part (of the ecliptic beginning with) Libra, it is always night for the dwellers of *Meru.* When it moves along the north (of the ecliptic beginning with) Aries, (it is continuous night) for the demons. When the Sun is hidden by a part of the earth, it is night for the men also.

Just as the shadow of a tree is longer when it is directly opposite to the Sun than at other times, so is the shadow of the part of the earth (directly) opposite to the Sun. (That shadow causes night

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8-9. These verses refute the first belief. According to this belief, the observers at the north pole can see the Sun travelling from Capricorn to Cancer and not from Aries to Libra. Lalla's argument is that if the observers at the north pole can see the Sun ascending from Aries to Gemini, they should also see it descending from Cancer to Virgo when the Sun travels exactly along the same diurnal circles. So the current belief cannot be correct. See, *SŚe,* xv. 59 cd-60 and *SŚi,* II. vii. 11-12.

10-13. These refute the second belief. Lalla maintains that the night is not caused by Meru but by the shadow of the earth. See *SŚi,* II. iii. 12.

in the part away from the Sun). (I have now explained how) the nights of the gods, demons and men come about.

[Doubts on Meru clarified]

14. How can *Meru*, the dwelling of the gods, have directions when the Sun rises all round it. If the point where the Sun rises first is called the east point, what will it be called, when the Sun sets there.

15. The Sun is simultaneously to the north of some and to the south of others. It is again in front of some and behind others. Thus, in reality, there are no directions (for the *Meru*).

[Visibility of the Moon to the manes]

16. The manes, living in the upper part of the Moon, see the Sun on their zenith on the day of the New Moon. That being so, how is it that they do not see it from the middle of the light half of the lunar month.

[Doubts on the eclipses cleared]

17. Men see a solar eclipse either at Sunrise or at sunset, when the Moon is a little to the north. But the gods cannot see it then.

14-15. These refute the third belief. Lalla says that no direction can be determined at Meru, because there the observer's horizon coincides with the celestial equator and hence there is no east point. See *SŚe*, xvi. 63-64.

16. This refutes the fourth belief. Lalla states that the whole of the light half of the lunar month cannot be the day of the manes, because they do not see the Sun after the eighth day. (See v. 15-18)

17-27. These refute the fifth belief. The Paurāṇika story goes that a demon in disguise was drinking the nectar churned by the gods from the milky ocean. The Sun and Moon pointed him out to Hari

18. It is known that when a demon, an enemy of the gods, was drinking the nectar, his head was chopped off by his enemy Hari. But the head did not die. Some say this is Rāhu. The Sun (and the Moon) are devoured by it.

19. (Some say that) the Rāhu is round like the discs of the Sun and Moon, and being dark cannot (always) be seen in the sky. It is seen only on the New and Full Moon days and that too due to the boon of Brahma.

20. Some say that the cause (of an eclipse) is a snake with head and tail. Others say it is only darkness. Again, some say that it is a moving demon, and others that there is a pair of them. And some say it goes wherever it wishes.

21. If the Rāhu has a body or is a disc, or a head or is a planet in the sky, and if it is always moving, why should it swallow (the Moon) only at a distance of 6 signs (from the Sun).

22. If you are of the opinion that an artful demon is always the cause of an eclipse by swallowing (the Sun or Moon), then how is it that an eclipse can be determined by means of calculation. Moreover, why is there not an eclipse on a day other than the day of New or Full Moon.

23. If a lunar eclipse takes place on the western horizon and is caused by Rāhu, then why does not the Sun's disc appear to be swallowed by the second Rāhu of the same speed.

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who severed his head from his body. But the head did not die. It remained immortal since it had drunk the nectar. Out of spite this head swallows the Sun and the Moon and thus causes an eclipse. Another belief is that a snake causes an eclipse.

Lalla refutes them both by pointing out that since there are differences with regard to the duration, obscured portion, etc., between a solar and a lunar eclipse, the obscuring body in both the cases cannot be the same. Moreover, why should Rāhu choose only the

24. If the opponents say that it is a snake which causes an eclipse by its head and its tail, then why does it not cover half of the circle between the head and the tail.

25. An eclipse cannot be caused by Rāhu, because the sides of the discs of the Sun and Moon, which are first to be eclipsed, are not the same ; nor are the portions eclipsed the same ; and nor even are the durations the same.

26. In a solar eclipse, people at different parts (of the earth) see different portions of the Sun eclipsed. Some do not see (the eclipse) at all. Knowing this, who can maintain that an eclipse is caused by Rāhu.

27. Because of the great authority of Brahmā, at the time of eclipse the Sun is near Rāhu. So, in the Vedas, Smṛtis and Saṃhitās it has come to be known that Rāhu is the cause of eclipses.

[Assertion that the Moon is below the Sun]

28. If the Moon is above the Sun, why does it not always appear illuminated like a star. (If the Moon were above the Sun) there

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New and Full Moon days for eclipsing the Sun and the Moon, respectively. Again, if Rāhu were the cause, calculation would be of no use in determining an eclipse.

However, since the scriptures consider Rāhu as the cause of an eclipse, Lalla, not willing to show any disrespect towards their statement, justifies it by saying that as in a solar eclipse, Rāhu (Moon's node) is near the Sun, the former is considered by the sacred books as the cause of an eclipse.

Śrīpati has also given the beliefs and has refuted them in *SŚe.*, xvii. See also *SŚi*, II. viii. 7-10.

28. This refutes the sixth belief. Lalla says that if the Moon were above the Sun, it would always be illuminated like a star.

would not be a solar eclipse ; there would not even be a lunar eclipse, as the Moon would then be (further) away from the extremity of the earth's shadow.

[Doubts about the Moon clarified]

29. If the Moon decreases because of some curse or because it is being sucked by the gods, how can the increase and decrease (in the illuminated portion) be determined by calculation: (And, moreover, if it is being sucked), it would appear completely dark when fully sucked.

[Doubts on the size and shape of the Earth clarified]

30. If the earth is infinite (in size) or if it extends up to innumerable *yojanas*, how can the celestial sphere go round it once a day.

Moreover, Avantī could not then be at a distance of 1/16 of the circumference (of the earth) from the meridian of Laṅkā.

31. The eclipse, the conjunction and rising (and setting) of planets, the cusps of the Moon and the length of the shadow (of the gnomon) at any time—the calculation of all these five (phenomena) depends upon the measurement of the earth. (And the calculated result) agrees with the observed result. So how can the earth be (infinitely) large.

32. By using the *grahaphala* (*grahagati*) and the above circumference of the earth in *yojanas*, it is found by means of simple

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Moreover, then it could neither cause a solar eclipse nor could it be obscured by the earth's shadow.

29. This refutes the seventh belief. If the Moon's daily decrease were due to its being sucked by the gods, mathematics would be of no use in computing its light and dark portions.

30-33. These refute the eighth belief. Lalla says that the earth could not be infinitely large, as then the sphere of the fixed stars could not go round it in one day. Moreover, the measurements of

proportion that the circumference of the circle of one revolution of a planet consists of 21600 minutes. Then how can the earth be of infinite size.

33. The earth may appear to be of infinite size or extending up to innumerable *yojanas* because of its being spherical. But its circumference and diameter are the same as given before and in no way different.

34. If the earth is plane as a mirror, why does not the water falling from the sky remain static, instead of, alas, flowing with a great speed in one direction.

35. Mathematicians say that one hundredth of the circumference of the earth appears to be plane. So, that portion of the earth *appears* to be plane to an observer.

36. If the earth is level, why cannot tall trees, like *tala* (date palm) alas, be seen by man, though at a very great distance from the observer.

37. To men the sky appears to meet (the earth) all round the horizon. So, the earth thus surrounded appears to be plane like a mirror.

[Doubts on the Earth's motion clarified]

38. If the earth goes downwards, (anything) thrown towards

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the earth, as given in i. 43 are correct, because the calculated results relating to the various phenomena, based on these measurements, tally with the observed results. See also *SŚe.*, xv. 24-25 and *SŚi.*, II. iii. 14-16.

34-37. These refute the ninth belief. Lalla maintains that the earth is spherical and not a plane. But as only a small portion of it is visible at a time, that may be the reason for its appearing as level as a mirror. If it were level, the tops of high trees could be seen even from a great distance. See *SŚe.*, xv. 8-10 and *SŚi.*, II, iii. 11.

38. 42-43. These refute the eleventh belief. The Bauddhas maintained that the earth was falling down in space. So Lalla says

the sky will not fall on the earth. If the earth is (continually) moving upwards, why does not the sphere of constellations come nearer.

[Doubts on the Earth's situation clarified]

39. Clay is destroyed by water. So it is not possible for the earth to remain in water. Again, (the statement) that the earth (floats on water) like a boat, cannot be correct, as the earth is itself made of clay.

40. Supposing that the earth is on water, the water like the sky is also unsupported. If the heavy sphere of earth can remain on water, (which is unsupported), why can it not remain in space ?

41. If the earth is supported by tortoise and other things, by whom are they supported in space ? If these can remain in space (unsupported), what prevents the earth from remaining thus (*i.e.* unsupported).

42. If the earth rotates, how could birds come back to their nests, Moreover, arrows shot towards the sky, would fall towards the west.

43. If the earth rotates to the east, the clouds would move to the west. If (it is said) that the earth moves slowly, then how can it go round (the universe) in one day ?

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that if it were so, how could a thing when thrown up come down again on the same piece of a ground. Again, if the earth were continuously moving up, the constellations would be nearer every moment. Neither could the earth be moving from east to west nor from west to east. If it did, the birds would not be able to find their nests. Lalla tries to refute Āryabhaṭa's theory that the earth rotates from west to east. See *SŚe.*, xv. 11-13, 15-19 and *SŚi.*, II. iii. 7-9.

39-41, 45-46. These refute the tenth belief that the earth is supported by an external ageney. Lalla says that the earth remains unsupported, suspended in space. If it were supported by something, the latter, in its turn, would have to be supported by something else and

[Doubt about double Sun and Moon clarified]

44. If there are two Suns and two Moons, which rise alternately, how can the circumpolar constellations complete their revolution in one day.

[Situation of the Earth in space]

45-46. Just as the branch of a tree struck by lightning, can be replaced by means of medicines; just as a (plucked) lotus (can live in water) by means of chemical applications (made of juices and roots plants); just as the astronomical instrument *Svyaṃvaha* is made of mercury, oil or water, though the construction appears inconceivable; in the same manner, (the earth) stays in space by the will of God. (Again, by His will) the universe moves.

[Creation of the Earth]

47. At first there was darkness. Then there came into existence an egg in the water in its natural form. Brahmā was the first to come out of the egg. His eyes were the Sun and the Moon. He created the universe.

The followers of Kaṇāda say that the *padārthas* are the cause of creation; the Sāṁkhya says that the *Puruṣa* and *Prakṛti* are the cause; some say that it is Nature ; while others say that it may be *Karma*, *Artha* or *Kāla*.

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so on. Then, there would be no end of supporters, and that is not possible. Many wonderful things happen in this world ; so there is nothing to be surprised at if the earth hangs in space. See *SŚe.*, xv. 7, 20-22 and *SŚi.*, II. iii. 4-6.

44. This refutes the twelfth belief held by the Jainas. See *SŚe.*, xv. 14 and *SŚi.*, II. iii. 8-10.

47. The first explanation for the cause of creation is found in the *Ṛgveda*. Kaṇāda, author of the *Vaiśeṣikasūtra*, is the propounder of the Vaiśeṣika system of philosophy or doctrine of atoms. *Dravya* or

48. When Brahmā is fully active, living creatures are born of Him. Lord Hari protects them with His quality of goodness. Hara, entering the state of darkness, destroys them. Others say that Brahmā alone takes all the three (steps) by His will.

49. I bow to Him, who has created this whole universe which has plains, mountains, oceans, gods, demons who are exorcised by means of oblations, men, horses, donkeys, elephants, goats, aquatic birds, lions, deer, cows, buffaloes, monkeys, bulls, camels, birds, serpents, mongees, jackals, cats, alligators, hares, tigers, rhinoceros, aquatic animals, insects, plants, precious stones, silver, gold, copper, etc. etc., this world where are childhood, old age and desires, etc.

Thus ends chapter Twenty

on

FALSE NOTIONS

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elementary substance is one of the categories in this system, the knowledge of which brings salvation. There are nine *dravyas*, earth, water, fire, air, ether, time, space, soul and mind.

According to the Sāṁkhya system of philosophy, the *Puruṣa* or Soul and the *Prakṛti* or Root principle of nature, are the cause of creation.

Various philosophical views were maintained with regard to the cause of the creation. Some attributed it to the *Svabhāva* or nature ; some to *Karman* or action ; some to *Artha* or desire ; and some to *Kāla* or time.

CHAPTER XXI

ASTRONOMICAL INSTRUMENTS*

[Automatic rotation by means of Mercury, Oil and Water]

1. The armillary sphere, with radius of one's liking, having perfect circles, made of light and dry timber by a skilled carpenter who is proficient in the various crafts, should be set to rotate uniformly with time, with the help of water, oil and mercury, by applying one's intellect.

2 (a-b). A weight which floats for 30 *kṣaṇas* (=24 hours) in mercury, oil or water rotates it (*i.e.*, the armillary sphere) with the help of the three (fluids—mercury, oil and water). This is the basic principle of self-rotation.

MATHEMATICAL NOTES

1. The word *vividhaśilpavadātatakṣṇa*, in the Sanskrit text, should be read as *vividhaśilpavidāptatakṣṇā.*

2 (a-b). The method may be stated in full as follows :

"One should fasten one end of a string to the west cardinal point of the armillary sphere and carry the string round the armillary sphere along the equator (from below). Coming to the west point again, he should tie to it a dry gourd containing mercury and throw it into the cylindrical basin, full of water, kept below the west point, and then open the hole in the bottom of the basin. With the outflow of the water, the gourd goes down and the armillary sphere rotates. In 24 hours the gourd reaches the bottom of the basin and the

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*This chapter was left untranslated by Dr. Bina Chatterjee. The translation given below was supplied by Dr. K. S. Shukla.

[Construction of Circle, Triangle, Quadrilateral and tests for Verticality and Level]

2 (c-d). A circle is constructed by means of a compass (or a circular plate is made by means of a revolving machine), and a triangle or quadrilateral by means of hypotenuses (*karṇa*) ; verticality is achieved by means of a plumb, and ground is levelled by means of water.

[1. The Gola-yantra]

3. Now, in order to determine the *lagna* (*i.e.*, the longitude of the rising point of the ecliptic) and the *kāla* (*i.e.*, the time elapsed since sunrise in the forenoon, or to elapse before sunset in the afternoon), one should get a *Bhagola* ('Sphere of the asterisms') constructed with the help of nine circles, the prime vertical etc. The ecliptic should also be exhibited in it, in the manner stated before

4. Outside the *Bhagola* should be constructed the *Khagola* ('Sphere of the sky' or the Celestial Sphere) by means of the prime vertical, the horizon, the meridian, and the 6 O' clock circle, and having the polar axis and in its middle the globe of the Earth, as before.

5. (The equator lying on) the *Bhagola* should be graduated with the 60 marks (of *ghaṭīs*), whereas the prime vertical and the other circles should be graduated with the 360 marks (of degrees). The *Khagola* should be fastened firmly (to the polar axis), whereas the *Bhagola* should be kept loose by tying it to two loose pipes inserted into the polar axis.

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armillary sphere makes one complete rotation. When necessary, one should apply oil to the ends of the axis of the armillary sphere, so that the axis may rotate smoothly."

The weight of mercury put into the hollow of the dry gourd should be so adjusted that the gourd may float on water and with its motion downwards the armillary sphere may rotate.

The word *palaṁ* should be read as *kṣaṇaṁ*.

2 (c-d). The same is stated by Āryabhaṭa I (*A-Bh.*, ii. 13), Brahmagupta (*BSS*, xxii. 7), and Śrīpati (*SŚe*, xix. 2).

6. Now, one should fix one end of a pin to that point of the equator which rises with the Sun and should point the other end of the pin towards the degree-mark of the ecliptic occupied by the Sun.

7. Now, rotate the *Bhagola* continuously until the shadow of the pin passes through (*lit.* does not leave) the centre (of the *Bhagola*). This being done, the degrees of the ecliptic lying between the point denoting the Sun and the horizon denote the degrees of the ecliptic which have risen since sunrise, and the *ghaṭīs* (on the equator) lying between the pin and the horizon denote the *ghaṭīs* (elapsed since sunrise).

[Alternative method]

8. Or, the *Khagola* should be mounted on four vertical pillars of equal height erected at the four cardinal points, whereas the *Bhagola* should be rotated by the instruments of action (such as rope etc.) or *bījas* (such as mercury, oil and water) by (the astronomer) hiding himself in the hemispherical cavity underneath.

9. In the polar axis one should fix a pin in the plane of the equator, which should be free to move with the *Bhagola.* The *ghaṭī*-marks of the equator with which its two ends are in contact (at the moment) indicate the *ghaṭīs* elapsed (since sunrise).

[Various time-measuring devices]

10. (One should fasten one end of a string to the west cardinal point and carry it round the *Bhagola* along the equator from below). Where the string completes one round of the *Bhagola*, tie to it a (dry hollow) gourd containing mercury and throw it into the (cylindrical) basin of water (kept below the west point), having a

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7. *Cf. SŚe*, xix. 3-6 ; *SŚi*, II. xi. 3-4.

8. The word *lambaḥ* in the Sanskrit text should be read as *golaḥ*.

hole in the bottom and graduated with the marks of *ghaṭīs* on the basis of the outflow of water.

11. As the water flows out (through the hole in the bottom of the basin), the gourd, going downwards, rotates the *Bhagola* ; and the *nāḍi* marks left behind by the water denote the *ghaṭīs* elapsed.

[Man with a rosary of beads]

12. Knowing the number of *aṅgulas* corresponding to one *ghaṭī* (from the fall in water level by the outflow of water in one *ghaṭī*), insert *ghaṭikā* beads at that distance in a strip of cloth and keep them in position by applying wax (or other adhesive) and throw it inside the body of the (figure of a) man with face bent down.

13. To one end of the cloth-strip tie a string and carry it round a rod fitted horizontally within the mouth of the man, and take it out through a certain hole in the belly of the man.

14. There tie to it a dry gourd (containing mercury) (and throw it into the basin of water kept underneath, as before). As the gourd goes down, the man, pulling the cloth-strip, releases one new *nāḍī* bead after the lapse of every *ghaṭī*.

[Tortoise, Fighting rams, Husband and wife, Peacock and serpent, Drum-beater, Eclipser and eclipsed, etc.]

15. The Tortoise, etc., also, in the same way, release the *ghaṭīkā* (beads) at the desired intervals of time. Similarly, there occurs a fight between two rams (after every *ghaṭī*), when they are connected by the string (carrying beads).

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11. See *SŚe*, xix. 7-8 ; *BSS*, xxii. 46.

12-13. In verse 12, the word *tadanena* should be read as *madanena*, and the word *udareṇa* as *udare*. In verse 13, *cīryeta baddha-sūtre* should be read as *cīryantabaddhasūtram*.

14. See *BSS*, xxii. 47-48 ; *SŚi*, xix. 9-10.

15. The word *kuryādayo* in verse 15 should be read as *kūrmādayo*.

16-17. By the proper coordination of time and distance, there occurs a union of husband and wife (after every *ghaṭī*), a peacock gradually devours a serpent marked with the divisions of *aṅgulas* denoting *ghaṭīs*; a man beats a drum (after every *ghaṭī*), and an eclipser eclipses the body to be eclipsed (after the lapse of every *ghaṭī*). Several other (time-measuring) instruments like these may be (designed and) constructed.

[Self-rotating wheel and Self-rotating sphere]

18. The 'Wheel', made of the timber of Śrīparṇī (the silk-cotton tree), having a hole in the centre, bearing excellent rim and beautiful nave, and skilfully half-filled with mercury in its (curly) spokes, rotates by itself.

19. Let the polar axis, which is the same as the axis of rotation of the *Bhagola*, be made the Wheel's axle passing through its nave. This being done, the *Bhagola* rotates automatically due to the rotation of the Wheel.

[2. The Cakra-yantra, the Circle]

20. Construct a circular board, graduate it with the 360 divisions (of degrees) as well as the 60 divisions (of *ghaṭīs*), fix a vertical needle at its centre and set it up in such a way it that the Sun may lie centrally in its plane.

21. Then the signs etc. left behind (since sunrise) by the shadow of the vertical needle denote the signs etc. by which the Sun has ascended (above the horizon). The *nāḍīs* left behind denote the *nāḍīs* elapsed since sunrise, provided the Sun is in the eastern hemi-

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16-17. *Cf.*, *BSS*, xxii. 50-52 ; *SŚe*, xix. 11.

18. See *SŚi*, II. xi. 50-51 a-b.

21. See *SŚe*, xix. 12-13a-b. This rule is true for places on the equator only. At other places, it will give the Sun's altitude and the *ghaṭīs* corresponding to it.

There are reasons to believe that when observation was made

sphere. When the Sun is in the western hemisphere, the *nāḍīs* thus obtained should be subtracted from half the duration of the day (to get the *nāḍīs* to elapse before sunset).

[3. The Dhanur-yantra, the Semi-circle]

22. The above is the so-called *Cakra-yantra* ('The Circle'). Half of it is called *Dhanuryantra* ('The Semi-circle'). This latter instrument should be held (with its chord horizontal) in such a way that a ray of the Sun passing through the hole in the middle of the chord may fall on the arc.

23. Then the *nāḍīs* left behind by the shadow of the central needle, as measured from the arc-end, denote the *nāḍīs* elapsed during the day (since sunrise), and the signs and degrees left behind denote the signs and degrees by which the Sun has (ascended above the horizon).

[4. The Kartarī-yantra, the Scissors]

24. When the (*Dhanur-yantra*) is permanently fixed in the plane of the equator (forming the lower half of the equator) (and a needle pointing towards the north pole is fixed at the middle of the chord),

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at a place away from the equator, the *nāḍīs* elapsed since sunrise were deduced from the Sun's altitude by applying the following formula :

$$\textit{nāḍīs}\ \text{elapsed} = \frac{\text{semi-duration of day} \times \text{Sun's altitude}}{\text{Sun's meridian altitude}}.$$

Brahmagupta has criticised the astronomers who used this formula. See *BSS*, xxii. 11.

23. See *BSS*, xxi. 8 ; *SŚe*, xix. 13c-d.

The remark made in connection with the previous rule applies to this rule also.

24. See *BSS*, xxii. 44 ; *SŚe*, xix. 14. This instrument is a modification of Brahmagupta's *Kartarī-yantra*. Lalla's *Kartarī-yantra*, however, does not look like a *kartarī* (=scissors), and its name *kartarī* is indeed a misnomer. The real *Kartarī-yantra* has one semi-circular

it is known as *Kartarī-yantra.* In this case, the *ghaṭīs* left behind by the shadow of the needle fixed at the middle of the chord denote the *ghaṭīs* elapsed since sunrise.

[5-6. The Kapāla-yantra and the Pītha Yantra]

25-26. This very *Kartarī-yantra,* with its dial set horizontally on the ground and its needle vertical, is called *Kapāla-yantra* ; and the *Cakra-yantra,* with its dial having the directions marked in its rim, and set on the ground with its axis vertical, is called *Pītha-yantra.*

In the case of these two instruments, the *ghaṭīs* left behind up to the west point by the shadow of the needle denote the *ghaṭīs* elapsed. Moreover, in the case of the *Pītha-yantra,* the degrees on the rim from the point where the Sun is observed at sunrise (up to the east point) give (the degrees of) the Sun's *agrā.*

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plate in the lower half of the equatorial plane and another semi-circular plate in the meridian plane below the polar axis, the diameter of the latter being evidently along the polar axis. The semi-circular plate in the equatorial plane serves the purpose of the dial, its circumference being graduated with the *ghaṭikā* marks.

Brahmagupta describes his *Kartarī-yantra* as follows : "The *Kartarī-yantra* consists of two fixed (semi-circular) plates, with a needle fixed at the intersection of their diameters. By observing the shadow (of the needle) in this *Kartarī-yantra,* one can have a rough estimate of the *ghaṭikās* elapsed."

One can easily see that the *Kartarī-yantra* is a kind of equatorial sundial.

25-26. *BSS,* xxii. 42-43 ; *SŚe,* xix. 15-16. The *Kapāla-yantra* is evidently a horizontal sundial. The *Pīṭha-yantra* is a circular platform, as high as one's eye, having its circumference marked with the directions and the graduations of signs, degrees, minutes, etc. See *BSS,* xxii. 45.

The word *cakre* in verse 25 should be read as *cakraṁ* and the word *mīnārkodayavedhāt* in verse 26 should be read as *pīṭhārkodaya-vedhāt.*

[7. The Bhagaṇa-yantra]

27. Or, alternatively, on the rim of the circular plate of the *Cakra-yantra,* mark the sign segments, large or small, as the case may be, depending on the time-degrees obtained by dividing the *vināḍīs* of the oblique ascensions of the signs by 10, and also graduate each sign segment with the divisions of degrees.

28-29. Keep it horizontally on well levelled ground in open space and fix a vertical needle at its centre. Then hold the *yantra* in the equatorial plane with its needle pointing to the north pole in such a way that at sunrise the Sun is in the seventh sign (just 6 signs ahead of its actual position) (so that the shadow of the needle is directed towards the Sun's position at sunrise). Then the degrees left behind by the shadow of the needle on the *Bhagaṇa-yantra* denote the degrees traversed by the Sun since sunrise.

[Conversion of observed ghaṭīs into true ones]

30. The *ghaṭīs* elapsed during the day (since sunrise) as indicated by the instrument (used) when multiplied by the actual measure of the day and divided by the number of *ghaṭīs* in a day as adopted in the graduation of the instrument give the true *ghaṭīs* elapsed since sunrise. Otherwise, they are gross.

[8. The Śaṅku-yantra, the Gnomon]

31-32. The gnomon, constructed with the help of the revolving machine, having equal periphery at the top and bottom, made of very heavy and strong timber, perfectly straight and free from scars, streaks

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27-29. The *Bhagaṇa-yantra* is evidently a variant of the *Cakra-yantra.* The word *svasthasya* in verse 28 should be read as *khasthasya.*

30. In graduating the *ghaṭī* divisions in the instrument it is generally assumed that there are 60 *ghaṭīs* in a day. Actually, it is not exactly the case. Hence the above rule.

Śrīpati, too, gives this rule. See *SŚe,* xix. 17.

31-32. See *SŚe,* xix. 18.

and spots, uniformly circular, six *aṅgulas* in circumference and twelve *aṅgulas* in height, should be set up on a (horizontal) board, (already) levelled by means of water, in the vertical direction with the help of four plumbs.

33. The shadow of this (cylindrical gnomon), resembling the tail of a cow, should be observed outside the circular base and should be measured from (the circular base at) the centre. From the tip of the shadow up to the (upper) end of the gnomon lies obliquely the hypotenuse (of the shadow).

[9. The Ghaṭī-yantra]

34-35. The *Ghaṭikā* vessel, looking like one-half of a (spherical water-pot called) *Kalaśa*, made of ten *palas* of copper, half a hand in diameter at the top and half as high, and having a hole bored (at the bottom) by a uniformly circular needle of 4 *aṅgulas* in length made of $3\frac{1}{3}$ *māṣas* of gold, sinks into limpid water in one *nāḍī* exactly.

36-37. Or, it is a vessel manufactured according to one's liking (with the hole in the bottom) later adjusted by the measure of a *ghaṭī*.

Or, it is a vessel having a finger-wide hole (in the bottom) and of size determined by proportion in such a way that it may sink (in water) as many times in a day as there are *ghaṭīs* in that day.

A *ghaṭī* is also equal to 360 *asus*, each *asu* (being equal to the time of pronunciation) of 20 short syllables (or 10 long syllables).

[10. The Śalākā-yantra, the Needle]

[Agrā]

38-39. Having made a needle of as many *aṅgulas* as there are (minutes) in the radius, stretch it from east to west; and at its

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32. The word *syāpyaḥ* in verse 32 should be read as *sthāpyaḥ*.

34-35. See *SŚe*, xix. 19-20.

(western) end, at right angles to it, lay off another needle of arbitrary length such that an observer, with his eye at the tip of it and his line of sight passing through the other end of (the former needle equal to) the radius, sees a planet exactly on the horizon. Then that (latter needle) is the *agrā* (of that planet).

[Angular distance between two Planets]

40. Observe one planet in the direction of (the needle equal) to the radius, and another planet from the tip of the vertical needle. Then the square root of the squares of the two (needles), which is really equal to the radius, is the hypotenuse (of the needle-triangle).

41. Of the two needles, the (latter) vertical needle stands directed north-to-south ; the other one, east-to-west. (The degrees in) the arc corresponding to that (vertical needle) should be known as the degrees between the two planets, as situated at the time of observation.

[11. The Śakaṭa-yantra, the Cart]

[Tithi From Observation]

42. (Construct a circle on level ground and graduate its circumference with the 360 divisions of degrees). Then having made the observation of the Sun and the Moon through the V-shaped (lit. cart-shaped) tubes, (the Sun through one tube and the Moon through the other), let the tubes be placed (on the ground) with their tips on the circumference of the circle graduated with the 360 divisions of degrees (and the common end of the tubes at the centre of the circle), and show by means of points the places (where the tips fall).

43. The degrees between these (two points) are the degrees between the Sun and the Moon. These, when divided by 12, give the *tithis* elapsed in the light half of the month, or the *tithis* to elapse in the dark half of the month.

[Time From Observation]

44-45. (Severally) multiply the Rsines of the (Sun's meridian)

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43. See *PS*, xiv. 12 ; *SŚe*, xix. 26.

zenith distance and *agrā* by 60 and divide (each product) by the radius. Lay them off, in their own directions, in a circle drawn (on level ground) with 60 *aṅgulas* as radius (and having the directions marked in it), the former from the centre and the latter from the east as well as the west points. And through the three points (thus obtained) draw, as before, the circle denoting the path of the gnomon. By laying off the same in the contrary directions, one may draw the circle (denoting the path of the tip) of the shadow (of the gnomon).

46. Beginning with the end of the *agrā* (laid off in the east), graduate the circle denoting the path of the gnomon with the (appropriate) signs by means of the degrees of time (of their oblique ascension). And then set up a gnomon (on that circle) in such a way that the tip of the shadow may fall at the centre.

47. Then the degrees of time lying from the end of the *agrā* (in the east) up to the foot of the gnomon are the degrees of time of the (Sun's) ascension. These degrees of time as divided by six are the *ghaṭīs* elapsed in the day (since sunrise).

[12. The Yaṣṭi-yantra, the Graduated Tube]

[Altitude, Zenith Distance and Bāhu]

48. Hold the *Yaṣṭi*, equal to the radius (of the circle drawn on level ground), with its lower end at the centre (of the circle), in such a way that it may not cast any shadow (on the ground). Then the degrees in the arc corresponding to the perpendicular (dropped on the ground from the other end) of it are the degrees of the (Sun's) altitude.

49. The *yaṣṭi*, equal to the radius, is the hypotenuse; the perpendicular (dropped on the ground from the upper end of the *Yaṣṭī*)

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47. See *ŚŚe*, xix. 24-25.

is the (great) gnomon (*i.e.*, the Rsine of the Sun's altitude); the square root of the difference of their squares is the (great) shadow, *i.e.*, the Rsine of the (Sun's) zenith distance; and the distance between the east-west line and the foot of the perpendicular is the *bāhu*.

[Equinoctial midday shadow]

50. Multiply the distance between the rising-setting line (lit. the line joining the ends of the *agrās* laid off in the east and west) and the gnomon by 12 and divide by the perpendicular dropped from the tip of the *Yaṣṭi*. This is how the equinoctial midday shadow is obtained from the perpendicular dropped from the tip of the *Yaṣṭi*.

[Distance between two planets and their longitudes]

51. Determine the distance between the two planets with the help of the gnomon or the *Yaṣṭi*, like that between the Sun and the Moon. This added to (the longitude of) the westward planet gives (the longitude of) the eastward planet and the same subtracted from the latter gives the former.

[Sun's altitude from the Śakaṭa-yantra]

52. The *Śakaṭa-yantra*, of which one tube is directed towards the position of the gnomon in the circle denoting its path and the other towards the Sun in its diurnal circle, enables one to know the signs etc. of the Sun's altitude.

[Instruments of Observation]

53. The *Gola*, the *Bhagaṇa*, the *Cakra*, the *Dhanu*, the *Ghaṭī*,

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49. See *SŚe*, xix. 21-22.

50. *Cf. BSS*, xxii. 29; *SŚe.*, xix. 23. *Prāgaparāgrāsaktaṁ sūtraṁ śaṅkvantaraṁ*, occurring in the Sanskrit text, should be read as *prāgaparāgrāsāktasūtra-śaṅkvantaraṁ* and *ṣaṣṭyavalambavibhaktaṁ* as *yaṣṭyavalambavibhaktaṁ*.

51. See *PS*, xiv. 13.

53. Brahmagupta omits the *Bhagaṇa*, the *Śakaṭa* and the *Śalākā*, but adds the *Turīya* (*i.e.*, the Quadrant). Śrīpati omits the *Bhagaṇa* and the *Śalākā*.

the *Śaṅku*, the *Śakaṭa*, the *Kartarī*, the *Pītha*, the *Kapāla*, the *Śalāka*, together with the *Yaṣṭi*, are the twelve (astronomical) instruments.

[Things required during observation]

54. The hypotenuse (*karṇa*), the shadow, the semi-duration of the day, (the longitude of) the Sun, the latitude (of the place), the plumb, the compass (or the revolving machine, *bhrama*), water, as well as intelligence and effort are the things required in the use of (astronomical) instruments.

Thus ends chapter Twentyone

on

ASTRONOMICAL INSTRUMENTS

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54. See *BSS*, xxii. 6. The word *karṇacchāyā*, occurring in the Sanskrit text, should be read as *karṇaśchāyā*.

CHAPTER XXII

ASTRONOMICAL PROBLEMS

1. Why are the following corrections applied to the mean longitude of a planet : Correction for difference in terrestrial longitude ; Correction for equation of time (calculated) by using the Sun's equations of centre ; and the Correction for ascensional difference (calculated) from the Sun's longitude ? Why is Visibility correction necessary for the Moon and other planets ? What should be the mean anomaly of the Moon when its mean motion is equal to its true motion ?

2. Why are the days and nights always equal at the equator ? Why does the day increase and so also the night (at certain times) at other places ?

3. Why do some signs of the zodiac take a shorter time to rise while others take a longer time ? Why do the six signs beginning with Cancer take a longer time to rise at some places and the six signs beginning with Capricorn take a (comparatively) shorter time ?

4. What are the signs which are always visible at a place and what are those which are not always visible ?

Why does the Moon (appear to) shine more brightly when on the horizon than when in the middle of the sky ?

5. Why are corrections for longitudinal and latitudinal parallax made repeatedly in a solar eclipse. Why are (the same corrections) not made in an eclipse of the Moon which is an enemy of the lotus and which is as lustrous as the face of a beautiful damsel ?

MATHEMATICAL NOTES

Most of these have been answered in Sn. I. Mathematics.

6. Why could not the length of the day in Avanti be the same when the Sun is in the fourth degree of Taurus, as when the Sun *is* in the middle of Cancer ?

7. He will live long who can find the latitude of a place from the times of rising of the signs at his own place and its distance from the prime meridian by means of the times of the rising of the signs at Laṅkā ; and who can find the equinoctial shadow from the shadow at any time and the shadow when the Sun is on the prime vertical.

8. How can he who does not know the longitude and latitude of a place, the ascensional difference, neither the longitude nor the declination of the Sun, nor even the time elapsed since the *Kaliyuga*, quickly tell, when asked by others, the longitude and the R sine declination of the Sun, the latitude of the place, the earth-sine, the midday shadow and the directions ?

9. Tell me the times of rising of Cancer and Capricorn above the horizon, at a place where the length of the day is 32 *ghaṭikās* when the Sun is in the middle of Cancer.

10. Where will the length of the midday shadow, the Sun being in the middle of Taurus, be the same as that cast in Laṅkā, when the Sun is in the twenty-fifth degree of Aries ?

11. O astronomer ! tell me quickly the latitude of a place, without knowing the time elapsed since the beginning of the *Kaliyuga* and hence the longitude of the Sun and its declination without looking at the polar star.

12. Of what quality is the Sun (*i.e.*, what is the longitude of the Sun) as seen from a latitude of 32° when it enters the prime vertical 7 *ghaṭikās* after its rising ?

13. What is the latitude of the place which a man reaches after covering a distance of 100 *yojanas* from Laṅkā in a south-east direction ? Again, (what is the latitude of the place if he now goes) in a southern direction and where the length of the midday shadow is 8 *aṅgulas* ?

14. At what latitude does the Sun set 80 days after its rising ? Once set, where does it again rise 80 days after ?

15. Suppose one has come out of a well and does not know the direction of the Sun, and the latitude of the place. If he is suddenly asked where the Sun is, tell me what will he say ?

16. Tell the correction for longitude (of a place) without knowing its latitude, the mean Sun and the (distance from the) prime meridian.

17. Will you, who know about the sphere of universe, tell me the length of the equinoctial midday shadow, after ascertaining the day when the Sun is at the first point of Aries, by observing its midday position in the sky by means of the instrument *Drkśaradala.*

18. What is the latitude of the place, where, when the Sun is at the end of Gemini, the night is one-third of the duration of the day ?

19. What is the latitude of the place where the length of the day is 34 *ghaṭikās*, when the Sun is at the beginning of Cancer ?

20. You, who know about the sphere of universe, please tell me correctly the latitude of the place where the Sun is seen due north-west by the people, when it is at the end of Taurus and on the horizon.

21. O friend, tell me after calculation the degrees in the colatitude of the place where the duration of the night is 1 *muhūrta* when the Sun is at the end of Gemini.

22. What are the R sine latitude and R sine colatitude, if their sum is 1308 and their difference is 538 ?

23. O friend, after careful and long deliberation, tell me when and where the Moon is simultaneously visible to the men, to the gods and to the demons.

Thus ends chapter Twentytwo

on

ASTRONOMICAL PROBLEMS

APPENDICES

TABLE I

REVOLUTIONS OF SUN, ETC. IN A YUGA OF 4320000 SOLAR YEARS

	Āryabhaṭīya (*ABh.*)	*Mahābhāskarīya* (*MB*)	*Laghubhāskarīya* (*LB*)	*Śiṣyaddhīvṛddhida* (*ŚD*)
Sun	4320000			
Moon	57753336			
Moon's Apogee	488219			
Moon's node	232226			
Mars	2296824	*Same*	*Same*	*Same*
Mercury (*śīghrocca*)	17937020			
Venus (*śīghrocca*)	7022388			
Saturn	146564			
Asterisms	1582237500 (earth)			
	(i. 3-4)	(vii. 1-5)	(i. 9-14)	(i. 4-6 ab, 8 cd)

TABLE II

SOLAR MONTHS, ETC. IN A YUGA

	ĀBh.	*MB*	*LB*	*ŚD*
Solar years	4320000			
Solar months	51840000			
Solar days	1555200000			
Lunar months	53433336			
Lunar days	1603000080	*Same*	*Same*	*Same*
Intercalary months	1593336			
Omitted *tithis*	25082580			
Civil days	1577917500			
	(iii. 5-6)	(vii. 6-8)	(i. 4-8)	(i. 6 cd, 8 ab

TABLE III

APOGEES OF SUN ETC.

	ĀBh.	*MB*	*LB*	*ŚD*
Sun	78°			
Mars	118°			
Mercury	210°	*Same*	*Same*	*Same*
Jupiter	180°			
Venus	90°			
Saturn	236°			
	(i. 9 cd)	(vii. 11)	(i. 18, 22)	(ii. 9 cd, iii. 1)

TABLE IV

MANDA EPICYCLES OF SUN, ETC.

	$\bar{A}$Bh.	MB	LB	ŚD
Sun	13½°			
Moon	31½°			
Mars	63°-81°			
Mercury	31½°-22½°	*Same*	*Same*	*Same*
Jupiter	31½°-36°			
Venus	18°-9°			
Saturn	40½°-58½°			
	odd-even			
	(i. 10-11)	(vii. 13-16)	(i. 19-21)	(iii. 2)

TABLE V

ŚIGHRA EPICYCLES OF SUN, ETC.

	ĀBh.	*MB*	*LB*	*ŚD*
Mars	$238\frac{1}{2}°$-$229\frac{1}{2}°$			
Mercury	$139\frac{1}{2}°$-$130\frac{1}{2}°$			
Jupiter	$72°$-$67\frac{1}{2}°$	*Same*	*Same*	*Same*
Venus	$265\frac{1}{2}°$-$256\frac{1}{2}°$			
Saturn	$40\frac{1}{2}°$-$36°$			
	odd-even			
	(i. 10-11)	(vii. 13-16)	(i. 19-21)	(iii. 2)

TABLE VI

ŚIGHRA ANGLES FOR REGRESSION

	ŚD	*BSS*	*ŚSe*	*ŚSi*	*SS*
Mars	163°				164°
Mercury	145°				144°
Jupiter	125°	*Same*	*Same*	*Same*	130°
Venus	165°				163°
Saturn	113°				115°
	(iii. 20)	(ii. 48)	(iii. 58)	(ii. 41)	(ii. 53-54)

ŚIGHRA ANGLES FOR RISING IN THE EAST

	ŚD	*BSS*	*ŚSe*	*ŚSi*
Mars	28°			
Mercury	205°			
Jupiter	14°	*Same*	*Same*	*Same*
Venus	183°			
Saturn	20°			
	(iii. 22)	(ii. 52-53)	(iii. 61)	(ii. 42-43)

ŚIGHRA ANGLES FOR RISING IN THE WEST

	ŚD	*BSS*	*ŚSe*	*ŚSi*
Mercury	51°	50°	50°	25°
Venus	23°	24°	24°	24°
	(iii. 23)	(ii. 52)	(iii. 62)	(ii. 43)

TABLE VII

KĀLAṂŚAS OF THE PLANETS

	ĀBh.	*MB*	*LB*	*ŚD*	*BSS*	*ŚSe*	*ŚSi*	*SS*
Moon	12°						12°	12°
Mars	17°						17°	17°
Mercury	13°	S	A	M	E		14°	14°
Jupiter	11°						11°	11°
Venus	9°						10°	10°
Saturn	15°						15°	15°
	(iv. 4)	(vi. 44)	(vi. 5; vii. 1)	(viii. 5)	(vi. 6)	(ix. 8)	(viii. 6)	(ix. 6-8; x. 1)

KĀLAṂŚAS FOR REGRESSION

	MB	*ŚD*	*BSS*	*ŚSe*	*ŚSi*	*SS*
Mercury	—	12°	—	12°	12°	12°
Venus	4°-4½°	8°	10°	8°	8°	8°
	(vi. 45)	(viii. 5)	(vi. 11)	(ix. 9)	(viii. 6)	(ix. 7-8)

TABLE VIII

DIAMETERS OF SUN, ETC. IN YOJANAS

	ĀBh.	*MB*	*LB*	*ŚD*
Sun	4410	*Same*	4410	4410
Moon	315		315	320 or 315
Earth	1050		1050 (iv. 4)	1050 or 1250 (v. 6·7)
Mars	$12\frac{3}{5}$			$12\frac{4}{5}$
Mercury	21			$21\frac{1}{3}$
Jupiter	$31\frac{1}{2}$			32
Venus	63			64
Saturn	$15\frac{3}{4}$			16
	(i. 7)	(v. 4, vi. 56)		(x. 2)

TABLE IX

MEAN LATITUDES OF PLANETS

Moon	270′			
Mars	90′			
Mercury	120′		Same	
Jupiter	60′	Same		Same
Venus	120′			
Saturn	120′			
	(i. 8)	(v. 30; vii. 9)	(iv. 8, vii. 7 ab)	(x. 5 ab)

TABLE X

MODES OF PLANETS

	ĀBh.	*MB*	*LB*	*ŚD*
Mars	40°			
Mercury	20°			
Jupiter	80°	*Same*	*Same*	*Same*
Venus	60°			
Saturn	100°			
	(i. 9)	(vii. 10)	(vii. 6 cd)	(x. 5 cd)

TABLE XI

POLAR LONGITUDES OF STARS

Junction Star of	*MB*	*LB*	*ŚD*	*BSS*	*SŚe*	*SŚi*	*SS*
Aśvinī	8°	8°	8°	8°	8°	8°	8°
Bharaṇī	27°	26°30′	20°	20°	20°	20°	20°
Kṛttikā	36°	36°	36°	37°28′	37°28′	37°28′	37°30′
Rohiṇī	49°	50°	49°	49°28′	49°28′	49°28′	49°30′
Mṛgaśirā	62°	62°	62°	63°	63°	63°	63°
Ārdrā	70°	70°	70°	67°	67°	67°	67°20′
Punarvasu	92°	92°	92°	93°	93°	93°	93°
Puṣya	105°	105°	105°	106°	106°	106°	106°
Āśleṣā	114°	114°	114°	108°	108°	108°	109°
Maghā	128°30′	128°80′	128°	129°	129°	129°	129°
P. Phalgunī	141°	141°	139°20′	147°	147°	147°	144°

Junction Star of	*MB*	*LB*	*ŚD*	*BSS*	*SŚe*	*SŚi*	*SS*
U. Phalguni	154°	154°	154°	155°	155°	155°	155°
Hasta	173°	173°	173°	170°	170°	170°	170°
Citrā	185°	185°	184°20′	183°	183°	183°	180°
Svāti	197°	197°	197°	199°	199°	199°	199°
Viśākhā	212°	212°	212°	212°5′	212°5′	212°5′	213°
Anurādhā	222°	222°	222°	224°5′	224°5′	224°5′	224°
Jyeṣṭhā	228°	228°	228°	229°5′	229°5′	229°5′	229°
Mūla	241°	241°30′	241°	241°	241°	241°	241°
P. Āṣāḍha	254°	254°30′	254°	254°	254°	254°	254°
U. Āṣāḍha	267°	266°30′	267°20′	260°	260°	260°	260°
Śravaṇa	285°	284°30′	283°10′	278°	278°	278°	280°
Dhaniṣṭhā	296°	295°30′	296°20′	290°	290°	290°	290°
Śatabhiṣā	307°	307°	313°20′	322°	320°	320°	320°
P. Bhādrapada	328°	328°	327°	326°	326°	326°	326°
U. Bhādrapada	345°	345°	335°20′	337°	337°	337°	337°
Revatī	363°	360°	359°	0	0	0	359°50′
	(iii. 63-66ab)	(viii. 1-4)	(xi. 1-3)	(x. 1-3)	(xii. 1-2)	(xi. 1-3)	(viii. 2-6ab)

TABLE XII

POLAR LATITUDES OF STARS

Junction Star of	*MB*	*LB*	*ŚD*	*BSS*	*SŚe*	*SŚi*	*SS*
Aśvinī	10°N	10°N	10°N	10°N	10°N	10°N	10°N
Bharaṇī	12°N	12°N	12°N	12°N	12°N	12°N	12°N
Kṛttikā	5°N	5°N	5°N	4°31′N	4°31′N	4°30′N	5°N
Rohiṇī	5°S	5°S	5°S	4°33′S	4°33′S	4°30′S	5°S
Mṛgaśirā	10°S	10°S	10°S	10°S	10°S	10°S	10°S
Ārdrā	11°S	11°S	11°S	11°S	11°S	11°S	9°S
Punarvasu	6°N	6°N	6°N	6°N	6°N	6°N	6°N
Puṣya	0	0	0	0	0	0	0
Āśleṣā	7°S	7°S	7°S	7°S	7°S	7°S	7°S
Maghā	0	0	0	0	0	0	0

Junction Star of	*MB*	*LB*	*ŚD*	*BSS*	*SŚe*	*SŚi*	*SS*
P. Phalgunī	12°N	12°N	12°N	12°N	12°N	12°N	12°N
U. Phalgunī	13°N	13°N	13°N	13°N	13°N	13°N	13°N
Hasta	7°S	7°S	8°S	11°S	11°S	11°S	11°S
Citrā	2°S	2°S	2°S	1°45′S	1°45′S	1°45′S	2°S
Svāti	37°N	37°N	37°N	37°N	37°N	37°N	37°N
Viśākhā	1°30′S	1°30′S	1°30′S	1°23′S	1°23′S	1°20′S	1°30′S
Anurādhā	3°S	3°S	3°S	1°44′S	1°44′S	1°45′S	3°S
Jyeṣṭhā	4°S	4°S	4°S	3°30′S	3°30′S	3°30′S	4°S
Mūla	8°20′S	8°30′S	8°30′S	8°30′S	8°30′S	8°30′S	9°S
P. Āṣāḍha	7°S	7°S	5°20′S	5°20′S	5°20′S	5°20′S	5°30′S
U. Āṣāḍha	7°20′S	7°S	5°S	5°S	5°S	5°S	5°S
Sravaṇa	30°N	30°N	30°N	30°N	30°N	30°N	30°N
Dhaniṣṭhā	36°N	36°N	36°N	36°N	36°N	36°N	36°N
Śatabhiṣā	18′S	18′S	20′S	18′S	18′S	20′S	30′S
P. Bhādrapada	24°N	24°N	24°N	24°N	24°N	24°N	24°N
U. Bhādrapada	26°N	26°N	26°N	26°N	26°N	26°N	26°N
Revatī	0	0	0	0	0	0	0
	(iii. 66 cd-71 ab)	(viii. 6-9)	(xi. 5-7, 9-10 ab)	(x. 5-9)	(xii. 4-7)	(xi. 4-6)	(ii. 6 cd-9)

APPENDIX XIII

TRUE LONGITUDES OF THE SUN AND THE MOON

Verse 14 of ch. Two of the *Śiṣyadhīvṛddhida* gives formulas for determining the true longitudes of the Sun and the Moon. This can be achieved either by the epicyclic method or by the eccentric method.

First the epicyclic method or *nīcoccavṛttabhaṅgi*. According to this method, the body is supposed to describe with uniform motion the circumference of a circle called *nīcoccavṛtta* or epicycle, the centre of which is supposed to move along the *kakṣāvṛtta* or deferent or circular orbit of the body, with a motion equal to the mean motion of the body but in a reverse direction. The time the body takes to make one revolution about the centre of the epicycle is the same as the time the epicycle takes to revolve once round the orbit.

This method is illustrated below.

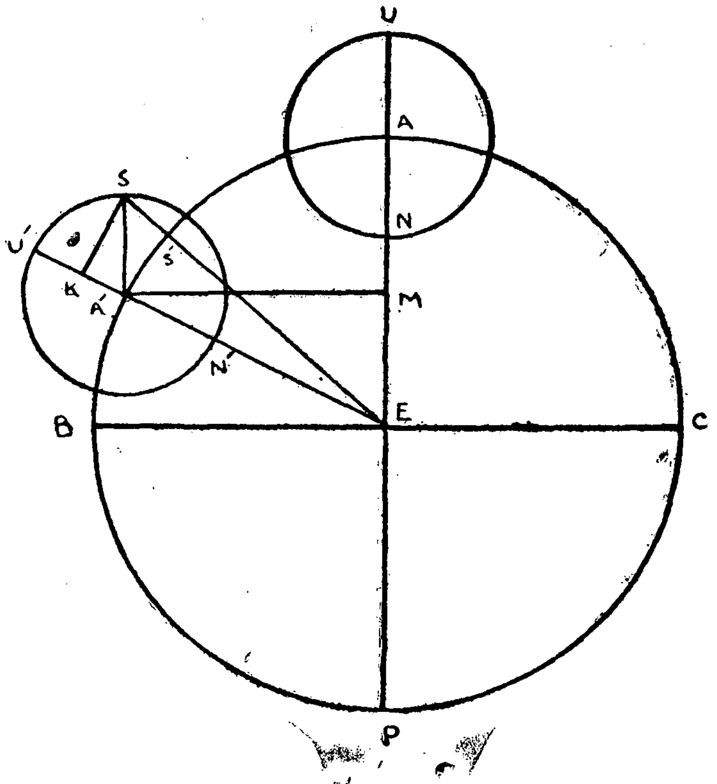

Fig. 31

Let ABPC be the deferent of the Sun, with centre E, the centre of earth. Let AEP be the apse line. With centre A and radius equal to *mandāntyaphalajyā* or radius of the Sun's epicycle describe a circle. Let the apse line cut it in U and N, which are the *mandocca* or apogee and *mandanīca* or perigee of the Sun on the epicycle, respectively. When the centre of the epicycle is at A, the Sun is at U. Let A move with the mean motion of the Sun up to A′ along the orbit and let the Sun move from U′ to S along the epicycle so that arc U′S=arc A′A. Join SE cutting the orbit at S′. Then A′ is the *madhyamasūrya* or mean place of the Sun and S′ is the *sphuṭasūrya* or true place of the Sun in the orbit. Thus the correction to be given to the mean longitude of the Sun to get its true longitude is the value of the arc A′S′. This value is called *mandaphala* or equation of the centre of the Sun and the process is called *mandakarma*.

Now to find the arc A′S′. Join SA′. Draw SK and A′M perpendiculars to U′A′E and UAE, respectively.

The angle A′EA, that is the angle between the apogee A and the mean Sun A′ is called *mandakendra* or mean anomaly.

A′M is the *mandakendrajya* or R sine mean anomaly. Now, since arc U′S=arc A′A, angle U′A′S=angle A′EA. So, SA′ is parallel to AE.

Then, from the similar triangles SA′K and A′EM,

$$SK=\frac{A'M\times SA'}{A'E}$$

$$=\frac{\text{R sin mean anomaly}\times\text{radius of epicycle}}{R}$$

$$=\frac{\text{R sin mean anomaly}\times\text{circumference of epicycle}}{360}$$

$$=\frac{\text{R sin mean anomaly}\times 13\tfrac{1}{2}}{360}$$

$$=\frac{\text{R sin mean anomaly}\times 3}{80} \qquad (1)$$

3 is called *guṇaka* (multiplier). SR is *doḥphala*.

The arc corresponding to (1) as R sine is approximately taken to be the value of arc A′S′ or mandaphala ; which is

$$\frac{\text{R sin mean anomaly}\times 3}{80}\ \text{minutes, nearly,}$$

from $225 : 225' : : \frac{\text{R sin mean anomaly}\times 3}{80} : ?$.

The *mandaphala* is subtracted from the Sun's mean longitude if its mean place is in advance of its true place, that is, when the mean anomaly is in the first or second quadrant ; and added, if its true place is in advance of its mean place, that is, in the third or fourth quadrant.

SE is called *mandakarṇa* or *manda* hypotenuse.

The second method is called *prativṛttabhaṅgī* or method of concentric and eccentric. This method supposes that the mean Sun or Moon moves with a uniform velocity along its circular orbit or concentric and the true Sun moves with the same velocity and in the same direction along the circumference of an equal circle called *prativṛtta* or eccentric, the centre of which is situated on the line joining the apogee to the centre of the earth, at a distance equal to *mandāntyaphalajyā* or radius of the epicycle of the body, from the latter point.

The method is illustrated below.

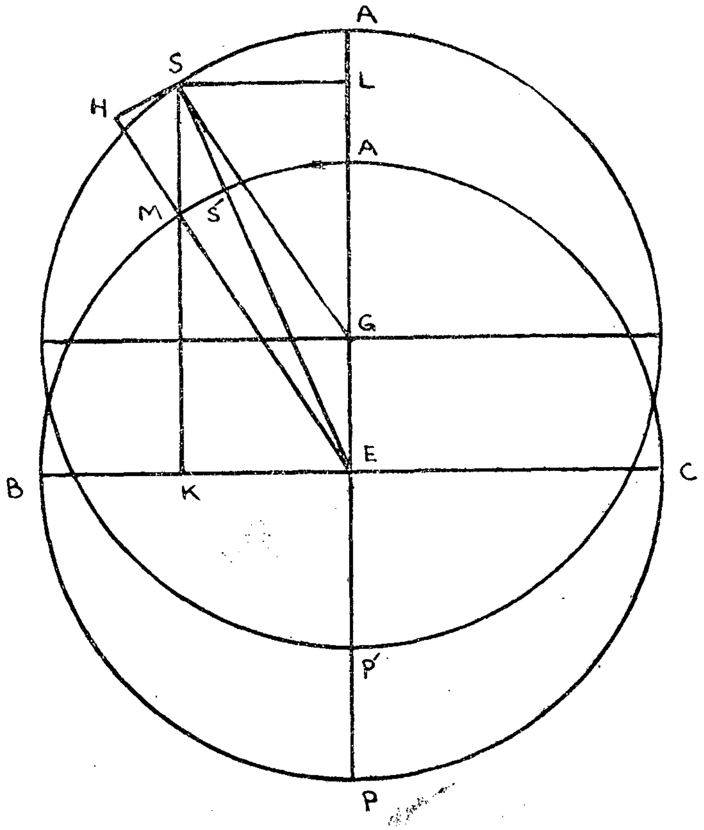

FIG., 32

Let ABPC be the Sun's circular orbit or concentric with centre E, the centre of the earth. Let AP be the apse line, so that A is the apogee and P the perigee on the orbit. Let GE be equal to the radius of the Sun's epicycle. With G as centre describe a circle equal to the orbit. This is the *mandaprativṛtta* or eccentric. Let AP cut it in A′ and P′, which are, respectively, the apogee and perigee on the eccentric. Let the mean Sun and true Sun start from A and A′, respectively, and move along the concentric and the eccentric with the same velocity and in the same direction. Let their new places be M and S. Join SE cutting the concentric at S′. Then M is the mean Sun and S′ is the true Sun. So the correction to be given to the mean longitude of the Sun to get its true longitude is the value of the arc MS′, which is the *mandaphala.*

Now to find this value. Join GS and EM. Draw SH perpendicular to EM produced. Join SM and produce it to meet BC at K. Draw SL perpendicular to A′P.

Now, since arc A′S=arc AM, angle A′GS=angle AEM. So GS is parallel to EM and GS is also equal to EM. Thus, SM is parallel and equal to GE or radius of the epicycle.

Now, angle A′GS=angle AEM, the angle between the apogee and the mean Sun or mean anomaly. Therefore SL=EK=R sin mean anomaly.

Now, from similar triangles SHM and EKM,

$$SH=\frac{KE\times SM}{EM}$$

$$=\frac{\text{R sin mean anomaly}\times\text{radius of epicycle}}{R}$$

$$=\frac{\text{R sin mean anomaly}\times\text{circumference of epicycle}}{360}$$

$$=\frac{\text{R sin mean anomaly}\times 13\tfrac{1}{2}}{360}$$

$$=\frac{\text{R sin mean anomaly}\times 3}{80} \qquad (1)$$

Hence the value of the *mandaphala*, which is the same as before.

Thus both methods give the same result.

The same methods are followed to find the true longitude of the Moon.

This *mandaphala* is the arc corresponding to

$$\frac{\text{R sin mean anomaly} \times 31\frac{1}{2}}{360} \text{ as R sin,}$$

$$\text{or to } \frac{\text{R sin mean anomaly} \times 7}{80} \text{ as R sine,}$$

$$\text{or } \frac{\text{R is mean anomaly} \times 7 \text{ minutes, nearly}}{80}$$

7 is called *guṇaka*.

As has already been pointed out, Lalla follows Āryabhaṭa when he takes the circumferences of the epicycles of the Sun and Moon as $13\frac{1}{2}°$ and $31\frac{1}{2}°$, respectively. Again, following him he gives the circumferences divided by $4\frac{1}{2}$.

Even from the time of Bhāskara I, if not earlier, the Indian astronomers were aware of the fact that the circumferences of the epicycles of the Sun, Moon and the planets as tabulated, are their mean circumferences and so the true positions calculated from these are not really the true positions. They, therefore, prescribed the method of successive approximations to obtain more correct results.

Lalla gives the method for the Sun and Moon in iii. 17, which is as follows.

Consider diagram on p. 312. SK or *dohphala* is already obtained. From the same similar triangles, KA′ or *koṭiphala*

$$= \frac{\text{R cos mean anomaly} \times 3}{80}$$

for the Sun.

Then, $KE = KA' + R$ is known, and hence

$$ES = \sqrt{SK^2 + KE^2}$$

$=$ *mandakarṇa* or *manda* hypotenuse.

Then $\frac{3 \times ES}{R}$ given the more correct *guṇaka* or circumference of the epicycle. Hence the *doḥphala, koṭiphala* and *karṇa* should again be calculated. Then again the *guṇaka.* The process should be repeated till the *karṇa* or the distance of the Sun is fixed and hence the correct epicycle.

Similarly, for the Moon, by using 7 as the *guṇaka,* the correct distance will be found.

The same procedure should be followed in the case of concentric and eccentric methods.

In this connection see *MB,* iv. 9-12, *LB,* ii. 6-7, *BSS,* xxi. 29, *SŚe,* xvi. 24 and *SŚi,* II. v. 36-37 and commentary.

Verse 15 of Ch. ii. gives formulas for determining the true motions of the Sun and Moon on any day.

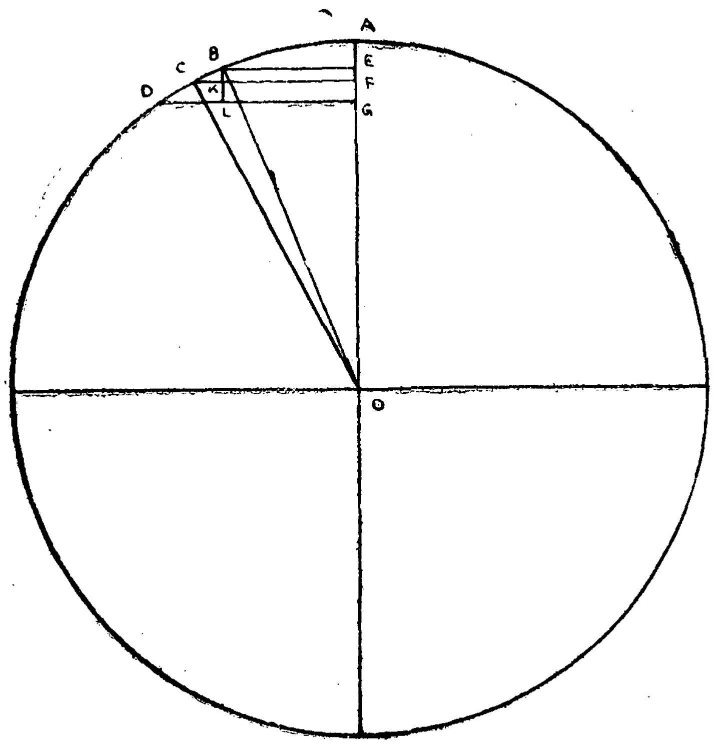

FIG. 33

Let O be the centre of the Sun's eccentric circle. Let the arc AB denote the Sun's *mandakendra* or mean anomaly for any day, and the arc AC that for the next day. Then the arc BC is the motion of the mean anomaly in 1 day or *mandakendragati.* Measure off arc BD=225′. Draw BE, CF and DG as perpendiculars to AO. Draw BL perpendicular to DG cutting CF in K. Now BE is R sine of the arc AB and CF that of arc AC. Thus CK is their difference. DL is called *bhogyakhaṇḍa* by Lalla.

Now, from the proportion, arc BD : DL=arc BC : CK

$$CK = \frac{\text{arc BC} \times \text{DL}}{\text{arc BD}}$$

$$= \frac{\text{motion of anomaly} \times \textit{bhogyakhaṇḍa}}{225}$$

From above, the Sun's *mandaphala* on any day

$$=\frac{\text{R sin mean anomaly on that day} \times 3}{80}$$

and its *mandaphala* on the next day

$$=\frac{\text{R sin mean anomaly on the next day} \times 3}{80}$$

Thus, the difference between the *mandaphalas* on two consecutive days or *mandaphala* or correction to be given to the mean motion

$=\frac{3}{80}$ (R sine mean anomaly for one day $\sim$ R sin mean anomaly for the next day)

$=\frac{3}{80} \times \text{CK}$

$=\frac{3}{80} \times \frac{\text{motion of anomaly} \times \textit{bhogyakhaṇḍa}}{225}$

$=\frac{3}{80} \times \frac{\textit{bhogyakhaṇḍa}}{225} \times$ (Sun's motion—motion of its apogee)

$=\frac{3}{80} \times \frac{\textit{bhogyakhaṇḍa}}{225} \times$ Sun's motion

(since Sun's apogee is supposed to have no motion)

$=\frac{3}{80} \times \frac{\textit{bhogyakhaṇḍa}}{225} \times 59'8''$

$=\frac{\textit{bhogyakhaṇḍa}}{101}$ minutes, nearly.

Similarly, the Moon's *mandagatiphala*

$=\frac{7}{80} \times \frac{\textit{bhogyakhaṇḍa}}{225} \times (790'35''—6'41'')$

$=\frac{10}{33} \times$ *bhogyakhaṇḍa* minutes, nearly.

The *mandagatiphala* added to or subtracted from the mean motion of the Sun or Moon, gives its true motion subtracted when the mean anomaly is in first or fourth quadrant; otherwise it is added.

Lalla, however, finds fault with the rule in iii. 16. He says that as far as the motion of the Moon is concerned this rule gives the motion for the day elapsed and not for the current day.

Bhāskara II, while commenting on this verse, upholds Lalla's view.

APPENDIX XIV

TRUE LONGITUDES OF THE PLANETS

Verses 3-8 to ch. Three of the *Śiṣyadhīvṛddhida* give the method to determine true longitudes of planets. In the case of planets revolving round the Sun, two corrections are given : (1) *mandaphala* correction and (2) *śīghraphala* correction. The first is equivalent to the zodiacal inequality and the second to the solar inequality of Ptolemy. The *śīghraphala* roughly represents the elongation in the case of an inferior planet and the annual parallax in the case of a superior planet. Both these corrections are calculated by the epicyclic or eccentric method.

First the epicyclic method. In the case of the Sun and Moon, the tabulated epicycles are the same in the odd and even quadrants. In the case of the planets both the *manda* epicycles and the *śīghra* epicycle vary, and for every anomaly, *manda* or *śīghra*, the variation is calculated and the correct epicycle or epicycle$\div 4\frac{1}{2}$ or *sphuṭamandaguṇaka*, as it is called, is obtained.

The method of calculation of the *mandaphala* is the same as in the case of the Sun. From verse 14 of ch. ii,

$$\textit{doḥphala} = \frac{\text{R sin mean anomaly} \times \textit{sphuṭamandaguṇaka}}{80}$$

The *mandaphala* will, therefore, be the corresponding arc. As in the case of the Sun, this is added to or subtracted from the mean longitude of the planet. The corrected planet is called *mandasphuṭagraha* or *mandaspaṣṭagraha* or true mean planet.

Now the second correction or *śīghra* correction using this corrected planet.

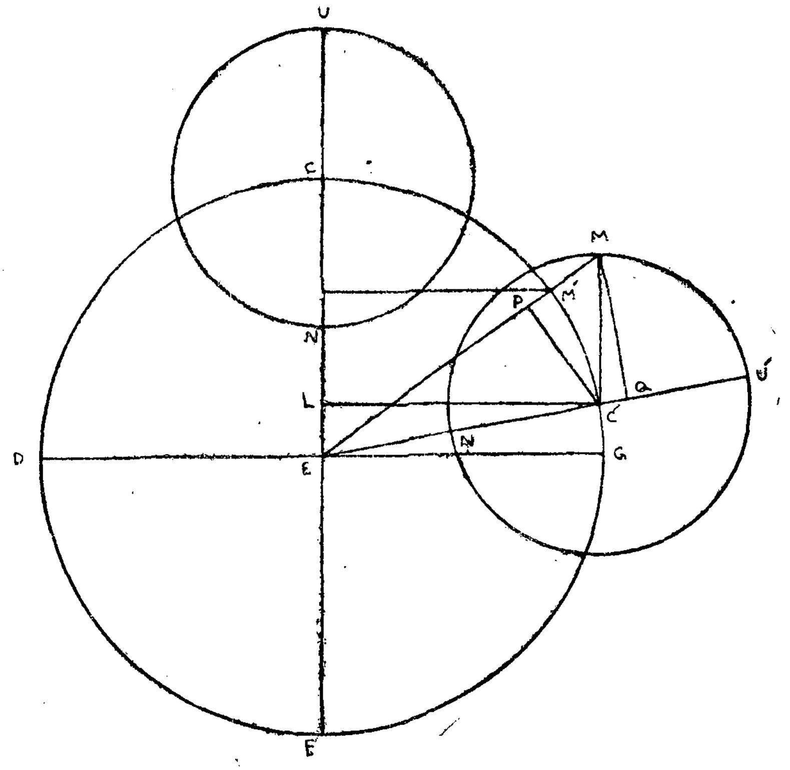

FIG. 34

Let CDFG be the deferent or *kakṣāvṛtta* of the planet, with centre E, centre of earth. With centre C and radius equal to *śīghrāntyaphalajyā* or radius of the *śīghra* epicycle of the planet, describe a circle. This is the *śīghranīcoccavṛtta*. Let CEF cut it at U and N, which are respectively *śīghrocca* or *śīghra* apogee and *śīghranīca* or *śīghra* perigee on the epicycle. Let C move with the velocity of the corrected planet or *mandaspaṣṭagraha* along the deferent and let the planet move from U′ to M along *śīghranīcoccavṛtta* so that arc U′M is equal to arc C′C. Join EM cutting the deferent at M′. Then C′ is the *mandaspaṣṭagraha* and M′ is true planet or *sphuṭagraha*. So the

correction to be given to the longitude of the true mean planet to find its true longitude is the value of arc C′M′. This value is called *Śīghraphala* and the process is called *śīghrakarma*.

Now to find arc C′M′. Join MC′. Draw C′L, C′P and MQ respectively perpendiculars to CE, EM and U′E.

Now angle C′EC, which is the angle between *śīghrocca* and *mandaspaṣṭagraha*, is *śīghrakendra* or *śighra* anomaly.

Therefore C′L is R sine *śīghra* anomaly and EL is R cosine *śīghra* anomaly.

Now, since arc U′M = arc C′C, angle U′C′M = angle C′EC. So, triangles M′CQ and C′EL are similar.

$$\text{So, } MQ = \frac{C'L \times MC'}{C'E}$$

$$= \frac{\text{R sin } \textit{śīghra} \text{ anomaly} \times \text{radius } \textit{śīghra} \text{ epicycle}}{\text{R}}$$

$$= \frac{\text{R sin } \textit{śīghra} \text{ anomaly} \times \text{circumference of } \textit{śīghra} \text{ epicycle}}{360}$$

$$= \frac{\text{R sin } \textit{śīghra} \text{ anomaly} \times \textit{sphuṭaśīghraguṇaka}}{80}$$

= *doḥphala*

Again, from the same triangles,

$$C'Q = \frac{EL \times MC'}{C'E}$$

$$= \frac{\text{R cos } \textit{śīghra} \text{ anomaly} \times \textit{sphuṭaśīghraguṇaka}}{80}$$

= *koṭiphala*.

Now, EQ, *sphuṭakoṭi* = C′E + C′Q

= R + *koṭiphala*.

In the first and fourth quadrants *sphuṭakoṭi* is equal to the sum of R and *koṭiphala*, but in the second and third quadrants it is equal to their difference.

EM, *śīghrakarṇa* = $\sqrt{EQ^2 + MQ^2}$

or *śīghra* hypotenuse = $\sqrt{(C''E + C'Q)^2 + MQ^2}$

Now, from similar triangles EC′P and EMQ,

$$\frac{C'P}{EC'} = \frac{MQ}{EM}$$

$$\text{So, } CP' = \frac{MQ \times EC'}{EM} = \frac{\textit{dohphala} \times R}{\textit{śīghra} \text{ hypotenuse}} \cdot$$

Arc C′M′, *śīghraphala,* is the arc corresponding to C′P as R sine. The *śīghraphala* is added to the corrected longitude, when *śīghrakendra* is in the first and second quadrants and subtracted when it is in the third and fourth quadrants. The *śīghraphala* can also be calculated in the same way by the eccentric method.

As regards their application to the mean longitude of a planet, the first operation should be to apply the amount of the first inequality to the mean longitude, getting thereby, in the case of a superior planet, its heliocentric longitude and in the case of an inferior planet, the centre of its circular orbit ; the second operation should be to apply the amount of the second inequality to the corrected mean longitude, which inequality is the annual parallax in the case of a superior planet and the elongation in the case of an inferior planet.

But Indian astronomers have given various methods of application, perhaps to synchronize calculation with observation. Lalla's rules are as follows :

1. Calculate the mean longitude of the planet at the observer's station and hence *mandaphala.* Apply half of it to the mean longitude. The result is the planet corrected once. Now calculate *śīghraphala* and apply half of it to the once-corrected planet. From this again calculate *mandaphala.* Apply the whole of it to the mean longitude. The result is *mandasphuṭa* planet. Calculate *śīghraphala* from it and apply the whole of it to the *mandasphuṭa* planet. The result is the true longitude of the planet.

2. According to the second method as given in verse 8, first the *śīghraphala* is calculated and half of it is applied and then the *mandaphala* is calculated and half of it applied. The third and fourth operations remain the same.

Here again, the tabulated circumferences of the epicycles, both *manda* and *śīghra*, give only the mean circumferences and as, in the case of the Sun and the Moon, the method of successive approximations should be followed.

While commending an iii. 17, Mallikārjuna says the same thing (see text, pp. 49 ff.).

APPENDIX XV

MEAN AND TRUE DAILY MOTION OF PLANETS

Verses 11-13 of ch. Three of the *Śiṣyadhīvṛddhida* give formulas for *mandagatiphala* and *śīghragatiphala* or two corrections to the mean daily motion of a planet to find its true motion on any day.

For the formula for *mandagatiphala* see above Appendix XIII, pp. 318-19, notes on ii. 15.

Now, the second formula. According to Lalla, R sine *śīghraphala* of a planet on any day

$$= \frac{\textit{doḥphala} \times R}{\textit{karṇa}\ (\textit{śīghra hypotenuse})} \qquad \text{(Appendix XIV, p. 323)}$$

$$= \frac{\text{R sin } \textit{śīghra} \text{ anomaly on that day} \times \text{correct } \textit{śīghraguṇaka} \times \text{R.}}{80 \times \textit{śīghra} \text{ hypotenuse}}$$

Again, R sine *śīghraphala* for the next day

$$= \frac{\text{R sine } \textit{śīghra} \text{ anomaly on the next day} \times \text{correct } \textit{śīghraguṇaka} \times \text{R}}{80 \times \textit{śīghra} \text{ hypotenuse}}$$

So *śīghragatiphala* or *śīghra* correction to motion

$$= \frac{\text{R}}{\text{hypotenuse}} \times \frac{\text{correct } \textit{śīghraguṇaka}}{80} \times \text{difference of the}$$

two R sines, approximately, according to Lalla,

$$= \frac{\text{R}}{\text{hypotenuse}} \times \frac{\text{correct } \textit{śīghraguṇaka}}{80} \times \frac{\textit{bhogyakhaṇḍa}}{225} \times$$

motion of *śīghra* anomaly minutes.

(See diagram 33 of Appendix XIII, verse ii. 15, p. 318.)

Now, motion of *śīghra* anomaly = motion of *śīghroccà* of the planet *minus* motion of the planet corrected by the *mandagatiphala*, found in the first part of the rule (iii. 12 ab).

As regards application, according to the text, the first correction or *mandagatiphala* should be applied positively or negatively, as the case may be, to the mean motion. The result is called *mandasphuṭagati.* Then to this corrected motion, the second correction, *śīghragātiphala,* should be applied positively or negatively, as the case may be. The result is the true motion. The *śīghrakendras* have been defined in verse iii. 14.

While commending on these verses Mallikārjuna has stated that the application of correction to motion should follow the same pattern as that of a planet.

ALTERNATE METHOD FOR TRUE MOTION

In verses iii. 18-19, Lalla gives another method to find the true motion of planets. Here, the first correction follows from the proportion,

mandakarṇa or *manda* hypotenuse : mean motion : : R : ?.

The result is the motion of the planet corrected once. It is called *mandasphuṭagati.* The *mandakarṇa* is found as in verse iii. 17. (pp. 55-56).

From this corrected motion the second correction or *śīghragatiphala* is calculated in the following manner :

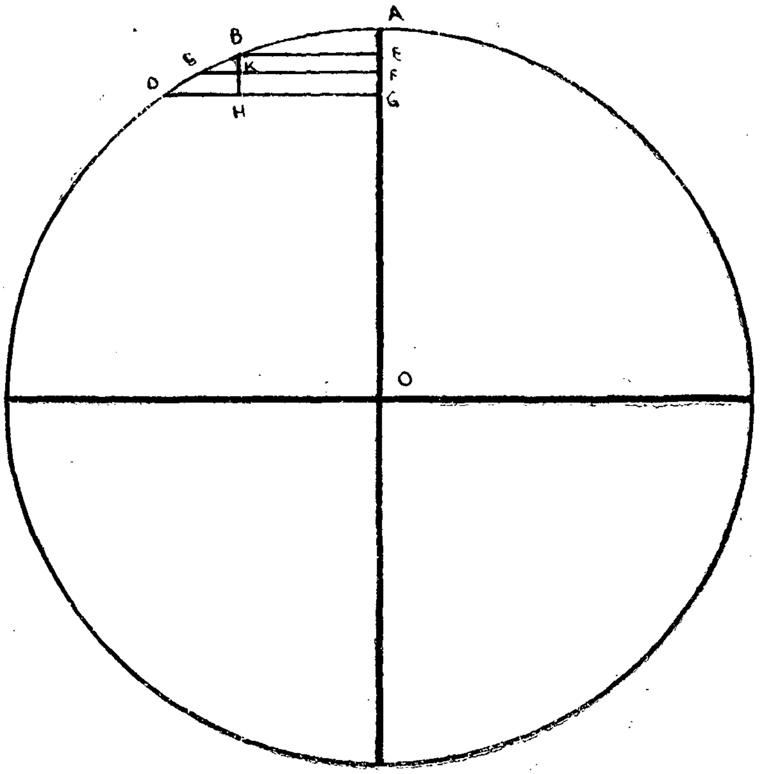

Fig. 35

Let O be the centre of a circle. Mark off arc AB equal to the *śīghraphala* of any planet and arc BC as *śīghrakendragati* or motion of *śīghra* anomaly. Let BD = 225′. Draw BE, CF, DG perpendiculars to AO from B, C and D, respectively. Draw BH perpendicular to DG meeting CF in K. Then DH is *bhogyakhaṇḍa* of *śīghraphala*.

Then, from the proportion, arc BD : DH : : arc BC : CK;

$$CK = \frac{\text{arc } BC \times DH}{\text{arc } BD}$$

$$= \frac{\text{motion of } \textit{śīghra} \text{ anomaly} \times \textit{bhogyakhaṇḍa of śīghraphala}}{225}$$

Now, CK is the difference between R sine *śīghraphala* and R sine of *śīghraphala* increased by *śīghrakendragati*. Then this difference is reduced to the *kakṣāvṛtta* or orbit of the planet by the proportion

śīghrakarṇa or *śīghra* hypotenuse : CK : : R : ?.

$$\text{The reduced result} = \frac{CK \times R}{\textit{śīghra}\text{ hypotenuse}}$$

This is considered by Lalla as approximately the difference between the R sines of *spaṣṭakendras* (*śīghrakendra*—*śīghraphala*) of two consecutive days. The angle MEC is *spaṣṭakendra* in diagram 34, p. 321.

Then, from the proportion,

$$225 : 225' : : \frac{CK \times R}{\textit{śīghra}\text{ hypotenuse}} : ?,$$

spaṣṭakendragati or motion of *spaṣṭakendra*

$$= \frac{CK \times R}{\textit{śīghra}\text{ hypotenuse}} \text{ minutes, nearly };$$

$$= \frac{\text{motion of } \textit{śīghrakendra} \times \textit{bhogyakhaṇḍa} \text{ of } \textit{śīghraphala} \times R}{225 \times \textit{śīghra}\text{ hypotenuse}}$$

(*śīghrakendragati* = motion of *śīghrocca*—*mandasphuṭagati* or motion corrected once as calculated above).

It is evident from the diagram 34 on p. 321, that *śīghrocca* on any day—*spaṣṭakendra* on that day = *sphuṭagraha* or true planet on that day, and *śīghrocca* on the next day—*spaṣṭakendra* on the next day = *sphuṭagraha*, or true planet on the next day. So, subtracting one from the other, *śīghroccagati* or motion of *śīghrocca*—*spaṣṭakendragat* = *sphuṭagrahagati* or true motion of the planet.

While commenting on these verses Mallikārjuna says that the method given here gives a more correct result.

Brahmagupta gives the same method in *BSS*, ii. 43-44 and *Śrīpati* in *ŚSe*, iii. 42-43.

Bhāskara II has criticized Lalla's formulas for correction to motion given in iii. 11-13 (see *ŚSi*, I. ii. 40), as they give only rough results. His formulas are as follows :

First correction or *mandagatiphala*

$$= \frac{\textit{koṭiphala} \times \text{motion of mean anomaly}}{R}$$

and second correction or *śīghragatiphala*

$$= \text{motion of } \textit{śīghra} \text{ anomaly} - \text{motion of } \textit{spaṣṭa} \text{ anomaly}$$

$$= \text{motion of } \textit{śīghra} \text{ anomaly}$$

$$- \frac{\text{motion of } \textit{śīghra} \text{ anomaly} \times R \cos \textit{śīghraphala}}{\text{hypotenuse}}$$

(*ŚSi*, I. ii. 36-39).

But in his commentary on the above verses of Lalla he says nothing. The above rules of Bhāskara are based on instantaneous velocities of planets or *tatkālikagati*. This subject was also dealt with by Muñjāla and Āryabhaṭa II.

In the case of the Moon, which has a great velocity, the above formula for the first correction will give better results. Bhāskara points this out in *ŚSi*, I. ii. 38.

APPENDIX XVI

GLOSSARY OF TECHNICAL TERMS

The following is a list of important astronomical terms in the text and their English equivalents, wherever possible. When there are no English equivalents, the number of the pages where they are explained are given. The words are arranged according to the Devanāgarī alphabet.

Aṃśa	Degree
Akṣajyā	R sine local latitude
Akṣāṃśa	Local latitude in degrees
Agrajyā	R sine amplitude
Agrā	Amplitude
Aṅgula	Unit of measurement of length
Adhidina	Intercalary day
Adhimāsa	Intercalary month
Adhimāsaśeṣa	Residue relating to above
Adhivarṣa	12 intercalary months
Antyaphala	Maximum correction due to mandocca or śīghrocca
Antyaphalajyā	R sine of maximum correction
Antyā	See pp. 75 ff.
Apamaṇḍala	Ecliptic
Amāvasyā	Day of conjunction of Sun and Moon
Avamadina	An omitted lunar day
Avamaśeṣa	Residue relating to the above
Asu	A unit of time
Astalagna	Setting point of ecliptic
Ahargaṇa	Number of civil days between any two dates
Ahorātravṛtta	Diurnal circle

Ākṣadṛkkarma	Visibility correction due to local latitude
Ākṣavalana	Deflection of equator from prime vertical on horizon of eclipsed body
Āyanadṛkkarma	Visibility correction due to ecliptic deviation
Āyanavalana	Deflection of ecliptic from the equator on horizon of eclipsed body
Iṣṭahṛti	See p. 61, 62
Iṣṭāntyā	Antyā at a given time
Ucca	Apex of slow or fast motion
Utkramajyā	R versed sine
Udayajyā	Sin amptitude of rising ecliptic point
Udayalagna	Rising point of ecliptic
Udayāstasūtra	Line of intersection of diurnal circle and horizon
Unnatakāla	Distance from horizon in time
Unnatāṃśa	Complement of zenith distance
Unnatāṃśajyā	R sine of bove
Unmaṇḍala	Six o'clock circle
Kakṣāvṛtta	Mean orbit of planet
Kadamba	Pole of ecliptic
Kadambaprata	Secondary to ecliptic
Karaṇa	Time during which Moon gains 6° over the Sun
Karṇa	Hypotenuse ; distance of planet from earth
Kālāṃśa	Degrees indicating time
Kujyā	Earth-sine
Kudina	Civil day
Kendra	Anomaly
Kendravṛtta	Eccentric circle of planet
Koṭi	Perpendicular side of a right-angled triangle
Koṭijyā	R cosine

Koṭiphala	See p. 322
Koṇamaṇḍala	Intermediary vertical circle
Koṇavṛtta	Do. Do.
Koṇaśaṅku	R sine altitude from intermediary vertical circle
Koṇaśaṅkutala	Base of above
Krānti	Declination
Krāntijyā	R sine declination
Krāntimaṇḍala	Ecliptic
Kṣitija	Horizon
Kṣepa	Additive or subtractive correction
Guṇaka	Multiplier
Ghaṭikā	Unit of time, 24 minutes
Candrakarṇa	Distance of Moon from earth
Candrabhagaṇa	Revolutions of Moon
Cara or *Caradala*	Ascensional difference
Carajyā	R sine asc. difference
Caraphala	Correction due to ascensional difference
Cāndradina	Lunar day
Cāndramāsa	Lunar month
Cāndravarṣa	12 lunar months
Chāyā	Shadow
Chāyākarṇa	Hypotenuse of right-angled triangle, two sides of which are the gnomon and its shadow
Jyā	R sine
Taddhṛti	See p. 61, 62, 65
Tantra	A type of astronomical treatise
Tithi	Lunar day
Trijyā	R sine of an arc of 90°
Dina	One day
Dinagata	Time of day passed
Dinadalāntyā	*Antyā* at midday
Dinaśeṣa	Time till sunset
Dinārdha	Half day

Dṛkkarma	Calculation of visibility correction
Dṛkkṣepa	Arc measured on vertical to ecliptic between zenith and non-agesimal point
Dṛkkṣepamaṇḍala	Secondary to ecliptic passing through zenith
Dṛgjyā	R sine of meridian zenith distance or simply zenith distance
Dṛṅnati	See p. 138
Dṛṅmaṇḍala	Circle through planet and zenith
Deśāntara	Distancee in longitude of any place from prime meridian
Deśāntaraphala	Correction due to difference in longitude
Dorjyā	R sine longitude
Doḥphala	See pp. 51 ff., 322
Dyujyā	Radius of diurnal circle
Dyujyāmaṇḍala	Diurnal circle
Dyujyāvṛtta	Same as above
Dyudalaśaṅku	R sine altitude at midday
Dhruva	Celestial pole
Dhruvaprota	Secondary to celestial equator
Nakṣatra	Asterism
Natakāla	Hour-angle
Natakālakoṭijya	R cosine of Natakāla
Natakālajya	R sine of Natakāla
Natakāletkramajyā	R versed sine of Natakāla
Natāṃśa	Zenith distance
Nati	Latitudinal parallax
Nakṣatradina	Sidereal day
Nāḍī	Unit of time, 24 minutes
Nāḍivalaya	Celestial equator
Nīca	Perigee of epicycle or eccentric of planet
Nīcoccarekhā	Line of apse
Nīcoccavṛtta	Epicycle

Pakṣa	Half of lunar month either from Full or New Moon
Pañcāṅga	Calendar
Paramakrānti	Greatest declination
Parilekha	Diagram ; Projection (of eclipse)
Pala	Unit of time, 24 seconds
Palakarṇa	Hypotenuse of right-angled triangle, two sides of which are gnomon and equinoctial shadow
Palabhā	Equinoctial shadow
Pāta	Moon's node ; time when sum of Sun's and Moon's longitudes is 6 or 12 signs
Pūrṇimā	Full Moon
Pūrvāparasūtra	East-west line
Prativṛtta	Eccentric circle of a planet
Bhagaṇa	Revolution
Bhabhrāma	Revolutions of fixed stars
Bhāga	Degree
Bhuktakāla	Time passed
Bhuja	Base of a triangle ; longitude
Bhujāntara	Correction for equation of time due to eccentricity of ecliptic
Bhogyakāla	Time to pass
Bhogyakhaṇḍa	Tabular difference of R sine, etc. to be passed in the meridian
Madhya	Zenith distance of ecliptic point
Madhyajyā	R sine of above
Madhyamarkrānti	Declination of place of planet on ecliptic
Madhyalagna	Meridian ecliptic point
Madhyalagnaśaṅku	R cosine of above
Madhyāhnacchāyākarṇa	Hypotenuse of right-angled triangle, two sides of which are gnomon and midday shadow
Mandakarṇa	See p. 316

Mandakendra	Mean anomaly
Mandakendrajyā	R sine of above
Mandagatiphala	Correction to mean motion of planet
Mandaguṇaka	See 50 ff., 316
Mandaparidhi	Circumference of manda epicycle
Mandaphala	Equation of centre
Mandaphalajyā	R sine of above
Mandaspaṣṭagati	Mean motion of a planet corrected by mandagatiphala
Mandasphuṭagati	Same as above
Mandāntyaphala	Maximum correction due to mandocca
Mandāntyaphalajyā	R sine of above
Mandocca	Apogee
Mahāyuga	4320000 solar years
Yamyottaramaṇḍala or *Yamyottaravṛtta*	Meridian
Yuga	Part of mahāyuga of 4320000 solar years
Yoga	Period during which Sun and Moon together describe 800′
Yojana	Unit of length used in ancient India ; 4, 5 or 9 English miles
Ravikarṇa	Distance of Sun from earth
Ravibhagaṇa	Revolutions of Sun
Rāśi	Sign of zodiac or 30°
Lagna	Orient ecliptic point
Laghujyā	Calculations of R sines of arcs of 10°
Laghvahargaṇa	Civil days between end of one solar year and any date in current year
Laṅkā	Imaginary place on equator with 0° longitude (and 0° latitude)
Lambajyā	R sine of colatitude

Lambana	Parallax in longitude ; in this text difference of Sun's and Moon's parallax
Lambāṃśa	Colatitude in degrees
Liptā	Minute (unit of angular measurement)
Varṣādhipa	Lord of year
Valana	Deflection
Vikṣepa	Celestial latitude of planet
Vikṣepamaṇḍala	Orbit of planet
Vitribha	Nonagesimal
Vināḍī	Unit of time, 24 seconds
Vipala	Unit of time, 24 seconds
Vimaṇḍala	Orbit of planet
Viliptā	Unit of angular measurement (second)
Vaidhṛta	Time when the sum of the longitudes of Sun and Moon is 12 signs
Vyatīpāta	Time when the sum of the longitudes of Sun and Moon is 6 signs
Śaṅku	Gnomon ; R sine altitude
Śaṅkutala	Base of R sine altitude
Śara	Celestial latitude
Śīghrakarṇa	See page 322
Śīghrakendra	Śīghra anomaly
Śīghrakendrakoṭijyā	R cosine of above
Śīghrakendrajyā	R sine of above
Śīghragatiphala	Second correction to mean motion of planet
Śīghraguṇaka	See pp. 50, 322
Śīghraparidhi	Circumference of Śīghra epicycle
Śīghraphala	Second correction to mean longitude
Śīghraphalajyā	R sine of above
Śīghrāntyaphala	Maximum śīghraphala
Śīghrāntyaphalajyā	R sine of above
Śīghrocca	Śīghra apogee

Śuddhi (p. 196)	Mean heliocentric position ; for superior planets, mean position of Sun.
Saṅkrānti	Last day of solar month
Samaprota	Secondary to prime vertical
Samamaṇḍala	Prime vertical
Samaśaṅku	R sine altitude from prime vertical
Sāvanadina	Civil day
Sāvanamāsa	30 civil days
Sauradina	Solar day
Sauramāsa	Solar month
Sauravarṣa	Solar year
Sphuṭakrānti	Declination of planet's centre
Sphuṭamandaguṇaka	See pp. 215, 375
Sphuṭavikṣepa	Celestial latitude corrected by parallax
Sphuṭaśīghraguṇaka	See pp. 216, 384
Hṛti	Hypotenuse
Horā	$\frac{1}{24}$ of one day and night

APPENDIX XVII

BIBLIOGRAPHY

A. SANSKRIT TEXTS

Āryabhaṭa I, *Āryabhaṭīya*, with the commentary of Paramādīśvara, (*sic*., Parameśvara), ed. by H. Kern, Leiden, 1874 ; with the com. of Nīlakaṇṭha Somasutvan, Pt. I, *Gaṇitapāda* and Pt. II, *Kālakriyāpāda*, ed. by K. Sambasiva Sastri, Trivandrum, 1930-31 ; Pt. III, *Golapāda*, ed. by S. K. Pillai, Trivandrum, 1957 ; ed. by Baladeva Misra, with his own commentary, Bihar Research Society, Patna, 1966 ; translated into English by W. E. Clark, Chicago, 1930.

Āryabhaṭa II, *Mahāsiddhānta*, ed. by S. Dvivedi, with his own com., Varanasi, 1910 ; ed. and tr. into English by S. R. Sarma, two parts, Marburg, 1966.

Jyautiṣa Siddhānta Saṅgraha, ed. by V. P. Dvivedi, Varanasi, 1912, 1917.

Brahmagupta, (1) *Brāhmasphuṭasiddhānta*, ed. by S. Dvivedi, with his own com., Varanasi, 1902 ; ed. with his own com. by R. S. Sharma, I-IV, New Delhi, 1966. (Ref. are to this edn.).

(2) *Khaṇḍakhādyaka*, with the com. of Āmarāja, ed. by Babua Misra, Calcutta, 1925 ; with the com. of Pṛthūdaka, ed. by P. C. Sen Gupta, Calcutta, 1941 ; tr. into English by P. C. Sen Gupta, Calcutta, 1934 ; ed. with the com. of Bhaṭṭotpala and tr. into English by Bina Chatterjee, Calcutta, 1970.

Brahmadeva, *Karaṇaprakāśa*, ed. by S. Dvivedi, with his own com., Varanasi, 1899.

Bhāskara I, (1) *Mahābhāskarīya*, ed. with the com. of Parameśvara, by Balavantaraya Apte, Poona, 1945 ; ed. and tr. into English by K. S. Shukla, Lucknow, 1960. (Ref. are to this edn.).

(2) *Laghubhāskarīya*, with the com. of Śaṅkaranārāyaṇa, University Manuscripts Library, Trivandrum, 1949 ; ed. and tr. into English by K. S. Shukla, Lucknow, 1963. (Ref. are to this edn.).

Bhāskara II, (1) *Līlāvatī*, ed. by S. Dvivedi, Varanasi, 1912 ; ed. with the com. of Gaṇeśa and Mahīdhara, by V. G Apte, Parts I and II, Anandasrama, Poona, 1937.

(2) *Bījagaṇita*, ed. by S. Dvivedi, Varanasi, 1888.

(3) *Siddhāntaśiromaṇi*, *Gaṇitādhyāya* and *Golādhyāya* with Vāsanābhāṣya, ed. by B. D. Sastri and revised by G.D. Sastri, Varanasi, 1929 ; (Ref. are to this edn.) ; *Gaṇitādhyāya* with the commentary *Śiromaṇiprakāśa*, ed. by V. G. Apte, Part I, Poona, 1939. Part II, 1941 ; *Gaṇitādhyāya* with the commentary *Marīci* and his own com., ed. by K.D. Joshi, Part I, Varanasi, 1961 ; Parts II and III, 1964 ; *Golādhyāya*, tr. into English by L. Wilkinson and B. D. Sastri, Calcutta, 1861.

(4) *Karaṇakutūhala*, ed. by S. Dvivedi, with his own com., Varanasi, 1881.

Muñjāla, *Laghumānasa*, ed. by B.D. Apte, Anandasrama, Poona, 1952.

Putumana Somayāji, *Karaṇapaddhati*, ed. by K. Sambasiva Sastri, Trivandrum, 1937.

Rāmadaivajña, *Muhūrtacintāmaṇi*, ed. with the com. Piyūṣadhārā by Govinda, by K. D. Joshi, Varanasi, 1972.

Lalla, *Śiṣyadhīvṛddhida*, ed. by S. Dvivedi, Varanasi, 1886.

Vaṭeśvara, *Vaṭeśvarasiddhānta*, ed. by R.S. Sharma, Vol. I, Delhi, 1962.

Varāhamihira, (1) *Pañcasiddhāntikā*, ed. and tr. into English by G. Thibaut and S. Dvivedi, Varanasi, 1889 (Ref. are to this edn.) ; ed. and tr. into English by O. Neugebauer and D. Pingree, Part I; Copenhagen, 1970; Part II, 1971.

(2) *Bṛhatsaṃhitā*, ed. with the com. of Bhaṭṭotpala, by A.V. Tripathi, Sampurnanand Sanskrit Vishvavidyalaya, Parts I and II, Varanasi, 1968.

(3) *Bṛhajjātaka*, with the com. of Bhaṭṭopala, ed. by S.R. Jha; Varanasi, 1973.

Vidyāmādhava, *Vidyāmādhavīya*, ed. by R. S. Sastry, Mysore, 1923.

Śrīpati, (1) *Siddhāntaśekhara*, ed. by Babua Misra, Part I, Calcutta, 1932 ; Part II, Calcutta, 1947.

(2) *Jyautiṣaratnamālā* with Śrīpati's com. in Marathi, ed. by M.G. Panse, Poona, 1957.

Sūryasiddhānta with the com. of Raṅganātha, ed. by B.P. Misra, Bombay, 1955 ; tr. into English by E. Burgess and ed. by P. Gangooly, Calcutta, 1935 ; with the com. of Parameśvara, ed. by K. S. Shukla, Lucknow, 1957.

B. OTHER SOURCES

Dikshit, S. B., *Bhāratīya Jyotiḥśāstra*, Poona, 1931; tr. into Hindi by S. Jharkhandi, Allahabad, 1957.

Dvivedi, S., *Gaṇakataraṅgiṇī*, (Sanskrit), Varanasi, 1892.

Sastri, K. A. N., *A history of South India*, Oxford, 1966.

Warren, J., *Kālasaṅkalita*, Madras, 1825.

Datta, B., 'Prācīna Bāṅgālī Jyotirvid Mallikārjuna Sūri', *Sahitya Parishad Patrika*, No. 2, Bangabda, 1340 (A.D. 1935).

Shukla, K. S., (1) 'Ācārya Jayadeva, the mathematician', *Gaṇita*, 5. i (June, 1954).

(2) 'Āryabhaṭa I's astronomy with midnight day-reckoning', *Gaṇita*, 18. i (June, 1967).

(3) 'Use of hypotenuse, etc.', *Indian Journal of History of Science*, 8. i-ii (1973).

(4) 'The *Pañcasiddhāntikā* of Varāhamihira' (1), *Gaṇita*, 24. i (June, 1973).